Measurement Theory, Nomological Machine And Measurement Uncertainties (In Classical Physics[1])

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Measurement is said to be the basis of exact sciences as the process of assigning numbers to matter (things or their attributes), thus making it possible to apply the mathematically formulated laws of nature to the empirical world. Mathematics and empiria are best accorded to each other in laboratory experiments which function as what Nancy Cartwright calls nomological machine: an arrangement generating (mathematical) regularities. On the basis of accounts of measurement errors and uncertainties, I will argue for two claims: 1) Both fundamental laws of physics, corresponding to ideal nomological machine, and phenomenological laws, corresponding to material nomological machine, lie, being highly idealised relative to the empirical reality; and also laboratory measurement data do not describe properties inherent to the world independently of human understanding of it. 2) Therefore the naïve, representational view of measurement and experimentation should be replaced with a more pragmatic or practice-based view.

Keywords: laboratory experiment, measurement theory, measurement uncertainties, classical physics, nomological machine

1. Introduction

Physics is a mathematical and empirical science—it treats the world as quantifiable and calculable. The basis of such treatment is said to be measurement, because measurement is the process of assigning numbers to matter,

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[1] I add "classical physics" to emphasise that I am not dealing with quantum physics here, which is usually regarded as the standard case for measurement problems, and that even in classical physics, where measurement is often cosidered to be unproblematic (see, e.g. the citation of Chris Isham in the Introduction), it does involve ambiguities. Considerations of (un)ambiguities, or even the meaning, of measurement in other areas (of physics) cannot be addressed in this paper.

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thus enabling to apply also mathematical tools to it. The importance of measurement for physics is expressed by Piret Kuusk thus:[2]

> Physics BEGINS with measurement, or more precisely—with choosing three standard objects in the world that are assigned the meaning of units by measuring three physical properties: space (length), time and mass. The relation of the corresponding property of the sample to the same property of the object under study yields the numerical ratio or measured value, and these are the numbers from which begins the mathematisation of physics. Physics is so mathematical because it is a science of measurement and measured values.

In order for the mathematical laws of physics to be applicable to matter, the matter must be expressed in a compatible form—numerical data. These are produced in laboratory experiments, where exact measurements can be carried out.

The representational view of measurement regards the assigned numbers and scale of the quantity as characterising the matter itself that is being measured, and science as exploring inherent properties of the world, and measurement as disclosing the real values of those properties. It is advocated both by scientists and by philosophers. Thus the physicist Chris Isham (1995, 57; first emphasis in original) says:

> From the perspective of classical physics, the separation of observer and system has no fundamental significance. Observer and observed are both parts of a single, objectively-existing world in which, ontologically speaking, both have equal status, and are potentially describable by the same physical laws. Similarly, there is nothing special about the concepts of *measurement* or *observable*. The reason why a measurement of an observable quantity yields one value rather than another is simply because the quantity *has* that value at the time the measurement is made. Thus *properties are intrinsically attached to the object as it exists in the world*, and *measurement is nothing more than a particular type of physical interaction designed to display the value of the specific quantity*. Of course, this presupposes that "perfect" measurements exist which have this desirable property of revealing what is actually the case; such an assumption is not totally trivial;

and Nancy Cartwright (2002b, 6) says: "Measuring instruments have this kind of ability to read nature." In my paper I consider some of the theoretical presumptions of measurement as the basis of physics and how the matter studied does not conform to those presumptions. Thus I will argue for a less representational and more pragmatic understanding of measurement.

[2] In personal correspondence (my translation).

In the second section I will briefly introduce measurement theory and its purported relation to the mathematical theory and laws of physics. In the third section the notion of nomological machine will be considered, its role in physics and its relation to measurement theory. I will discern two modes and thence two roles of nomological machines (NM): the ideal NM corresponding to or enacting phenomena or fundamental laws of physics, and material NM corresponding to material experiment, producing data and giving rise to phenomenological laws. The fourth and fifth sections are expositions of my two main claims: 1) Not only fundamental, but also phenomenological laws lie in the sense as Cartwright argues; and moreover, measurement data do not describe the real properties of the world, not even of the laboratory world, as it is independently of human understanding of it. 2) Therefore the naïve representational view of measurement and experimentation should be replaced with a more pragmatic or practice-based view.

2. Measurement theory and its relation with the mathematical theory of (classical) physics

The two underlying notions of measurement theory are "empirical relational system" and "numerical relational system" (Suppes and Zinnes 1962, Hand 2004). Empirical relational system (ERS) is usually said to be a set of objects that are compared to one another as to some chosen property X. I prefer to regard the empirical relational system as different magnitudes of a property that are compared to one another, because things can only make up a system through a systematising idea. In measurement theory the idea is discriminating the intricate material reality into sets of well defined properties, for example length, duration or mass, and these properties having determinate relations concerning their intensity, like identity, more than or less than, or concatenation. A numerical relational system (NRS) is a set of numbers n assigned to an empirical relational system, and is supposed to preserve the structure of it; that is, the system of numbers must preserve the relations holding on the empirical relational system, and the operations that can be performed on it. It makes up an arithmetic or type of scale. Measurement theory deals with two fundamental problems: the problem of assigning numbers or a numerical scale to objects or phenomena on the basis of a chosen property (the problem of representation), and the problem of uniqueness of the assigned scale (the problem of uniqueness). Measurement, on this account, is finding a homomorphism between a system of objects and a numerical system: $n = M(X)$ (M stands for a measure function realising the homomorphism). It is often said that the relation between an ERS and a NRS is isomorphism, but this can only hold if the ERS is regarded as a set of different magnitudes of a property, hence as an abstract system (also Hand

2004, 29).

David Hand (2004) discerns between representational and pragmatic measurement. Representational measurement is about detecting the essential properties and relations of a real world system on the basis of empirical knowledge about it, and formulating an idealised model, a NRS, which preserves its inherent properties. Laws of physics, in this account, are the idealised models of real world properties, or the axiomatic which characterises the scale type of the quantity under study. For pragmatic measurement, the choice of the arithmetic, and accordingly axiomatic, is based mainly on considerations of practical usability for a particular purpose; the chosen attributes and their values do not necessarily represent any real world entities or relations. Hand thinks representational measurement to prevail in physics. How concretely is the representational aspect of measurement related to the mathematicalness of physics, that is—how the NRS or arithmetic of numbers relates to the mathematical laws of physics, is expressed by James Clerk Maxwell (1965, quotes in Boumans 2005, 853) thus: "all the mathematical sciences are founded on relations between physical laws and laws of numbers, so that the aim of exact science is to reduce the problems of nature to the determination of quantities by operations with numbers". This assignment of numbers to an examined material setting (for example, a laboratory setting in experiments), on the one hand gives to the part of material world such a form (numerical "data") which is comparable to the mathematical laws of physics, and on the other hand enables to manipulate this given form using mathematical principles true of that particular arithmetic system. This is, then, how measurement is the basis of physics: an ERS is an idealised model of the objects compared, and laws of nature (formulated in physics) are the axioms characterising the mathematical operations valid on the NRS or arithmetic assigned to it (Hand 2004, 28, 32). Briefly put, according to measurement theory, measurement as the basis of physics presumes that the world has clear discernible properties, which can take on determinate magnitudes detectable through mutual comparison of things which have that property.

3. Nomological machine and measurement in laboratory experiments

The reduction of material things to idealised properties or quantities requires a nomological machine (NM), as also Marcel Boumans (2005) has argued. He says that measurement theory only holds in a laboratory setting which functions like a NM, or measuring instrument itself must function as such. Cartwright (1999, 49) specifies:

> [A nomological machine] is a fixed (enough) arrangement of compo-

> nents, or factors, with stable (enough) capacities that in the right sort of stable (enough) environment will, with repeated operation, give rise to the kind of regular behaviour that we represent in our scientific laws.

I think the notion "nomological machine" can be understood in two ways: as an ideal model, or as a material setting. As an ideal model I take it to correspond to a mathematical law of physics, particularly what Cartwright calls fundamental laws of physics; as such, it is a schema or model which guides the enactment of a law in an experiment, but is also the aim of experimentation. As a material setting it is a laboratory experiment designed according to the guidance of the fundamental law, and is to help establish certain mathematical laws. This distinction roughly sidesteps James Woodward's account of the split between phenomena—"relatively stable and general features of the world" reproducible with various material arrangements, corresponding to ideal NMs—and data—"what registers on a measurement or recording device", susceptible to confounding circumstances and noise, corresponding to outcomes of material NMs. The repeatability or regularity of a NM pertains, respectively, either to phenomena, which must be generatable with various material arrangements (replicability), referring to their alleged ubiquitousness (see e.g. Woodward 1989); or to reliability of material (measuring) devices. The two converge in the requirement that a purported law must be reproducible both on the same and on differing experimental set-ups.[3]

I take the ideal NM to correspond to a mathematical law of physics $X = f(Z_{1...n}, a_{1...m})$, "a necessary regular association between properties antecedently regarded as OK" (Cartwright 1999, 49), or the properties $(X, Z_{1...n})$ which determine the "problems of nature" in scientific context. According to this understanding, each Z_i represents an attribute—a quantity —of the real world system, and the formula shows how they can co-vary. By substituting the variables with concrete numbers, representing concrete magnitudes of those variables, one learns how the unknown X varies when a system determined by those (and only those) properties is manipulated accordingly. In this sense the ideal NM guides how a material NM, or a physical experiment, is to be designed: which conditions $(Z_{1...n})$ must be realised, and what it means that possible confounding circumstances are shielded off (what are the *ceteris paribus* conditions to be met, or rather *ceteris neglectis*—that the remaining circumstances affect the phenomenon examined to a minor or negligible degree, as they cannot in any way be totally excluded, Boumans 2005).

[3] Jaak Kikas in personal communication. See also (Radder 2003).

I take NM as a material set-up to constitute the essence of laboratory experimentation in physics: "we expect the experimental design to be a design for a nomological machine" (Cartwright 1999, 88). Experimentation has a double task with respect to its guiding purported mathematical law:[4] to materially enact the phenomenon described by the law, and to provide empirical data to test the law. The aim is to construct an apparatus or a stand that would approximate the ideal NM as close as possible by realising the interaction of the essential properties making up the phenomenon as free from other circumstances as possible. Experiment is intimately linked to measurement: "Whatever the nature of the experiment, it [the experiment] will be constructed out of measurements…" (Baird 1964, 89). This claim holds in various senses, implicating different NMs:

Firstly and most essentially, measurements of the target quantity X for varying values of independent (defining) quantities ($Z_{1...n}$) provide the empirical part of science—the data for testing the purported mathematical law. Thereby the sample of experimentation is rendered into an ERS (possible magnitudes of X) by constructing the material NM, i.e. a measuring set-up, approximating to the (or eventually an) ideal NM by creating *ceteris neglectis* conditions. "Unless the nature of measurement is clear to the experimenter, the benefit which is sought from carrying out the experiment cannot be fully realized" (Baird 1964, v). This claim must be seen on the background of the mathematisation process mentioned above (in the quotation by Maxwell): experiments are carried out in physics also when the nature of the phenomenon of interest, hence also the nature of measurements (comparisons of capacities or attributes with each other and/or with standards) performed, are not yet clear[5]—otherwise there could be no evolvement of physical theory and its mathematical structure in relation to the material world; the benefit sought from experimentation is realised when the new phenomenon is brought into the theory, under the mathematical and theoretical concepts of physics. As soon as this is reached—as soon as one has a reliable mathematical representation for the phenomenon—experimentation has fulfilled its task and finished.[6] This assessed and qualified mathematics, I surmise, can then be treated as a NM in the ideal sense of the notion. As such it can instruct about stable correlations to underlie further measuring instruments, that is—comparison systems.

[4] Experiments are also carried out without prior clear theoretical guidance, or the theoretical question pursued may substantially change during experimentation (Hacking 1988, Hon 1989, Piret Kuusk and Jaak Kikas in personal communications); see also the next section about this.

[5] Piret Kuusk in personal communication.

[6] Jaak Kikas in personal communication.

Secondly, numbers are usually assigned not immediately to X, but to a directly observable property Y, which is lawfully correlated with X: $Y = F(X)$; as it is also a function of other circumstances of the test situation ($n = M(Y) = M(F(X, OC))$, the measure function M being realised by a measuring instrument; OC stands for 'other circumstances'), the influence of those on its state and on the changing of its state must be small in relation to the influence of the target quantity X (Boumans 2005, 855, 861).[7] Hence the target and observable quantities, X and Y, must relate to each other like a NM, $Y = F(X, OC)$, and so must the observable quantity and the scale realising the assignment of a NRS to the created ERS, $n = M(Y, OC)$, OC being negligible, the NM assigning the same number to a quantity each time the latter appears with the same intensity; this is provided by stabilisation of the phenomenon[8] and calibration of the apparatus.

Thirdly, each of the defining quantities ($Z_{1...n}$) is measured for ensuring its purported value. Hence the apparatus that create and check for the conditions must function as NMs—these are usually provided by previous theoretical-experimental practices on those particular phenomena or quantities.[9]

Fourthly, although elements of experiment are accounted for by some theory or set of theories (depending on the element's complexity), the material realisations do not function exactly as the theory says, but instead of analytic models phenomenological laws or even purely numerical models often have to be exploited in a mathematical description of the experimental set-up where many parameters due to materials have to be taken into account. So sometimes the "other circumstances" are also measured, which is necessary for numerically determining their influence on the phenomenon and hence on the relation of what is considered as the essential features of a phenomenon to "merely contingent factors" resulting of a particular material realisation of it. It is even here possible to apply the notion of NM accounting for the other factors influencing the phenomenon: according to Demetris Portides, in some cases the terms of the disturbing factors are reinserted into the idealised formula of the phenomenon. His account is based on Cartwright's notions of fundamental and phenomenological laws. Each secondary factor (disturbing material effect) would be accountable for by some fundamental law of some physical theory, and combining them would

7 Boumans' own example is about thermometer, where X stands for temperature and Y for the height of the mercury column, OC includes properties of the material that the mercury tube is made of.

8 See (Hacking 1988) about plasticity and stability of scientific laboratory practice.

9 E.g. (Hacking 1988), Piret Kuusk and Jaak Kikas in personal communications.

provide an accurate description of what is going on in the experiment.[10] To wit, in the pendulum example analysed by Portides (2006) the following factors are enumerated:

> (i) finite amplitude, (ii) finite radius of bob, (iii) mass of ring, (iv) mass of cap, (v) mass of cap screw, (vi) mass of wire, (vii) flexibility of wire, (viii) rotation of bob, (ix) double pendulum, (x) buoyancy, (xi) linear damping, (xii) quadratic damping (xiii) decay of finite amplitude, (xiv) added mass, (xv) stretching of wire, (xvi) motion of support.

For such a composing of factors in describing a material situation to be possible, there should be known mathematical descriptions of each phenomenon or factor in the composition, received as Cartwright (2002b, 190–191) conjectures:

> What *is* an ideal situation for studying a particular factor? It is a situation in which all other 'disturbing' factors are missing. And what is special about that? *When all other disturbances are absent, the factor manifests its power explicitly in its behaviour.* When nothing else is going on, you can see what tendencies a factor has by looking at what it does. This tells you something about what will happen in very different, mixed circumstances…

So, allegedly, when separate factors have been abducted, then they can be (re)combined according to the requirements of particular situations. But, as already mentioned, these factors cannot be extracted in pure form: there is no such situation as "all other 'disturbing' factors [being] missing" (Boumans 2005, Portides 2009, D'Agostini 1999, Baird 1964).

The notion of NM is congruent with the representational understanding of measurement: they both presuppose existence and discernibility of clear mutually exclusive properties of real world objects and phenomena in terms of quantities (quantifiable attributes) or capacities correspondingly, that can be represented with a mathematical formula. I see Woodward's concept of phenomenon to comply with Cartwright's understanding of a law of nature as conceived by an ideal NM: phenomena are independent of particular detection device, that is, of contingencies of concrete material set-up, hence their mathematical (numerical) composition must discount (it must be extracted from) noise produced by incidental side-effects in material situations (Woodward 1989, 396–397). The ideal NM embodying the mathematical law of nature would constitute the general conditions for the appearance of a phenomenon and is thus a part of the theoretical explanation of it. A NM

[10] Reinserting factors as described by Portides above may count as shifting some OCs from $n = M(F(X, OC))$ under Z_i-s in $X = f(Z_i)$.

is the precondition for factors to display their capacities in stable phenomena that enable to (quantitatively) compare them without disturbing circumstances and thus detect the arithmetic to represent the capacities or quantitative attributes. Woodward and Cartwright differ in the ontological status that they assign to phenomena as so understood: whereas Woodward takes them to pertain to the world, to be general features of the world and due to their independence from contingent material circumstances thus non-idiosyncratic, Cartwright, in contrast, takes them to be idiosyncratic due to their dependence on highly artificial, constructed conditions of occurrence, the conditions fulfilled by a NM; thence her claim that fundamental laws of physics lie. Phenomenological laws are to describe numerically exactly what is going on in a material setting. Cartwright regards these laws as ontologically superior relative to fundamental laws describing phenomena, because they pertain to the actual material world. Woodward, in contrast, regards those as idiosyncratic, prone to errors and contingent factors in measurement results, hence as ontologically inferior. To get phenomena out of data, one must discount the errors as noise; phenomena must be extracted by tuning to detect signal "in the sea of noise", that includes experimental design, control for errors and bias, measurement techniques and data analysis (Woodward 1989, 396–397). In the next section I will consider those errors relating to measurement activity in laboratory, and their import on data, and hence also on the NM that the experiment providing those data is to be.

4. Measurement error and uncertainty in experiment: how laws and data lie

I agree with Cartwright that the simple fundamental laws of physics do not pertain to the real material world. They describe something stable and simple, but the material world is intricate and changing. I also agree with Woodward that data produced by material experiments are idiosyncratic. Due to their idiosyncrasies, also phenomenological laws do not pertain to the material reality. Their theoretical mathematical components are as idealised as fundamental laws, and their empirical numerical components depend on particular measurement situations and presuppose the representational view of measurement. If a concrete measurement act would embody a material NM, it could be regarded as revealing the real properties of that concrete material situation. However, numerous sources of uncertainties in real measurement render their results doubly fuzzy: as to their numerical magnitude, and as to the property to which it is assigned. The outcome of a single measurement event can be influenced by various factors which are considered as disturbing noise, "other circumstances", error, that confound detection of the value of the measurand, thus rendering measurement process uncertain. Uncertainties are said to be either of systematic nature (systematic error), or statistical nature (random error). Random error (listed 10 hereinafter) is reflected in the reading only if a large number of small perturbations are present; systematic error is a "perturbation which influences all measurements of a particular quantity equally" (Baird 1964, 10). However, a type of error can travel from one category to the other (Baird 1964, 10), and as D'Agostini (1999, 8) states, there is no clear distinction between systematic and statistical errors: all those considered as systematic effects on measurement contribute to statistical effects; but moreover, this distinction is theory- or model-laden (Tal 2011b), and is based on purely numerical grounds of discrimination (or mathematical, in contrast to the source of error as a discrimination criterion Hon 1989, 477).

Let us consider some of the various sources of errors and uncertainties in measurement.[11]

(a) Instrument calibration uncertainty, which can only be removed through comparison with a standard instrument;[12] this should be checked for before every experiment (this, however, is usually not done in physical laboratory experiments);[13] and 7) "inexact values of measurement standards and reference materials". As measurement (in the case of classical physics) is comparison of the target object with the standard object (the unit, realised in a form of an object like standard kilogram or metre, or process like standard second and metre), variability of the unit implies variability of the magnitudes of the same target measurand at different times, given that everything else remains the same.[14]

3) "Non-representative sampling—the sample measured may not rep-

[11] The lettered sources in this list are from Baird (1964, 11–14), numbered ones from D'Agostini (1999, 7–8, quoting the ISO Guide); here I consider only few of the error and uncertainty sources indicated by Baird and D'Agostini. The others are: (b) Instrument reproducibility: the value of calibration under certain circumstances can be removed by mechanical defects in slightly different circumstances; (c) Observer skill; (e) Fineness of scale division; 1) "Incomplete definition of the measurand"; 2) "Imperfect realisation of the definition of the measurand"; 4) "Inadequate knowledge of the effects of the environmental conditions on the measurement, or imperfect measurement of environmental conditions"; 5) "personal bias in reading analogue instruments"; 8) "Inexact values of constants and other parameters obtained from external sources and used in the data-reduction algorithm"; 9) "Approximations and assumptions incorporated in the measurement method and procedure".

[12] Here, of course, the experimenter depends on the reliability of the standard instrument, which is also just a material being, as given by D'Agostini as error source 7.

[13] Jaak Kikas in personal communication.

[14] (Tal 2011b,a) about the variability of standard second, (Riordan 2011) about the etalon of the kilogram.

resent the defined measurand"; this is based on the assumption that there is something in the world (a possible material sample) that exactly corresponds to the definition of the measurand, $X = f(Z_i)$. That could only be the standard object, mentioned in the introduction, which itself defines the measurand. However, it has itself more properties ("OC") than just the quantity X defined on its basis and is thus subject to material uncertainties as referred to by (7) above.

6) "Finite instrument resolution or discrimination threshold", or "intrinsic instrument inertia", as an engineer would call it,[15] that all measuring instruments are said to have: by this it is meant that the instrument is unable to register all minute changes; instead, it integrates the measurement inputs over small differences—that also Cartwright concedes:

> [W]e do not actually observe the real value of a quantity at a time. Instead what we see is a long-time average over these values—long compared to relaxation times in the objects themselves. By a coarse-grained averaging, we can construct new quantities that are all compatible. We then claim that these new quantities, and not the originals, are the macroscopic observables that concern us. (Cartwright 2002a, 166)[16]

Hence the number received as an outcome of a measurement act does not pertain to one concrete magnitude of the measured attribute, but rather to a set or an interval of (possible) magnitudes.

(d) Miscellaneous error: "In an experiment involving more than one or two variables or factors that influence the final measurement, there are bound to be perturbations which influence the final reading." Baird (1964, 94) expresses the same yet thus: very often the "independent" variables that are controlled (manipulated) in an experiment are in fact dependent on each other; due to this *ceteris paribus* conditions cannot be met. But something yet stronger can be claimed: that always, even in the seemingly simplest, paradigmatic cases, there are plenty of physical factors that are not included in the mathematical model but do interfere with the experiment situation, affecting measurement results, and that cannot in any way be removed because they are inherent in the material realisation of the phenomenon studied (e.g. Portides, example above). Hence the number does not pertain to a concrete single distinct property either, but rather to a set or "system" of properties. Thus the reference of the number or reading is doubly fuzzy: as

[15] Tõnis Mets in personal communication.

[16] Here Cartwright seems to diverge from her statement about measuring instruments as reading nature: measuring instruments appear to have no epistemic superiority in recognising neither isolate properties of phenomena, nor their values.

to the magnitude, or measured value (due to errors sources a, 7 and 6), and as to the property that it is to be the magnitude of (error source d).

10) "Variations in repeated observations of the measurand under apparently identical conditions", also pointed out by Baird: that by repeating the same measurement different outcomes or readings are received. This necessitates statistical analysis of data and, as I see it, compels both D'Agostini and Baird to urge that the true value of the measurand cannot be known. The statistical analysis treats the set of readings as a statistical collection, of which different indicators are calculated, like mean, standard deviation etc. Thus it provides an idealised data model, which in nicest cases shows a Gaussian curve, which helps to provide an estimate of the true value of the measurand (mean), precision of measurements on the particular instrument in the particular conditions (standard deviation), so that further measurements in other conditions (other values of the measured quantity) can be estimated for precision and accuracy (Baird 1964, Hon 1989).

4.1 Ideality of measurement concepts

Let us notice in which ways the treatment and formulations of measurement errors and uncertainties as given above are idealised:

Firstly, they presuppose that there are clear distinct properties—perhaps even quantities—in the world. Cartwright expresses this presumption explicitly. This presumption is further manifested by the mathematical notion 'variable' applied on a material set-up, and the notion 'discrimination threshold', which presupposes pre-existent values and properties to be discriminated, or read out of nature. The theoretical notion 'inertia' that is said to characterise each measuring instrument, however fine, seems to hint to the background belief that there is an ideal toward which the fineness of instrument procedure can strive—namely perfectly displaying the true value of the measured quantity. Similarly in Portides' account of composition of factors: 'linear' and 'quadratic' are characteristics of mathematical systems, but here used to qualify empirical processes ("damping") as if the material system itself counted various kinds of damping in its behaviour, and as if those were clearly separable from each other. That is, there are presumed to be ERSs, or systems of measurable properties in the world. However, terms making up a mathematical formula do not all correspond to distinguishable or defined properties of the real world system[17] (ERSs), not even in a laboratory experiment which should serve to extract them from the rest of the material world. So also Cartwright (1999, 152) admits:

[17] Thanks to prof. Rafaela Hillerbrand on a discussion on this issue and on Portides' approach given above.

> Contrary to strict empiricist demands, the concepts of physics generally do not pick out independently identifiable or measurable properties. Its concepts, rather, build in—and thus cover up—the whole apparatus necessary to get a nomological machine to work properly: the arrangement of parts, the shielding and whatever it takes to set the machine running.

But this statement makes it unclear, in what sense do NMs enact and test fundamental laws about phenomena, and that a "factor manifests its power explicitly in its behaviour." In addition, it cannot count for the dispersion of measurement outcomes received by repeating a test in apparently same conditions, as it would still provide just one possible numerical value for given initial and boundary conditions.

Secondly, attributes are *defined* as quantities in the theories tested, where they have certain functions and aims, rather than mapped from the matter, as its properties appear, into numbers: they are connected in mathematical formulae that gain their meanings due to the assumption of numerically-valuedness of their terms; this numericalness due to there being something defined as a unit. Similarly, the nature of "noise" is defined (Rothbart 2007, 85, referring to Skoog and Leary 1992; italics added): "A noisy signal, *by definition*, has a contaminating effect on the accuracy and precision of an analytic signal by negatively influencing the evolution of the signal within the instrument." To be sure, those mathematical definitions are accorded with practical handling of the matter with respect to those attributes, but the theoretical background accompanies also this handling and interpretation of its results.[18] That is, NRSs are ideal theoretical definitions guiding measurement and defining data and noise.

Thirdly, the outcome of one measurement is just one number, not a set of numbers each pertaining to exactly one perturbing or perturbed "variable" or factor involved; the one number does not by itself split up into "the true value" of the measurand and "confounding circumstances", "error" or "noise". The splitting is done on the basis of theoretical preconceptions about the measured quantity, the measuring arrangement and its functioning,[19] and of the statistically received idealised data model. Hence not only 'quantity' and 'value', but also 'error' and 'noise' are theoretically laden idealised terms.

Fourthly, as for a Gaussian normal distribution of measurement results an infinite set of measurements would be needed, which is trivially impossible, the (back)inference from the mathematical model of the instrument reliability (the Gaussian) to further, yet unrealised material measurement situations amounts to in good faith extrapolating the idealised model of in-

[18] See also (Radder 2003, Baird 1964, 100, 136).
[19] See also (Tal 2011b).

strument performance. Baird (1964, 30) is optimistic: a population of a particular reading "refers to the infinite set of readings which could be made with the apparatus, and thus provides a link between actual observations and the statistical theory." So he is treating an apparatus as if it could work like an ideal NM—always identical with itself, not susceptible to material decay and contingent confounding circumstances—which comes to a contradiction, because it is exactly this susceptibility to confounding circumstances that the statistical model is to measure. Or he treats the "idiosyncrasies" of an apparatus as a NM—in a way similar to Portides' account of the composition of secondary factors.

And fifthly, taking these uncertainties into account, what exactly does replicability of results mean? How is it decided, given the scattering of individual measurement results, that a result has been replicated? Also Baird (1964, 135–136) poses the question: "What [...] is the justification for accepting any postulated ideal variation to describe a set of observations?", and also answers it: "the cause of science is best advanced by interpreting the results [of measurement] in terms of [the theory tested]." Jaak Kikas acknowledges a similar approach:[20] leaning on the principle of Occam's razor, the mathematical formula is retained which conforms to the theory, is more convenient to handle and implement in practice. Baird does warn about too easily rejecting data that seem not to conform to the Gaussian: as Gaussian curve approximates the axis infinitely, there is some probability of legitimate data lying outside the standard deviations. So the idealised Gaussian—a particular mathematical-statistical tool—is taken to legitimately represent empirical input for a physical theory; moreover, pure mathematics (the Gaussian), which is an even more highly idealised theory than a physical theory, is deemed to be a legitimate advisor on material possibilities:[21] here particularly on the possible material variation of "one and the same" process or interaction.

5. How measurement is pragmatic

Taking into account the described fuzziness of the notions underlying the assumptions of measurement theory, what is the reference system, the ERS, that the measurement assigns a NRS to? It may be the operations executed (Suppes and Zinnes 1962, 4); or the averaged numerical outcome of measurements, which is to instantiate one element of the constructed or enacted ERS (that is, one particular magnitude of the target attribute X). I take it

[20] In personal communication.

[21] Here the NRS, whose axiomatic is statistics, is assigned to an ERS of a laboratory artefact—a model of a measuring device.

to be the operation of the apparatus as a whole that has here received an idealised notation $n = M(F(X, OC))$: more narrowly, the instrument in a sense measures the entirety of interactions taking place in it, the intended inputs, the unintended "noise", through their mutual influences to the n we read on the dial; more broadly, an ERS refers to the whole history and practice standing behind the particular measurement and giving sense and meaning to it. Measurement as comparison of objects is guided by practical needs and possibilities—how the objects are perceived and used, how they are conceptualised through this perception and use, what roles they play in handling and understanding the world around us. As Daniel Rothbart (2007, 59) mentions, it was at first in practical matters such as trade, navigation and the like, not theories and science, that instruments realising assignments of numbers—"mathematical instruments"—came to be applied. But things and processes were compared and counted long before, thereby brought to understanding by units defined on the basis of immediate and activity-related cognition, like anatomical units (digit, hand, foot, ...) or procedural units (certain travelling distances, sowing areas, ...) (Hand 2004, Ch. 7 on the (pre-)history of units of physics). The great variety of units and their denoted magnitudes (e.g. different lengths of 'foot') accords varying local or temporal customs and circumstances (e.g. 'foot' measures not only length, but also the stature of the folk: health, bodily fitness due to social status or fertility of arable land, thereby conditioning people's anatomical idiosyncrasies like height and length of feet). What is measured (the ERS) depends on the "axiomatic" of the measurement procedure and arrangement, as different procedures and arrangements comprise different idiosyncrasies that will be reflected in the outcome.[22] So for example a distance when measured in travelling time will mean something else than when measured in feet; similarly the scale (NRS) of an attribute meaningful for one thing may be meaningless for another (Hand 2004, 11): the weight of apples is not the same as the weight of grain; the area of a poor agricultural land is not the same as the area of a rich agricultural land. Here the notion of NM as a mechanism requiring *ceteris neglectis* conditions, and their dependence on practical aims becomes apparent: 'a travelling distance' presumes certain culture and ways of travelling, certain fitness of the traveller, a certain kind of landscape, a certain load to be carried, favourable weather, absence of threats to the progression etc.—conditions presumably (implicitly) provided for by the particular social and natural conditions; measuring an area in furrows or scheffels only makes sense if the measurand is an agricultural land, and perhaps has a known fertility.

[22] See e.g. (Tal 2011b) for an overview of the operationalist understanding of measurement; he rejects it as not accordable with scientific practice.

Science is one kind of practice, with mathematics as its ideal unique kernel (Vihalemm 1979, 1989). Evolution of this kernel is intertwined with the practices of instrument making, like optics for sciences and clock making, and the introduction of measuring components into scientific (optical) instruments, thus rendering phenomena mathematical and introducing precision into the intricate world (Rothbart 2007, 59–63, Gooding 2003, 274). But its beginning—measurement—has evolved during thousands of years before these scientific instruments came into being, as hinted in the previous section and reported by Hand (2004, Ch. 7). Gooding (2003, 279) says: units and quantification bring unity, regularity and unambiguous meaning into nature—or rather, into our conception of nature, which guides us in handling nature, whereby this unity and meaning is built into it, nature is (re)constructed to bear meaningful and useful (for us) regularities,[23] for example in forms of experiments or various kinds of technology. So there can be—it has a sense that there are—unified systems of measurement and units only if there is technology to support such a system, if the units can be multiply realised (by reproductions of the standard object or phenomenon, and/or by different "material" definitions of the unit)[24] and measurement procedures with them carried out in stable ways. The assumption of isomorphism or homomorphism between empirical and numerical operations is to ensure stability and certitude, as arithmetic or mathematics do not depend on matter and its idiosyncrasies, but are assumed stable and apodictic (Vihalemm 1979). In order to legitimately claim this stability and apodicity, I surmise, physics must also introduce unified measurement system over and above the multifarious local measuring systems. That is, it must treat the world as if it was uniform (the assumption of uniform space and time in Newtonian physics is to this end) and as if it made sense to measure everything according to the same standards, that is—compare all things with the same etalons. There are at least two kinds of abstraction in the evolving practices of measurement: to numbers (e.g. that three apples, three seconds and three acres of land are all instances of the same number: *three*) and to attributes (e.g. that a nanometre, a yard or four yards, "seven lands and seas" and a light year are all instances of the same attribute: *length*). These are part of abstraction from concrete things or processes to phenomena (as Woodward understands them).[25] Similarly to pre-scientific conceptualisation and measurement practices, different material settings may thereby give rise to

[23] E.g. (Vihalemm 1979) about interdependence of theory (also pre-theoretical world view) and practice.

[24] See (Tal 2011a) for an example of multiple realisability; the case of standard second.

[25] See also (Heidelberger 2003, 148–149) about the generalisation of electricity as a phenomenon.

slightly different units.[26] I would also contend the reverse: that different settings meant to measure the same attribute still literally refer to different properties with their outcomes—in the sense indicated in the previous section as sets of properties comprising the target measurand and all the "confounding circumstances"—either due to different axiomatics (in terms of measurement theory) underlying the instruments (a ruler directly measuring length, or an acoustic oscillation indirectly measuring length),[27] or due to material idiosyncrasies (two rulers of different or even of the same material are not quite the same). But even if different measuring instruments aimed at one and the same attribute operate quite differently and thus literally measure different properties, due to the unifying scientific approach and stabilising practice they can, and for the practical and pragmatic aims of science they must, be treated as measuring the same property (Tal 2011b).

I put forward that treating the world as ERSs, or as NMs, is in two ways pragmatic. Firstly, this justifies the use of handy mathematical theories in accounting for and manipulating with the world, hence assuming apodictic calculability and predictability of the world, an "underlying ideal reality" (Hand 2004, 36). Also Woodward's preference to link reality to stability—as he takes simple phenomena to be ontically primary to noisy data—is pragmatic in the sense that it assumes a simple world and thence a simple treatment of the world. Instead of arithmetics of things (Suppes and Zinnes 1962, 3)—of all the varied kinds of things that would provoke various individual accountings—there are a few arithmetics of numbers that can account for sundry things irrespective of their idiosyncrasies.

Secondly, this approach justifies bringing ever more of the material world under mathematical treatment. Matter is thought of as fuzzy because of involving numerous confounding circumstances. The quantification of uncertainties and noise, conceptual and mathematical modelling of measurement errors, serve the assumption that the world really consists of simple phenomena and can be treated as potentially comprehensible with well defined models. Thus by mathematising the treatment of what is called noise and uncertainties, ever fuzzier matter is brought under mathematical treatment on theoretical and practical levels. To this end, D'Agostini (1999, 7, emphasis added) presents the definition in ISO Guide of "*a* true value of the measurand" guiding determination of error: "*a* value compatible with the definition of a given particular quantity." According to this definition, there is no absolute basis of comparison for discerning genuine results from

[26] See, e.g. (Hand 2004, 236–238) for units related to electricity.

[27] This undermines, besides the notions of representational and pragmatic measurement, also the distinctions between direct and indirect measurement, fundamental and derived measurement, and intrinsic and extrinsic attributes given by (Hand 2004, 16, 28).

noise. D'Agostini considers this definition more useful in actual measuring and test situations than the idealised one of "*the* true value" for permitting more of the measured matter to be interpreted in terms of the theory applied to it—also in cases where measurements are difficult or impossible to reproduce—thus enhancing plasticity of experimental practices and stability of scientific theories.

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