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# On $\alpha$-order $\lambda$-statistical convergence and some inclusion theorems in non-Archimedean 2-normed spaces

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Abstract. This paper introduces the notion of $\alpha$-order $\lambda$-statistical convergence over non-Archimedean 2-normed spaces, with $\mathbb{K}$ representing a non-trivially valued, complete non-Archimedean field. Further, properties like linearity, uniqueness of limits, and certain properties of $\alpha$-order $\lambda$-statistical convergence sequences, $\alpha$-order $\lambda$-statistical Cauchy sequences are established in non-classical analysis. Some inclusion theorems are proved, and the concepts of $\alpha$-order $\lambda$-statistical limit superior and $\alpha$-order $\lambda$-statistical limit inferior for sequences in non-Archimedean 2-normed spaces are introduced, along with a discussion of related results.

## 1. Introduction

Statistical convergence in the classical analysis was initially put forth independently by Fast [6], and Steinhaus [15]. In 1959, Schoenberg [14] presented some properties of statistical convergence and summability methods. Many authors, such as Connor [5], Fridy [7, 8], along with Orhan [10, 9], Rath and Tripathy [13], Móricz [11], and Çakallı [3], made notable contributions to the growth of the realm of statistical convergence.

According to [6], a sequence $x = \{x_u\}$ is said to be *statistically convergent* to the limit $\mathfrak{l}$, if for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n} \left|\{u \leq n; n \in \mathbb{N} : |x_u - \mathfrak{l}| \geq \varepsilon\}\right| = 0,$$

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Corresponding author: Suja Krishnamurthy where the bars represent the total number of elements in the set contained within it. This is written as

$$\operatorname{stat}\text{-}\lim x_u = \mathfrak{l}.$$

Çolak [4] analyzed the $\alpha$-order statistical convergence for number sequences. Let $0 < \alpha \leq 1$. A sequence $x = \{x_u\}$ is said to be *$\alpha$-order statistically convergent* to the limit $\mathfrak{l}$ if, for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n^\alpha} \left|\{u \leq n; n \in \mathbb{N} : |x_u - \mathfrak{l}| \geq \varepsilon\}\right| = 0.$$

Mursaleen [12] introduced the notion of $\lambda$-statistical convergence. Let $\lambda = \{\lambda_n\}$ be an increasing sequence of positive numbers tending to infinity, where $\lambda_{n+1} \leq \lambda_n + 1$ and $\lambda_1 = 1$, $n \in \mathbb{N}$. Let $I_n$ denote the interval $[n-\lambda_n+1, n]$. The sequence $x = \{x_u\}$ is said to be *$\lambda$-statistically convergent* to the limit $\mathfrak{l}$, if for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \left|\{u \in I_n : |x_u - \mathfrak{l}| \geq \varepsilon\}\right| = 0.$$

We write

$$\operatorname{stat}_\lambda\text{-}\lim x_u = \mathfrak{l}.$$

Çolak [4] extended this further and defined $\alpha$-order $\lambda$-statistical convergence. The sequence $x = \{x_u\}$ is called *$\alpha$-order $\lambda$-statistically convergent* to the limit $\mathfrak{l}$ if, for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : |x_u - \mathfrak{l}| \geq \varepsilon\}\right| = 0.$$

We write

$$\operatorname{stat}_\lambda^\alpha\text{-}\lim x_u = \mathfrak{l}.$$

Non-Archimedean valued space, or simply a non-Archimedean space, refers to the mathematical structure where the order of the elements does not follow the Archimedean property. Archimedean property states that, for any positive real numbers $a$ and $b$, there exists a positive integer $n$ such that $n \cdot a > b$, which is true for real numbers.

In classical analysis, concepts such as limits, continuity, and differentiability are generally examined within the fields of real numbers $\mathbb{R}$ or complex numbers $\mathbb{C}$, which naturally satisfy the Archimedean property. But in the case of advanced mathematical research, particularly in number theory and algebra, it becomes necessary to extend this framework. This extension naturally leads to the study of $p$-adic numbers, which form a non-Archimedean field and expand the scope of analysis to include elements not present within $\mathbb{R}$ or $\mathbb{C}$. The field of $p$-adic numbers $\mathbb{Q}_p$ is the most popular non-Archimedean field, obtained by the completion of the field $\mathbb{Q}$ using the $p$-adic valuation [2]. Non-Archimedean analysis, including its functional analysis variant, considers function spaces, operators, and other structures within these non-Archimedean fields.

In a non-Archimedean space, the metric satisfies the stronger triangle inequality $d(x, y) \leq \max\{d(x, z), d(z, y)\}$, also known as the ultrametric inequality, meaning that the distance between any two elements is never greater than the maximum of the distances from a third element to the other two. This results in unusual geometric properties, such as all triangles being isosceles and every point on a circle being a centre. These contrast with Archimedean spaces, where the triangular inequality is less restrictive. For example, in the $p$-adic analysis, the distance between two numbers is defined in terms of their $p$-adic valuation rather than their absolute difference. This leads to counterintuitive properties such as sequences converging to zero without their terms becoming arbitrarily small in the usual sense. These developments provide powerful tools for tackling problems in number theory, algebraic geometry, and theoretical physics that lie beyond the reach of classical analysis.

Suja and Srinivasan [16] put forth the concept of statistically convergent sequence and statistical Cauchy sequence in non-Archimedean fields. As in [1], let $\mathbb{K}$ represent a non-trivially valued, complete non-Archimedean field and $\mathcal{F}$ be a vector space of $\dim(\mathcal{F}) \geq 2$. The mapping $\|\cdot, \cdot\| : \mathcal{F} \times \mathcal{F} \to \mathbb{R}$ satisfies the following properties for all $s, t, q \in \mathcal{F}$:

- $\|s, t\| = 0$ if and only if $s$ and $t$ are not linearly independent;
- $\|s, t\| = \|t, s\|$;
- $\|c\ s, t\| = |c|\|s, t\|$ for $c \in \mathbb{K}$;
- $\|s + q, t\| \leq max\,\{\|s, t\|, \|q, t\|\}$.

$(\mathcal{F}, \|\cdot, \cdot\|)$ is called an *ultrametric* 2-*normed space*, also known as *non-Archimedean* 2-*normed space.*

The results herein concentrate on $\alpha$-order $\lambda$-statistical convergence within ultrametric 2-normed spaces.

## 2. Main results

**Definition 1.** A sequence $x = \{x_u\}$ is said to have $\alpha$-order $\lambda$-statistical convergence, where $0 < \alpha \leq 1$, to the limit $\mathfrak{l}$ in an ultrametric 2-normed space $(\mathcal{F}, \|\cdot, \cdot\|)$ if, for all $\varepsilon > 0$, there exists non-zero $z \in \mathcal{F}$ satisfying

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right| = 0.$$

This is written as

$$\mathrm{stat}_\lambda^\alpha\text{-}\lim \|x_u - \mathfrak{l}, z\| = 0 \quad \text{or} \quad \mathrm{stat}_\lambda^\alpha\text{-}\lim x_u = \mathfrak{l},$$

where $\lambda = \{\lambda_n\}$ is an increasing sequence of non-negative numbers approaching infinity and $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$ and $I_n$ denotes the interval $[n - \lambda_n + 1, n]$, where $n \in \mathbb{N}$.

*Remark* 1. Every convergent sequence is an $\alpha$-order $\lambda$-statistically convergent sequence in an ultrametric 2-normed space.

**Example 1.** Let $(\mathcal{F}, \|.,.\|)$ be an ultrametric 2-normed space over the field $\mathbb{K}$ with $\dim \mathcal{F} \geq 2$ and $z \in \mathcal{F}\backslash\{0\}$. Let $w$ denote the sequence $\{w_u\}$ in $\mathcal{F}$ given by

$$w_u = \begin{cases} y, & \text{if } n - \lfloor\sqrt{\frac{\lambda_n}{2}}\rfloor + 1 \leq u \leq n, \\ 0, & \text{otherwise,} \end{cases}$$

where $y \in \mathcal{F}\backslash\{0\}$. Thus we have

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|w_u - 0, z\| \geq \varepsilon\}| = 0 \quad \text{for} \quad \frac{1}{2} < \alpha \leq 1.$$

Hence the sequence is $\alpha$-order $\lambda$-statistically convergent to the limit 0 in $\mathcal{F}$.

**Example 2.** Let $(\mathcal{F}, \|.,.\|)$ be an ultrametric 2-normed space over the field $\mathbb{K}$ with $\dim \mathcal{F} \geq 2$ and $z \in \mathcal{F}\backslash\{0\}$. Let $x$ denote the sequence $\{x_u\}$ in $\mathcal{F}$ given by

$$x_u = \begin{cases} y, & \text{if } n - \lfloor\lambda_n^\beta\rfloor + 1 \leq u \leq n, \\ 0, & \text{otherwise,} \end{cases}$$

where $y \in \mathcal{F}\backslash\{0\}$, $0 < \beta < 1$ and $\beta < \alpha$. Thus we have

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u - 0, z\| \geq \varepsilon\}| = 0 \quad \text{for} \quad \beta < \alpha \leq 1.$$

Hence the sequence is $\alpha$-order $\lambda$-statistically convergent to the limit 0 in $\mathcal{F}$.

**Example 3.** Let $(\mathcal{F}, \|.,.\|)$ be an ultrametric 2-normed space over the field $\mathbb{K}$ with $\dim \mathcal{F} \geq 2$ and $z \in \mathcal{F}\backslash\{0\}$. Let $\lambda$ denote the sequence $\{\lambda_n\}$ given by

$$\lambda_n = \begin{cases} 1, & \text{if } n = 1, \\ n/2, & \text{if } n \geq 2, \end{cases}$$

and $x = \{x_u\}$ be the sequence in $\mathcal{F}$ given by

$$x_u = \begin{cases} \mathcal{M}, & \text{for } u = q^3 \text{ and } q \in \mathbb{N}, \\ 0, & \text{otherwise,} \end{cases}$$

where $\mathcal{M} \in \mathcal{F}\backslash\{0\}$ is fixed. Then, for every $\epsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u - 0, z\| \geq \varepsilon\}| = 0 \quad \text{for} \quad \frac{1}{3} < \alpha \leq 1.$$

Therefore, the sequence exhibits $\alpha$-order $\lambda$-statistical convergence with a limit being 0 in $\mathcal{F}$.

**Definition 2.** In an ultrametric 2-normed space $(\mathcal{F}, \|\cdot,\cdot\|)$, the sequence $x = \{x_u\}$ is called an *$\alpha$-order $\lambda$-statistical Cauchy sequence* if, for every $\varepsilon > 0$, and $z \in \mathcal{F}\backslash\{0\}$ there exists $p \in \mathbb{N}$ such that

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u - x_p, z\| \geq \varepsilon\}| = 0.$$

**Definition 3.** Let $\mathbb{B}_x$ stand for the set

$$\mathbb{B}_x := \left\{ b \in \mathbb{K} : \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| > b\}\right| \neq 0 \right\}$$

and let $\mathbb{A}_x$ denote the set

$$\mathbb{A}_x := \left\{ a \in \mathbb{K} : \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| < a\}\right| \neq 0 \right\}.$$

**Definition 4.** The *$\alpha$-order $\lambda$-statistical limit superior* of the sequence $x = \{x_u\}$ in an ultrametric 2-normed space is defined by

$$\operatorname{stat}_\lambda^\alpha\text{-}\limsup x = \begin{cases} -\infty, & \text{if } \mathbb{B}_x = \emptyset, \\ \sup\ \mathbb{B}_x, & \text{if } \mathbb{B}_x \neq \emptyset. \end{cases}$$

**Definition 5.** The *$\alpha$-order $\lambda$-statistical limit inferior* of the sequence $x = \{x_u\}$ in an ultrametric 2-normed space is given by

$$\operatorname{stat}_\lambda^\alpha\text{-}\liminf x = \begin{cases} +\infty, & \text{if } \mathbb{A}_x = \emptyset, \\ \inf\ \mathbb{A}_x, & \text{if } \mathbb{A}_x \neq \emptyset. \end{cases}$$

**Definition 6.** The sequence $x = \{x_u\}$ is said to be *$\alpha$-order $\lambda$-statistical null sequence* for $0 < \alpha \leq 1$ in an ultrametric 2-normed space $(\mathcal{F}, \|\cdot,\cdot\|)$ if, for all $\varepsilon > 0$ and non-zero $z \in \mathcal{F}$,

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| \geq \varepsilon\}\right| = 0.$$

**Theorem 1.** *A sequence in an ultrametric 2-normed space is $\alpha$-order $\lambda$-statistically convergent if and only if it is an $\alpha$-order $\lambda$-statistical Cauchy sequence.*

*Proof.* Let the sequence $x = \{x_u\}$ be $\alpha$-order $\lambda$-statistically convergent in an ultrametric 2-normed space $(\mathcal{F}, \|\cdot,\cdot\|)$, that is, $\operatorname{stat}_\lambda^\alpha\text{-}\lim x_u = \mathfrak{l}$. Let

$$\mathcal{A}(\varepsilon) = \{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\} \ \text{ and } \ \mathcal{B}(\varepsilon) = \{u \in I_n : \|x_u - x_s, z\| \geq \varepsilon\}$$

$$\implies \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\mathcal{A}^C(\varepsilon)\right| = 1$$

for $s \in \mathcal{A}^C(\varepsilon)$, $\|x_s - \mathfrak{l}, z\| < \varepsilon$. To show $\mathcal{B}(\varepsilon) \subseteq \mathcal{A}(\varepsilon)$, for $u \in \mathcal{B}(\varepsilon)$, we have $\|x_u - x_s, z\| \geq \varepsilon$ and $\|x_u - \mathfrak{l}, z\| \geq \varepsilon$, that is $u \in \mathcal{A}(\varepsilon)$. Otherwise, if $\|x_u - \mathfrak{l}, z\| < \varepsilon$, then

$$\varepsilon \leq \|x_u - x_s, z\| \leq \max\{\|x_u - \mathfrak{l}, z\|, \|x_s - \mathfrak{l}, z\|\} < \varepsilon,$$

which is not possible. Thus $\mathcal{B}(\varepsilon) \subseteq \mathcal{A}(\varepsilon)$. Therefore $x = \{x_u\}$ is an $\alpha$-order $\lambda$-statistical Cauchy sequence.

Conversely, let $x = \{x_u\}$ be an $\alpha$-order $\lambda$-statistical Cauchy sequence. Then, for all $\varepsilon > 0$, $z \in \mathcal{F}\backslash\{0\}$, there exists $s \in \mathbb{N}$ such that

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\mathcal{C}(\varepsilon)| = 0,$$

where $\mathcal{C}(\varepsilon) := \{u \in I_n : \|x_u - x_s, z\| \geq \varepsilon\}$.

Suppose that the sequence $x = \{x_u\}$ is not $\alpha$-order $\lambda$-statistically convergent. That is, for every $\mathfrak{l} \in \mathcal{F}$,

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}| \neq 0.$$

Thus, for $x_s$ in $\mathcal{F}$,

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u - x_s, z\| \geq \varepsilon\}| \neq 0,$$

contradicting the assumption of $x = \{x_u\}$ being an $\alpha$-order $\lambda$-statistical Cauchy sequence. Thus $x = \{x_u\}$ is an $\alpha$-order $\lambda$-statistically convergent sequence. ☐

**Theorem 2.** *Let $x$ denote a sequence $\{x_u\}$ in an ultrametric 2-normed space $(\mathcal{F}, \|\cdot,\cdot\|)$. If, for all non-zero $z \in \mathcal{F}$,*

$$\operatorname{stat}_\lambda^\alpha\text{-}\lim_{u\to\infty} \|x_u - \mathfrak{l}, z\| = 0$$

*and*

$$\operatorname{stat}_\lambda^\alpha\text{-}\lim_{u\to\infty} \|x_u - \mathfrak{l}^*, z\| = 0,$$

*where $\mathfrak{l}, \mathfrak{l}^* \in \mathcal{F}$, then $\mathfrak{l} = \mathfrak{l}^*$.*

*Proof.* Assume there exist $\mathfrak{l}, \mathfrak{l}^* \in \mathcal{F}$ such that for every non-zero $z \in \mathcal{F}$,

$$\operatorname{stat}_\lambda^\alpha\text{-}\lim_{u\to\infty} \|x_u - \mathfrak{l}, z\| = 0$$

and

$$\operatorname{stat}_\lambda^\alpha\text{-}\lim_{u\to\infty} \|x_u - \mathfrak{l}^*, z\| = 0.$$

That is,

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}| = 0 \tag{1}$$

and

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u - \mathfrak{l}^*, z\| \geq \varepsilon\}| = 0. \tag{2}$$

Let us assume that $\mathfrak{l} \neq \mathfrak{l}^*$. Hence, there exists a $v \neq 0$ in $\mathcal{F}$ such that $\mathfrak{l} - \mathfrak{l}^*$ and $v$ are linearly independent.

Observe that

$$\begin{aligned}
&\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|\mathfrak{l} - \mathfrak{l}^*, v\| \geq \varepsilon\}| \\
&\qquad = \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|\mathfrak{l} - x_u + x_u - \mathfrak{l}^*, v\| \geq \varepsilon\}| \\
&\qquad = \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|(x_u - \mathfrak{l}^*) - (x_u - \mathfrak{l}), v\| \geq \varepsilon\}|,
\end{aligned}$$

$$\begin{aligned}
&\lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|\mathfrak{l}-\mathfrak{l}^*, v\|\geq\varepsilon\}\right|\\
&\qquad\qquad\qquad\leq max\begin{cases}\lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|x_u-\mathfrak{l}^*, v\|\geq\varepsilon\}\right|,\\[2ex] \lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|x_u-\mathfrak{l}, v\|\geq\varepsilon\}\right|\end{cases}\\[2ex]
&\qquad\qquad\qquad= 0 \text{ [from equations (1) and (2)].}
\end{aligned}$$

This implies $\mathfrak{l}=\mathfrak{l}^*$. Thus, the limits are unique, if they exist. $\square$

**Theorem 3.** *Let $x, y$ denote the sequences $\{x_u\}$, $\{y_u\}$, respectively, in an ultrametric 2-normed space $(\mathcal{F}, \|\cdot,\cdot\|)$. If, for $z\in\mathcal{F}\backslash\{0\}$,*

$$\mathrm{stat}_\lambda^\alpha\text{-}\lim_{u\to\infty}\|x_u, z\| = \|x', z\| \quad \textit{and} \quad \mathrm{stat}_\lambda^\alpha\text{-}\lim_{u\to\infty}\|y_u, z\| = \|y', z\|,$$

*then*

*(a)* $\mathrm{stat}_\lambda^\alpha\text{-}\lim_{u\to\infty}\|cx_u, z\| = \|cx', z\|,$ *for all* $c\in\mathbb{K}$,
*(b)* $\mathrm{stat}_\lambda^\alpha\text{-}\lim_{u\to\infty}\|x_u\pm y_u, z\| = \|x'\pm y', z\|.$

*Proof.* We have

$$\lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|x_u-x', z\|\geq\varepsilon\}\right| = 0, \tag{3}$$

$$\lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|y_u-y', z\|\geq\varepsilon\}\right| = 0. \tag{4}$$

Let $c\in\mathbb{K}$. Observe that

$$\begin{aligned}
&\lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|cx_u-cx', z\|\geq\varepsilon\}\right|\\
&\qquad\qquad\qquad= \lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : |c|\|x_u-x', z\|\geq\varepsilon\}\right|\\
&\qquad\qquad\qquad= \lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|x_u-x', z\|\geq\frac{\varepsilon}{|c|}\}\right|\\
&\qquad\qquad\qquad= 0.
\end{aligned}$$

Hence $\mathrm{stat}_\lambda^\alpha\text{-}\lim_{u\to\infty}\|cx_u, z\| = \|cx', z\|$.

Consider

$$\begin{aligned}
&\lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|x_u+y_u-x'-y', z\|\geq\varepsilon\}\right|\\
&\qquad\qquad\qquad= \lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|(x_u-x')+(y_u-y'), z\|\geq\varepsilon\}\right|,
\end{aligned}$$

$$\begin{aligned}
&\lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|x_u\ +y_u-x'-y',z\|\geq\varepsilon\}\right| \\
&\qquad\qquad\qquad\leq max\begin{cases}\lim\limits_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|x_u-x',z\|\geq\varepsilon\}\right|,\\ \\ \lim\limits_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|y_u-y',z\|\geq\varepsilon\}\right|\end{cases}\\
&\qquad\qquad\qquad= 0 \text{ [from equations (3) and (4)].}
\end{aligned}$$

Hence $\text{stat}_\lambda^\alpha\text{-}\lim\limits_{u\to\infty}\|x_u+y_u,z\| = \|x'+y',z\|$. Similarly, we can show that $\text{stat}_\lambda^\alpha\text{-}\lim\limits_{u\to\infty}\|x_u-y_u,z\| = \|x'-y',z\|$.

Thus $\text{stat}_\lambda^\alpha\text{-}\lim\limits_{u\to\infty}\|x_u\pm y_u,z\| = \|x'\pm y',z\|$. ☐

**Theorem 4.** *Let* $x=\{x_u\}$ *be an* $\alpha$*-order* $\lambda$*-statistically convergent sequence with limit* $\mathfrak{l}$ *in an ultrametric* $2$*-normed space* $(\mathcal{F},\|\cdot,\cdot\|)$*. Let* $y=\{y_u\}$ *and* $r=\{r_u\}$ *be sequences such that* $\lim\limits_{u\to\infty}y_u=\mathfrak{l}$ *and* $x=y+r$*. Then* $r=\{r_u\}$ *is an* $\alpha$*-order* $\lambda$*-statistically null sequence if*

$$\lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : x_u\neq y_u\}\right| = 0.$$

*Proof.* Let the sequence $x=\{x_u\}$ be $\alpha$-order $\lambda$-statistically convergent to $\mathfrak{l}$ in an ultrametric 2-normed space $(\mathcal{F},\|\cdot,\cdot\|)$. We have

$$\lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|x_u-\mathfrak{l},z\|\geq\varepsilon\}\right| = 0.$$

Let $y=\{y_u\}$ and $r=\{r_u\}$ be sequences such that $x=y+r$ and $\lim\limits_{u\to\infty}y_u=\mathfrak{l}$. From Remark 1, the sequence $y=\{y_u\}$ is also an $\alpha$-order $\lambda$-statistically convergent sequence in the ultrametric 2-normed space. From $x=y+r$, we get $r=x-y$. Also, if $x_u=y_u$, then $r_u=0$, and if $x_u\neq y_u$, then $r_u\neq 0$. Thus $\{u\in I_n : x_u\neq y_u\}=\{u\in I_n : r_u\neq 0\}$. Hence

$$\lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : r_u\neq 0\}\right| = 0. \tag{5}$$

If $\|r_u,z\|\geq\varepsilon$, then we have $r_u\neq 0$,

$$\begin{aligned}
&\Longrightarrow \{u\in I_n : \|r_u,z\|\geq\varepsilon\}\subseteq\{u\in I_n : r_u\neq 0\},\\
&\Longrightarrow |\{u\in I_n : \|r_u,z\|\geq\varepsilon\}|\leq|\{u\in I_n : r_u\neq 0\}|,\\
&\Longrightarrow \frac{1}{\lambda_n^\alpha}|\{u\in I_n : \|r_u,z\|\geq\varepsilon\}|\leq\frac{1}{\lambda_n^\alpha}|\{u\in I_n : r_u\neq 0\}|,\\
&\Longrightarrow \lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{u\in I_n : \|r_u,z\|\geq\varepsilon\}\right| = 0 \text{ [from equation (5)].}
\end{aligned}$$

Therefore $r=\{r_u\}$ is an $\alpha$-order $\lambda$-statistically null sequence. ☐

**Theorem 5.** *A sequence* $x = \{x_u\}$ *in an ultrametric* 2*-normed space is an* $\alpha$*-order* $\lambda$*-statistically convergent sequence if and only if*

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n, u' \in I_n : \|x_u - x_{u'(s)}, z\| \geq \varepsilon\}\right| = 0, \tag{6}$$

*where* $\{x_{u'(s)}\}$ *denotes a subsequence of* $\{x_u\}$ *such that*

$$\lim_{s\to\infty} x_{u'(s)} = \mathfrak{l}.$$

*Proof.* Let $x = \{x_u\}$ be an $\alpha$-order $\lambda$-statistically convergent sequence in an ultrametric 2-normed space $(\mathcal{F}, \|\cdot,\cdot\|)$. By definition

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right| = 0. \tag{7}$$

Since convergence of a sequence implies $\alpha$-order $\lambda$-statistical convergence, we have

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u' \in I_n : \|x_{u'(s)} - \mathfrak{l}, z\| \geq \varepsilon\}\right| = 0. \tag{8}$$

Consider

$$\begin{aligned}
&\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n, u' \in I_n : \|x_u - x_{u'(s)}, z\| \geq \varepsilon\}\right| \\
&\quad = \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n, u' \in I_n : \|x_u - \mathfrak{l} + \mathfrak{l} - x_{u'(s)}, z\| \geq \varepsilon\}\right| \\
&\quad = \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n, u' \in I_n : \|(x_u - \mathfrak{l}) - (x_{u'(s)} - \mathfrak{l}), z\| \geq \varepsilon\}\right| \\
&\quad \leq max \begin{cases} \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right|, \\[2ex] \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u' \in I_n : \|x_{u'(s)} - \mathfrak{l}, z\| \geq \varepsilon\}\right| \end{cases} \\
&\quad = 0 \text{ [from equations (7) and (8)].}
\end{aligned}$$

Thus the equation (6) is true.

Conversely, assume that the equation (6) holds:

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n, u' \in I_n : \|x_u - x_{u'(s)}, z\| \geq \varepsilon\}\right| = 0.$$

Consider

$$\begin{aligned}
&\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right| \\
&\quad = \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n, u' \in I_n : \|x_u - x_{u'(s)} + x_{u'(s)} - \mathfrak{l}, z\| \geq \varepsilon\}\right| \\
&\quad \leq max \begin{cases} \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n, u' \in I_n : \|x_u - x_{u'(s)}, z\| \geq \varepsilon\}\right|, \\[2ex] \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u' \in I_n : \|x_{u'(s)} - \mathfrak{l}, z\| \geq \varepsilon\}\right| \end{cases}
\end{aligned}$$

$$= 0 \text{ [from equations (6) and (8)]}$$

$$\implies \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}| = 0.$$

Therefore $x = \{x_u\}$ is an $\alpha$-order $\lambda$-statistically convergent sequence. ☐

**Theorem 6.** *Let* $\beta = \mathrm{stat}_\lambda^\alpha\text{-}\limsup x_u$. *If* $\beta$ *is finite then, for every* $\varepsilon > 0$ *and non-zero* $z \in \mathcal{F}$,

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u, z\| > \beta - \varepsilon\}| \neq 0 \tag{9}$$

*and*

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u, z\| > \beta + \varepsilon\}| = 0. \tag{10}$$

*Conversely, if the above two equations hold, then* $\beta = \mathrm{stat}_\lambda^\alpha\text{-}\limsup x_u$.

*Proof.* Let us assume that, for any $\varepsilon > 0$, $\beta = \mathrm{stat}_\lambda^\alpha\text{-}\limsup x_u$ is finite. That is, $\beta = \sup \mathbb{B}_x$, where

$$\mathbb{B}_x = \left\{ b \in \mathbb{K} : \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u, z\| > b\}| \neq 0 \right\},$$

which implies, for any $b < \mathrm{stat}_\lambda^\alpha\text{-}\limsup x_u$,

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u, z\| > b\}| \neq 0,$$

and for any $b > \mathrm{stat}_\lambda^\alpha\text{-}\limsup x_u$,

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u, z\| > b\}| = 0.$$

From the definition of $\beta$, clearly $\beta - \varepsilon \in \mathbb{B}_x$ and $\beta + \varepsilon \notin \mathbb{B}_x$. Thus

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u, z\| > \beta - \varepsilon\}| \neq 0$$

and

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u, z\| > \beta + \varepsilon\}| = 0.$$

Conversely, if the equations (9) and (10) hold, then $\beta - \varepsilon \in \mathbb{B}_x$ and $\beta + \varepsilon \notin \mathbb{B}_x$, so it is clear that $\beta = \mathrm{stat}_\lambda^\alpha\text{-}\limsup x_u$. ☐

The following theorem is dual to the above theorem.

**Theorem 7.** *Let* $\gamma = \mathrm{stat}_\lambda^\alpha\text{-}\liminf x_u$. *If* $\gamma$ *is finite, then, for every* $\varepsilon > 0$ *and non-zero* $z \in \mathcal{F}$,

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u, z\| < \gamma + \varepsilon\}| \neq 0 \tag{11}$$

*and*

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} |\{u \in I_n : \|x_u, z\| < \gamma - \varepsilon\}| = 0. \tag{12}$$

*Conversely, if the above two equations hold, then* $\gamma = \mathrm{stat}_\lambda^\alpha\text{-}\liminf x_u$.

*Proof.* Let us assume that, for every $\varepsilon > 0$, $\gamma = \mathrm{stat}_\lambda^\alpha\text{-}\liminf x_u$ is finite. That is, $\gamma = \inf \mathbb{A}_x$, where

$$\mathbb{A}_x = \left\{ a \in \mathbb{K} : \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| < a\}\right| \neq 0 \right\},$$

which implies, for any $a > \mathrm{stat}_\lambda^\alpha\text{-}\liminf x_u$,

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| < a\}\right| \neq 0.$$

From the definition of $\gamma$, we have $\gamma + \varepsilon \in \mathbb{A}_x$ and $\gamma - \varepsilon \notin \mathbb{A}_x$. Thus

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| < \gamma + \varepsilon\}\right| \neq 0$$

and

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| < \gamma - \varepsilon\}\right| = 0.$$

Conversely, if the equations (11) and (12) hold, then $\gamma + \varepsilon \in \mathbb{A}_x$ and $\gamma - \varepsilon \notin \mathbb{A}_x$. Thus $\gamma = \inf \mathbb{A}_x = \mathrm{stat}_\lambda^\alpha\text{-}\liminf x_u$. ☐

**Theorem 8.** *In an ultrametric* 2*-normed space, for any sequence* $x = \{x_u\}$,

$$\mathrm{stat}_\lambda^\alpha\text{-}\liminf x_u \leq \mathrm{stat}_\lambda^\alpha\text{-}\limsup x_u.$$

*Proof.* **Case 1:** If $\mathrm{stat}_\lambda^\alpha\text{-}\limsup x_u = -\infty$, then the set $\mathbb{B}_x = \emptyset$, where

$$\mathbb{B}_x = \left\{ b \in \mathbb{K} : \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| > b\}\right| \neq 0 \right\}$$

$$\implies \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| > b\}\right| = 0$$

$$\implies \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| \leq b\}\right| = 1.$$

Thus, for every $a \in \mathbb{K}$,

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| < a\}\right| \neq 0.$$

Therefore $\mathrm{stat}_\lambda^\alpha\text{-}\liminf x_u = -\infty$, as the sequence is statistically unbounded below.

**Case 2:** If $\mathrm{stat}_\lambda^\alpha\text{-}\limsup x_u = \infty$, then the condition $\mathrm{stat}_\lambda^\alpha\text{-}\liminf x_u \leq \mathrm{stat}_\lambda^\alpha\text{-}\limsup x_u$ holds trivially.

**Case 3:** Let $\mathrm{stat}_\lambda^\alpha\text{-}\limsup x_u = \beta$ (finite). Now let $\beta = \sup \mathbb{B}_x = \mathrm{stat}_\lambda^\alpha\text{-}\limsup x_u$ and $\gamma = \inf \mathbb{A}_x = \mathrm{stat}_\lambda^\alpha\text{-}\liminf x_u$, where

$$\mathbb{A}_x = \left\{ a \in \mathbb{K} : \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| < a\}\right| \neq 0 \right\}$$

and

$$\mathbb{B}_x = \left\{ b \in \mathbb{K} : \lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| > b\}\right| \neq 0 \right\}.$$

By Definition 4,

$$\begin{aligned} &\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\left\{u \in I_n : \|x_u, z\| > \beta + \frac{\varepsilon}{2}\right\}\right| = 0, \\ \implies &\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\left\{u \in I_n : \|x_u, z\| \leq \beta + \frac{\varepsilon}{2}\right\}\right| = 1, \\ \implies &\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| < \beta + \varepsilon\}\right| = 1. \end{aligned}$$

By Definition 3, $\beta + \varepsilon \in \mathbb{A}_x$ for every $\varepsilon > 0$ and $\gamma = \inf \mathbb{A}_x$, hence $\gamma \leq \beta + \varepsilon$ for every $\varepsilon > 0$. As $\varepsilon \to 0$, we have $\gamma \leq \beta$. □

**Theorem 9.** *Let $x$ denote a sequence $\{x_u\}$ in an ultrametric 2-normed space $(\mathcal{F}, \|\cdot,\cdot\|)$, which is bounded statistically. The sequence $x = \{x_u\}$ is an $\alpha$-order $\lambda$-statistically convergent sequence if and only if* $\operatorname{stat}_\lambda^\alpha\text{-}\limsup x_u = \operatorname{stat}_\lambda^\alpha\text{-}\liminf x_u$.

*Proof.* Consider $\gamma = \operatorname{stat}_\lambda^\alpha\text{-}\liminf x_u$ and $\beta = \operatorname{stat}_\lambda^\alpha\text{-}\limsup x_u$. Suppose, for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right| = 0,$$

which implies

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| > \mathfrak{l} + \varepsilon\}\right| = 0 \tag{13}$$

and

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u, z\| < \mathfrak{l} - \varepsilon\}\right| = 0. \tag{14}$$

From the definition of $\mathbb{B}_x$, equation (13) implies $\beta = \operatorname{stat}_\lambda^\alpha\text{-}\limsup x_u \leq \mathfrak{l}$, and from the definition of $\mathbb{A}_x$, equation (14) implies $\gamma = \operatorname{stat}_\lambda^\alpha\text{-}\liminf x_u \geq \mathfrak{l}$. Therefore

$$\operatorname{stat}_\lambda^\alpha\text{-}\limsup x_u \leq \mathfrak{l} \leq \operatorname{stat}_\lambda^\alpha\text{-}\liminf x_u.$$

From Theorem 8, $\operatorname{stat}_\lambda^\alpha\text{-}\liminf x_u \leq \operatorname{stat}_\lambda^\alpha\text{-}\limsup x_u$, thus

$$\operatorname{stat}_\lambda^\alpha\text{-}\limsup x_u = \operatorname{stat}_\lambda^\alpha\text{-}\liminf x_u.$$

Conversely, assume $\operatorname{stat}_\lambda^\alpha\text{-}\liminf x_u = \operatorname{stat}_\lambda^\alpha\text{-}\limsup x_u$. Let $\gamma = \mathfrak{l} = \beta$, then

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\left\{u \in I_n : \|x_u, z\| > \mathfrak{l} + \frac{\varepsilon}{2}\right\}\right| = 0$$

and

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\left\{u \in I_n : \|x_u, z\| < \mathfrak{l} - \frac{\varepsilon}{2}\right\}\right| = 0.$$

This implies that

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right| = 0.$$

Therefore $x = \{x_u\}$ is an $\alpha$-order $\lambda$-statistically convergent sequence to the limit $\mathfrak{l}$. □

**Theorem 10.** *Consider a sequence $x = \{x_u\}$ in an ultrametric 2-normed space $(\mathcal{F}, \|\cdot,\cdot\|)$. Let $\lambda = \{\lambda_n\}$ be a sequence with $\lim\inf_{n\to\infty} \frac{\lambda_n^\alpha}{n} > 0$. Then* $\operatorname{stat}\text{-}\lim x_u \subseteq \operatorname{stat}_\lambda^\alpha\text{-}\lim x_u$.

*Proof.* Let $x = \{x_u\}$ be a statistically convergent sequence. By the definition of statistical convergence

$$\lim_{n\to\infty} \frac{1}{n} \left|\{u \leq n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right| = 0.$$

Since we have $\lim\inf_{n\to\infty} \frac{\lambda_n^\alpha}{n} > 0$, as $n \to \infty$:

$$\begin{aligned}
\frac{1}{n} \left|\{u \leq n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right| &\geq \frac{1}{n} \left|\{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right| \\
&\geq \frac{\lambda_n^\alpha}{n} \cdot \frac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right|.
\end{aligned}$$

Thus $x = \{x_u\}$ is $\alpha$-order $\lambda$-statistically convergent if it is statistically convergent. Hence

$$\operatorname{stat}\text{-}\lim x_u \subseteq \operatorname{stat}_\lambda^\alpha\text{-}\lim x_u.$$

□

**Theorem 11.** *Let $x = \{x_u\}$ and $\lambda = \{\lambda_n\}$ be sequences in an ultrametric 2-normed space $(\mathcal{F}, \|\cdot,\cdot\|)$. If $\lim_{n\to\infty} \frac{\lambda_n^\alpha}{n} = 1$, then* $\operatorname{stat}\text{-}\lim x_u = \operatorname{stat}_\lambda^\alpha\text{-}\lim x_u$.

*Proof.* Let $\lim_{n\to\infty} \frac{\lambda_n^\alpha}{n} = 1$. For every $\varepsilon > 0$,

$$\begin{aligned}
\frac{1}{n}\left|\{u \leq n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right| \leq\ & \tfrac{1}{n} \left|\{u \leq n - \lambda_n^\alpha : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right| \\
& + \tfrac{\lambda_n^\alpha}{n} . \tfrac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right|,
\end{aligned}$$

$$\frac{1}{n}\left|\{u \leq n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right| \leq \tfrac{n-\lambda_n^\alpha}{n} + \tfrac{\lambda_n^\alpha}{n} . \tfrac{1}{\lambda_n^\alpha} \left|\{u \in I_n : \|x_u - \mathfrak{l}, z\| \geq \varepsilon\}\right|.$$

Thus, if $x = \{x_u\}$ is $\alpha$-order $\lambda$-statistically convergent then it is statistically convergent. Hence, $\operatorname{stat}_\lambda^\alpha\text{-}\lim x_u \subseteq \operatorname{stat}\text{-}\lim x_u$. Also, since $\lim_{n\to\infty} \frac{\lambda_n^\alpha}{n} = 1$, we have $\lim\inf \frac{\lambda_n^\alpha}{n} > 0$. By Theorem 10, $\operatorname{stat}\text{-}\lim x_u \subseteq \operatorname{stat}_\lambda^\alpha\text{-}\lim x_u$. Hence $\operatorname{stat}\text{-}\lim x_u = \operatorname{stat}_\lambda^\alpha\text{-}\lim x_u$. □

## 3. Conclusion

In summary, this work develops the framework of $\alpha$-order $\lambda$-statistical convergence in an ultrametric 2-normed space and explores its behaviour in it. Examples are provided to understand the concepts introduced and emphasise use of statistical convergence over non-Archimedean fields when compared to classical convergence.

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