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A weak form of continuity

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Abstract. The aim of this paper is to introduce the notion of a new class of functions which generalize classes of α-irresolute, α-continuous, strongly α-irresolute, contra-continuous and contra α-continuous functions. This class of functions is called slightly α-continuous. Moreover, basic properties and preservation theorems of slightly α-continuous functions are obtained and relationships between slightly α-continuous functions and α-co-closed graphs are investigated.

1. Introduction

In recent years various classes of near functions were defined and studied by various authors. In this paper, we introduce and study a new class of functions generalizing α-irresolute functions [5], α-continuous functions [8], strongly α-irresolute functions [2], contra-continuous functions [1] and contra α-continuous functions [4]. Furthermore, basic properties and preservation theorems of slightly α-continuous functions are obtained.

In Section 3, we obtain characterizations and basic properties of slightly α-continuous functions. In Section 4 and in Section 5, we investigate relationships between slightly α-continuity and separation axioms and between slightly α-continuity and connectedness, respectively. In Section 6, we introduce α-co-closed graphs and we study relationships between slightly α-continuity and α-co-closed graphs. In Section 7, we investigate relationships between slightly α-continuity and compactness.

2. Preliminaries

Throughout the present paper, X and Y are always topological spaces. Let A be a subset of X. We denote the interior and the closure of a set A by $int(A)$ and $cl(A)$, respectively.

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A subset A of a space X is said to be preopen [7] if $A \subset int(cl(A))$, α-open [10] if $A \subset int(cl(int(A)))$. The complement of a preopen (α-open) set is said to be preclosed (respectively α-closed). The collection of all preopen (respectively preclosed, α-open, α-closed and clopen) subsets of X will be denoted by $PO(X)$ (respectively $PC(X)$, $\alpha O(X)$, $\alpha C(X)$ and $CO(X)$).

It is shown in [10] that $\alpha O(X)$ is a topology for X. By $\alpha\text{-}cl(A)$, we denote the closure of a subset A with respect to $\alpha O(X)$.

Definition 1 ([5]). A function $f : X \to Y$ is called α-irresolute if $f^{-1}(V)$ is α-open set in X for each α-open set V of Y.

Definition 2 ([8]). A function $f : X \to Y$ is said to be α-continuous if $f^{-1}(V)$ is α-open in X for every open set V of Y.

Definition 3 ([2]). A function $f : X \to Y$ is said to be strongly α-irresolute if for each $x \in X$ and each α-open subset V of Y containing $f(x)$, there exists a open subset U of X containing x such that $f(U) \subset V$.

Definition 4 ([1]). A function $f : X \to Y$ is called contra-continuous if $f^{-1}(V)$ is closed set in X for each open set V of Y.

Definition 5 ([4]). A function $f : X \to Y$ is called contra α-continuous if $f^{-1}(V)$ is α-closed set in X for each open set V of Y.

3. Slightly α-continuous functions

Definition 6. A function $f : X \to Y$ is said to be slightly α-continuous if for each $x \in X$ and each clopen subset V in Y containing $f(x)$, there exists an α-open subset U in X containing x such that $f(U) \subset V$.

Theorem 7. *Let (X, τ) and (Y, υ) be topological spaces. The following statements are equivalent for a function $f : X \to Y$:*

(1) *f is slightly α-continuous;*
(2) *for every clopen set $V \subset Y$, $f^{-1}(V)$ is α-open;*
(3) *for every clopen set $V \subset Y$, $f^{-1}(V)$ is α-closed;*
(4) *for every clopen set $V \subset Y$, $f^{-1}(V)$ is α-clopen.*

Proof. (1) $\Rightarrow$ (2) : Let V be a clopen subset of Y and let $x \in f^{-1}(V)$. Since $f(x) \in V$, by (1), there exists a α-open set U_x in X containing x such that $U_x \subset f^{-1}(V)$. We obtain that $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$. Thus, $f^{-1}(V)$ is α-open.

(2) $\Rightarrow$ (3) : Let V be a clopen subset of Y. Then, $Y \backslash V$ is clopen. By (2), $f^{-1}(Y \backslash V) = X \backslash f^{-1}(V)$ is α-open. Thus, $f^{-1}(V)$ is α-closed.

(3) $\Rightarrow$ (4) : It can be shown easily.

(4) $\Rightarrow$ (1) : Let V be a clopen subset in Y containing $f(x)$. By (4), $f^{-1}(V)$ is α-clopen. Take $U = f^{-1}(V)$. Then, $f(U) \subset V$. Hence, f is slightly α-continuous. □ □

Lemma 8 ([8]). *If $A \in PO(X)$ and $B \in \alpha O(X)$, then $A \cap B \in \alpha O(A)$.*

Theorem 9. *If $f : X \to Y$ is slightly α-continuous and $A \in PO(X)$, then the restriction $f\mid_A : A \to Y$ is slightly α-continuous.*

Proof. Let V be a clopen subset of Y. We have $(f\mid_A)^{-1}(V) = f^{-1}(V) \cap A$. Since $f^{-1}(V)$ is α-open and A is preopen, it follows from the previous lemma that $(f\mid_A)^{-1}(V)$ is α-open in the relative topology of A. Thus, $f\mid_A$ is slightly α-continuous. □

Lemma 10 ([8]). *If $A \in \alpha O(Y)$ and $Y \in \alpha O(X)$, then $A \in \alpha O(X)$.*

Theorem 11. *Let $f : X \to Y$ be a function and $\Sigma = \{U_\alpha : \alpha \in I\}$ be a cover of X such that $U_\alpha \in \alpha O(X)$ for each $\alpha \in I$. If $f\mid_{U_\alpha}$ is slightly α-continuous for each $\alpha \in I$, then f is a slightly α-continuous function.*

Proof. Suppose that V is any clopen set of Y. Since $f\mid_{U_\alpha}$ is slightly α-continuous for each $\alpha \in I$, it follows that $(f\mid_{U_\alpha})^{-1}(V) \in \alpha(U_\alpha)$. We have $f^{-1}(V) = \bigcup_{\alpha \in I}(f^{-1}(V) \cap U_\alpha) = \bigcup_{\alpha \in I}(f\mid_{U_\alpha})^{-1}(V)$. Then, as a direct consequence of the previous lemma we obtain that $f^{-1}(V) \in \alpha O(X)$ which means that f is slightly α-continuous. □

Theorem 12. *Let $f : X \to Y$ be a function and let $g : X \to X \times Y$ be the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is slightly α-continuous, then f is slightly α-continuous.*

Proof. Let $V \in CO(Y)$, then $X \times V \in CO(X \times Y)$. Since g is slightly α-continuous, then $f^{-1}(V) = g^{-1}(X \times V) \in \alpha O(X)$. Thus, f is slightly α-continuous. □

Remark 13. The following diagram holds:

contra-continuous $\Rightarrow$ contra α-continuous $\Rightarrow$ slightly α-continuous

$\Uparrow$

α-continuous

$\Uparrow$

α-irresolute

$\Uparrow$

strongly α-irresolute

The following examples show that these implications are not reversible.

Example 14. Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{d\}, \{a, c\}, \{a, c, d\}\}$. Let $Y = \{x, y, z\}$ and $\sigma = \{Y, \emptyset, \{x\}, \{z\}, \{x, z\}\}$. We define a function $f : (X, \tau) \to (X, \sigma)$ as follows: $f(a) = x$, $f(b) = f(c) = y$ and $f(d) = z$. Then f is slightly α-continuous, but it is not α-continuous.

Example 15. Let $\mathbb{R}$ and $\mathbb{Q}$ be the real numbers and rational numbers, respectively. Let $A = \{x \in \mathbb{R} : x \text{ is rational and } 0 < x < 1\}$. We define two topologies on $\mathbb{R}$ as $\tau = \{\mathbb{R}, \emptyset, A, \mathbb{R}\backslash A\}$ and $\upsilon = \{\mathbb{R}, \emptyset, \{0\}\}$. Let $f : (\mathbb{R}, \tau) \to (\mathbb{R}, \upsilon)$ be a function which is defined by $f(x) = 1$ if $x \in \mathbb{Q}$ and $f(x) = 0$ if $x \notin \mathbb{Q}$. Then, f is slightly α-continuous, but it is not contra α-continuous since for closed set $\mathbb{R}\backslash\{0\}$, $f^{-1}(\mathbb{R}\backslash\{0\}) = \mathbb{Q}$ is not α-open.

The other implications are not reversible as shown in several papers [4, 5, 8].

Recall that a space is 0-dimensional if its topology has a base consisting of clopen sets.

Theorem 16. *If $f : X \to Y$ is slightly α-continuous and Y is 0-dimensional space, then f is α-continuous.*

Proof. Let $x \in X$ and let V be an open subset of Y containing $f(x)$. Since Y is 0-dimensional, there exists a clopen set U containing $f(x)$ such that $U \subset V$. Since f is slightly α-continuous, then there exists an α-open subset G in X containing x such that $f(G) \subset U \subset V$. Thus, f is α-continuous. □

Theorem 17. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then, the following properties hold:*

(1) *If f is α-irresolute and g is slightly α-continuous, then $g \circ f : X \to Z$ is slightly α-continuous.*

(2) *If f is strongly α-irresolute and g is slightly α-continuous, then $g \circ f : X \to Z$ is slightly α-continuous.*

Proof. (1) Let V be any clopen set in Z. Since g is slightly α-continuous, $g^{-1}(V)$ is α-open. Since f is α-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is α-open. Therefore, $g \circ f$ is slightly α-continuous.

(2) It can be obtained similarly. □

Definition 18 ([14]). A function $f : X \to Y$ is called strongly α-open if for every α-open subset A of X, $f(A)$ is α-open in Y.

Theorem 19. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. If f is strongly α-open and surjective and $g \circ f : X \to Z$ is slightly α-continuous, then g is slightly α-continuous.*

Proof. Let V be any clopen set in Z. Since $g \circ f$ is slightly α-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is α-open. Since f is strongly α-open, then $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is α-open. Hence, g is slightly α-continuous. □

Combining the previous two theorems, we obtain the following result.

Theorem 20. *Let $f : X \to Y$ be surjective, α-irresolute and strongly α-open and $g : Y \to Z$ be a function. Then $g \circ f : X \to Z$ is slightly α-continuous if and only if g is slightly α-continuous.*

Definition 21 ([3]). A filter base Λ is said to be α-convergent to a point x in X if for any $U \in \alpha O(X)$ containing x, there exists a $B \in \Lambda$ such that $B \subset U$.

Definition 22. A filter base Λ is said to be co-convergent to a point x in X if for any $U \in CO(X)$ containing x, there exists a $B \in \Lambda$ such that $B \subset U$.

Theorem 23. *If a function $f : X \to Y$ is slightly α-continuous if and only if for each point $x \in X$ and each filter base Λ in X α-converging to x, the filter base $f(\Lambda)$ is co-convergent to $f(x)$.*

Proof. ($\Rightarrow$) Let $x \in X$ and Λ be any filter base in X α-converging to x. Since f is slightly α-continuous, then for any $V \in CO(Y)$ containing $f(x)$, there exists a $U \in \alpha O(X)$ containing x such that $f(U) \subset V$. Since Λ is α-converging to x, there exists a $B \in \Lambda$ such that $B \subset U$. This means that $f(B) \subset V$ and therefore the filter base $f(\Lambda)$ is co-convergent to $f(x)$.

($\Leftarrow$) Let $x \in X$ and $V \in CO(Y)$ containing $f(x)$. If we take Λ to be the set of all sets U such that $U \in \alpha O(X)$ containing x, then Λ will be a filter base which α-convergences to x. Thus, there exists $U \in \Lambda$ such that $f(U) \subset V$. Hence, we obtain that f is slightly α-continuous. □

4. Separation axioms

In this section, we investigate relationships between slightly α-continuous functions and separation axioms.

Definition 24 ([9]). A space X is said to be α-T_0 if for each pair of distinct points in X, there exists an α-open set of X containing one point but not the other.

Definition 25 ([9]). A space X is said to be α-T_1 if for each pair of distinct points x and y of X, there exist α-open sets U and V containing x and y, respectively, such that $y \notin U$ and $x \notin V$.

Definition 26. A space X is said to be co-T_0 if for each pair of distinct points in X, there exists a clopen set of X containing one point but not the other.

Definition 27. A space X is said to be co-T_1 if for each pair of distinct points x and y of X, there exist clopen sets U and V containing x and y, respectively, such that $y \notin U$ and $x \notin V$.

Theorem 28. *If $f : X \to Y$ is a slightly α-continuous injection and Y is co-T_1, then X is α-T_1.*

Proof. Suppose that Y is co-T_1. For any distict points x and y in X, there exist V, $W \in CO(Y)$ such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since f is slightly α-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are α-open subsets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that X is α-T_1. □

Definition 29 ([9]). A space X is said to be α-T_2 (α-Hausdorff) if for each pair of distinct points x and y in X, there exist disjoint α-open sets U and V in X such that $x \in U$ and $y \in V$.

Definition 30. A space X is said to be co-T_2 (co-Hausdorff) if for each pair of distinct points x and y in X, there exist disjoint clopen sets U and V in X such that $x \in U$ and $y \in V$.

Theorem 31. *If $f : X \to Y$ is a slightly α-continuous injection and Y is co-T_2, then X is α-T_2.*

Proof. For any pair of distict points x and y in X, there exist disjoint clopen sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. Since f is slightly α-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ is α-open in X containing x and y, respectively. We have $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. This shows that X is α-T_2. □

Theorem 32. *If $f, g : X \to Y$ are slightly α-continuous functions and Y is co-Hausdorff, then $E = \{x \in X : f(x) = g(x)\}$ is α-closed in X.*

Proof. If $x \in X \backslash E$, then it follows that $f(x) \neq g(x)$. Since Y is co-Hausdorff, there exist $f(x) \in V \in CO(Y)$ and $g(x) \in W \in CO(Y)$ such that $V \cap W = \emptyset$. Since f and g are slightly α-continuous, $f^{-1}(V)$ and $g^{-1}(W)$ are α-open in X with $x \in f^{-1}(V)$ and $x \in g^{-1}(W)$. Set $O = f^{-1}(V) \cap g^{-1}(W)$. Then, O is α-open and $f(O) \cap g(O) = \emptyset$. Hence, $x \notin \alpha\text{-}cl(E)$. This shows that E is α-closed in X. □

Theorem 33. *If $f : X \to Y$ is slightly α-continuous function and Y is co-Hausdorff, then $E = \{(x, y) \in X \times X : f(x) = f(y)\}$ is α-closed in $X \times X$.*

Proof. Let $(x, y) \in (X \times X) \backslash E$. It follows that $f(x) \neq f(y)$. Since Y is co-Hausdorff, there exist $f(x) \in V \in CO(Y)$ and $f(y) \in W \in CO(Y)$ such that $V \cap W = \emptyset$. Since f is slightly α-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are α-open in X with $x \in f^{-1}(V)$ and $y \in f^{-1}(W)$. Take $U = f^{-1}(V)$ and $G = f^{-1}(W)$. Hence, $(U \times G) \cap E = \emptyset$. We have that $U \times G$ is α-open in $X \times X$ and containing (x, y). This means that E is α-closed in $X \times X$. □

Definition 34. A space is called co-regular (respectively, strongly α-regular) if for each clopen (respectively, α-closed) set F and each point $x \notin F$, there exist disjoint open sets U and V such that $F \subset U$ and $x \in V$.

Definition 35. A space is said to be co-normal (respectively, strongly α-normal) if for every pair of disjoint clopen (respectively, α-closed) subsets F_1 and F_2 of X, there exist disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Theorem 36. *If f is slightly α-continuous injective open function from a strongly α-regular space X onto a space Y, then Y is co-regular.*

Proof. Let F be a clopen set in Y and $y \notin F$. Take $y = f(x)$. Since f is slightly α-continuous, $f^{-1}(F)$ is an α-closed set. Take $G = f^{-1}(F)$. We have $x \notin G$. Since X is strongly α-regular, there exist disjoint open sets U and V such that $G \subset U$ and $x \in V$. We obtain that $F = f(G) \subset f(U)$ and $y = f(x) \in f(V)$ such that $f(U)$ and $f(V)$ are disjoint open sets. This shows that Y is co-regular. □

Theorem 37. *If f is slightly α-continuous injective open function from a strongly α-normal space X onto a space Y, then Y is co-normal.*

Proof. Let F_1 and F_2 be disjoint clopen subsets of Y. Since f is slightly α-continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are α-closed sets. Take $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. We have $U \cap V = \emptyset$. Since X is strongly α-normal, there exist disjoint open sets A and B such that $U \subset A$ and $V \subset B$. We obtain that $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$ such that $f(A)$ and $f(B)$ are disjoint open sets. Thus, Y is co-normal. □

5. Connectedness

In this section, we study relationships between slightly α-continuous functions and connectedness.

Lemma 38. *The following properties are equivalent for a subset A of a space X:*

(1) *A is clopen;*

(2) *A is α-closed and α-open;*

(3) *A is α-closed and preopen.*

Theorem 39. *If $f : X \to Y$ is slightly α-continuous surjective function and X is connected space, then Y is connected space.*

Proof. Suppose that Y is not a connected space. Then there exists nonempty disjoint open sets U and V such that $Y = U \cup V$. Therefore, U and V are clopen sets in Y. Since f is slightly α-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are α-closed and α-open in X and hence clopen in X. Moreover, $f^{-1}(U)$ and $f^{-1}(V)$ are nonempty disjoint and $X = f^{-1}(U) \cup f^{-1}(V)$. This shows that X is not connected. This is a contradiction. By contradiction, Y is connected. □

Definition 40. A topological space X is called hyperconnected [13] if every open set is dense.

Remark 41. The following example shows that slightly α-continuous surjections do not necessarily preserve hyperconnectedness.

Example 42. Let $X = \{a, b, c\}$, $\sigma = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$ and $\tau = \{X, \emptyset, \{a\}\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is slightly α-continuous surjective, (X, τ) is hyperconnected, but (X, σ) is not hyperconnected.

6. α-Co-closed graphs

In this section, we investigate relationships between α-co-closed graphs and slightly α-continuous functions.

Recall that for a function $f : X \to Y$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 43. The graph $G(f)$ of a function $f : X \to Y$ is said to be α-co-closed in $X \times Y$ if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exist $U \in \alpha O(X)$ containing x and $V \in CO(Y)$ containing y such that $(U \times V) \cap G(f) = \emptyset$.

Lemma 44. *The graph $G(f)$ of a function $f : X \to Y$ is α-co-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exist $U \in \alpha O(X)$ containing x and $V \in CO(Y)$ containing y such that $f(U) \cap V = \emptyset$.*

Theorem 45. *If $f : X \to Y$ is slightly α-continuous and Y is co-Hausdorff, then $G(f)$ is α-co-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) \backslash G(f)$, then $f(x) \neq y$. Since Y is co-Hausdorff, there exist $U \in CO(Y)$ and $V \in CO(Y)$ with $f(x) \in U$ and $y \in V$ such that $U \cap V = \emptyset$. Since f is slightly α-continuous, there exists $A \in \alpha O(X)$ containing x such that $f(A) \subset U$. Therefore, we obtain $y \in V \in CO(Y)$ and $f(A) \cap V = \emptyset$. This shows that $G(f)$ is α-co-closed. □

Theorem 46. *If $f : X \to Y$ is α-continuous and Y is co-T_1, then $G(f)$ is α-co-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) \backslash G(f)$, then $f(x) \neq y$ and there exists a clopen set V of Y such that $f(x) \in V$ and $y \notin V$. Since f is α-continuous, there exists $U \in \alpha O(X)$ containing x such that $f(U) \subset V$. Therefore, we obtain $f(U) \cap (Y \backslash V) = \emptyset$ and $Y \backslash V \in CO(Y)$ containing y. This shows that $G(f)$ is α-co-closed in $X \times Y$. □

Theorem 47. *Let $f : X \to Y$ have an α-co-closed graph $G(f)$. If f is injective, then X is α-T_0.*

Proof. Let x and y be any two distinct points of X. Then, we have $(x, f(y)) \in (X \times Y) \backslash G(f)$. By definition of α-co-closed graph, there exist a α-open set U of X and $V \in CO(Y)$ such that $(x, f(y)) \in U \times V$ and $f(U) \cap V = \emptyset$; hence $U \cap f^{-1}(V) = \emptyset$. Therefore, we have $y \notin U$. This implies that X is α-T_0. □

Theorem 48. *Let $f : X \to Y$ have an α-co-closed graph $G(f)$. If f is a surjective strongly α-open function, then Y is α-T_2.*

Proof. Let y_1 and y_2 be any distinct points of Y. Since f is surjective $f(x) = y_1$ for some $x \in X$ and $(x, y_2) \in (X \times Y) \backslash G(f)$. By α-co-closedness of the graph $G(f)$, there exist an α-open set U of X and $V \in CO(Y)$ such that $(x, y_2) \in U \times V$ and $(U \times V) \cap G(f) = \emptyset$. Then, we have $f(U) \cap V = \emptyset$.

Since f is strongly α-open, $f(U)$ is α-open and $f(x) = y_1 \in f(U)$. This implies that Y is α-T_2. □

Definition 49. A space X is said to be mildly compact [12] if every clopen cover of X has a finite subcover. A subset A of a space X is said to be mildly compact relative to X if every cover of A by clopen sets of X has a finite subcover. A subset A of a space X is said to be mildly compact if the subspace A is mildly compact.

Theorem 50. *If a function $f : X \to Y$ has an α-co-closed graph $G(f)$, then $f^{-1}(K)$ is α-closed in X for every subset K which is mildly compact relative to Y.*

Proof. Assume that K is mildly compact relative to Y and $x \notin f^{-1}(K)$. For each $y \in K$, we have $(x, y) \in (X \times Y)\backslash G(f)$ and there exist $U_y \in \alpha O(X)$ containing x and $V_y \in CO(Y)$ containing y such that $f(U_y) \cap V_y = \emptyset$. Since $\{K \cap V_y : y \in K\}$ is a clopen cover of the subspace K, there exists a finite subset $K_1 \subset K$ such that $K \subset \bigcup\{V_{y_k} : k \in K_1\}$. Set $U = \bigcap\{U_{y_k} : k \in K_1\}$, then $U \in \alpha O(X)$ containing x and $f(U) \cap K = \emptyset$. Therefore $U \cap f^{-1}(K) = \emptyset$ and hence $x \notin \alpha\text{-}cl(f^{-1}(K))$. This shows that $f^{-1}(K)$ is α-closed in X. □

Theorem 51. *Let Y be a mildly compact space. If a function $f : X \to Y$ has an α-co-closed graph $G(f)$, then f is slightly α-continuous.*

Proof. Suppose that Y is mildly compact and $G(f)$ is α-co-closed. First, we show that a clopen set of Y is mildly compact. Let V be a clopen set of Y and let $\{H_\alpha : \alpha \in I\}$ be a cover of V by clopen sets H_α of V. For each $\alpha \in I$, there exists a clopen set K_α of X such that $H_\alpha = K_\alpha \cap V$. Then, the family $\{K_\alpha : \alpha \in I\} \cup (Y\backslash V)$ is a clopen cover of Y. Since Y is mildly compact, there exists a finite subset $I_0 \subset I$ such that $Y = \bigcup\{K_\alpha : \alpha \in I_0\} \cup (Y\backslash V)$. Therefore, we obtain $V = \bigcup\{H_\alpha : \alpha \in I_0\}$. This shows that V is mildly compact. For any clopen set V, $f^{-1}(V)$ is α-closed in X and hence f is slightly α-continuous. □

7. Covering properties

In this section, we investigate relationships between slightly α-continuous functions and compactness.

Definition 52. A space X is said to be α-compact [6] if every α-open cover of X has a finite subcover. A subset A of a space X is said to be α-compact relative to X [11] if every cover of A by α-open sets of X has a finite subcover. A subset A of a space X is said to be α-compact if the subspace A is α-compact.

Theorem 53. *If a function $f : X \to Y$ is slightly α-continuous and K is α-compact relative to X, then $f(K)$ is mildly compact in Y.*

Proof. Let $\{H_\alpha : \alpha \in I\}$ be any cover of $f(K)$ by clopen sets of the subspace $f(K)$. For each $\alpha \in I$, there exists a clopen set K_α of Y such that $H_\alpha = K_\alpha \cap f(K)$. For each $x \in K$, there exists $\alpha_x \in I$ such that $f(x) \in K_{\alpha_x}$ and there exists $U_x \in \alpha O(X)$ containing x such that $f(U_x) \subset K_{\alpha_x}$. Since the family $\{U_x : x \in K\}$ is a cover of K by α-open sets of K, there exists a finite subset K_0 of K such that $K \subset \{U_x : x \in K_0\}$. Therefore, we obtain $f(K) \subset \bigcup\{f(U_x) : x \in K_0\}$ which is a subset of $\bigcup\{K_{\alpha_x} : x \in K_0\}$. Thus $f(K) = \bigcup\{H_{\alpha_x} : x \in K_0\}$ and hence $f(K)$ is mildly compact. □

Corollary 54. *If $f : X \to Y$ is a slightly α-continuous surjection and X is α-compact, then Y is mildly compact.*

Definition 55. A space X is said to be:

(1) mildly countably compact [12] if every clopen countable cover of X has a finite subcover;

(2) mildly Lindelöf [12] if every cover of X by clopen sets has a countable subcover;

(3) countably α-compact if every α-open countable cover of X has a finite subcover;

(4) α-Lindelöf if every α-open cover of X has a countable subcover.

Theorem 56. *Let $f : X \to Y$ be a slightly α-continuous surjection. Then the following statements hold:*

(1) *If X is α-Lindelöf, then Y is mildly Lindelöf.*

(2) *If X is countably α-compact, then Y is mildly countably compact.*

Proof. We prove (1), the proof of (2) being entirely analogous.

Let $\{V_\alpha : \alpha \in I\}$ be any clopen cover of Y. Since f is slightly α-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is an α-open cover of X. Since X is α-Lindelöf, there exists a countable subset I_0 of I such that $X = \bigcup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Thus, we have $Y = \bigcup\{V_\alpha : \alpha \in I_0\}$ and Y is mildly Lindelöf. □

Definition 57. A space X is said to be:

(1) α-closed-compact if every α-closed cover of X has a finite subcover;

(2) countably α-closed-compact if every countable cover of X by α-closed sets has a finite subcover;

(3) α-closed-Lindelöf if every cover of X by α-closed sets has a countable subcover.

Theorem 58. *Let $f : X \to Y$ be a slightly α-continuous surjection. Then the following statements hold:*

(1) *If X is α-closed-compact, then Y is mildly compact.*

(2) *If X is α-closed-Lindelöf, then Y is mildly Lindelöf.*

(3) *If X is countably α-closed-compact, then Y is mildly countably compact.*

Proof. It can be obtained similarly as the previous theorem. □

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