ACTA ET COMMENTATIONES UNIVERSITATIS TARTUENSIS DE MATHEMATICA
Volume 12, 2008

Estimating main characteristics of processes with non-regular observations

Tatiana Varatnitskaya

Abstract. In this paper the amplitude modulated version of a random process is investigated. Two cases have been taken into consideration. When the irregularities in observations are defined as a Poisson sequence, the estimators of the covariance function and the spectral density have been constructed. When the irregularities in observations are defined as a stationary random process in the wide sense, the estimators of the mean and the covariance function have been constructed. Statistical properties of the estimators have been studied.

In different practical applications we frequently deal with stationary processes with non-regular observations. Estimates of main characteristics of processes give common information about studied phenomenon. Asymptotic methods of time series analysis allow us to find asymptotic distribution of the estimates when the number of observations tends to infinity.

Parzen (1963) introduced a sequence

$$Y(t) = X(t)d(t), t \in Z, \tag{1}$$

which is called an amplitude modulated version of $X(t)$. It is supposed that the processes $\{X(t) : t \in Z\}$ and $\{d(t) : t \in Z\}$ are independent.

A number of examples of $d(t)$ has been considered in the literature (Jiang, Hui (2004), Lee (2004) etc.). Many investigations are dedicated to the case where $d(t)$ is a sequence of independent Bernoulli trials.

Definition 1. The moment of n-th order of the random process $\{X(t) : t \in Z\}$ is a function

$$m_n(t_1, ..., t_n) = EX(t_1)...X(t_n),$$

$t_j \in Z$, $j = 1, 2, ..., n$.

Received September 24, 2007.
2000 *Mathematics Subject Classification.* 60G10.
Key words and phrases. Amplitude modulation, covariance function, mean, missing observations, spectral density.

It is possible to determine moments of n-th order by using formula

$$m_n(t_1,...,t_n) = i^{-n}\frac{\partial^n H_n(\alpha_1,...,\alpha_n;t_1,...,t_n)}{\partial\alpha_1...\partial\alpha_n}\Big|_{\alpha_1=...=\alpha_n=0},$$

where $H_n(\alpha_1,...,\alpha_n;t_1,...,t_n) = E\exp\left\{i\sum_{j=1}^{n}\alpha_j X(t_j)\right\}$ is the characteristic function, $t_j \in Z$, $j = 1,2,...,n$ and $(\alpha_1,...,\alpha_n)$ is a real non-zero vector.

Definition 2. The cumulant of n-th order of the random process $\{X(t) : t \in Z\}$ is a function

$$c_n(t_1,...,t_n) = i^{-n}\frac{\partial^n ln H_n(\alpha_1,...,\alpha_n;t_1,...,t_n)}{\partial\alpha_1...\partial\alpha_n}\Big|_{\alpha_1=...=\alpha_n=0},$$

$t_j \in Z$, $j = 1,2,...,n$.

There are relations between moments and cumulants of n-th order (Žurbenko, 1987)

$$m_n(I) = \sum_{I_1\cup...\cup I_q=I}\prod_{k=1}^{q} c_{l_k}(I_k),$$

$$c_n(I) = \sum_{I_1\cup...\cup I_q=I}(-1)^{q-1}(q-1)!\prod_{k=1}^{q} m_{l_k}(I_k),$$

where $I = \{1,2,...,n\}$, $I_k \subseteq I$, $I_k = \{i_1,...,i_{l_k}\}$, $1 \le k \le q$, $I_k \cap I_m = \emptyset$ for $k \ne m$, $m_n(I) = m_n(t_1,...,t_n)$, $c_n(I) = c_n(t_1,...,t_n)$, $m_{l_k}(I_k) = m_{l_k}(t_{i_1},...,t_{i_{l_k}})$, $c_{l_k}(I_k) = c_{l_k}(t_{i_1},...,t_{i_{l_k}})$, and $\sum\limits_{I_1\cup...\cup I_q=I}$ is a sum over all primitive partitions of the set I.

Definition 3. The function

$$f_n(\lambda_1,...,\lambda_n) = \frac{1}{(2\pi)^n}\sum_{t_1,...,t_n=-\infty}^{\infty} c_n(t_1,...,t_n)e^{-i\sum\limits_{j=1}^{n}\lambda_j t_j},$$

$\lambda_j \in R$, $j = 1,2,...,n$, is called the semi-invariant spectral density of n-th order of a random process if the series converges absolutely.

These functions are discussed by Brillinger (1975) and Leonov, Širjaev (1959).

Let $\{X(t) : t \in Z\}$ be a stationary random process in the wide sense. This process has the mean m^X, the covariance functions $R^X(\tau), \tau \in Z$, the spectral density $f^X(\lambda), \lambda \in \Pi = [-\pi,\pi]$, the semi-invariant spectral density of the fourth order $f_4^X(\lambda_1,\lambda_2,\lambda_3)$, $\lambda_i \in \Pi$, $i = 1,2,3$, the 4-th moment $m_4^X(t_1,t_2,t_3)$, $t_i \in Z$, $i = 1,2,3$, and the cumulant of the fourth order $c_4^X(\tau_1,\tau_2,\tau_3)$, $\tau_i \in Z$, $i = 1,2,3$. Let the irregularities in observations $\{d(t) : t \in Z\}$ be given as Poisson sequences. The parameter of the distribution is

$\alpha > 0$. Note that $d(t)$ is a sequence of independent random variables. We assume in this part that $m^X = 0$.

Let

$$Y(0), Y(1), ..., Y(T-1) \tag{2}$$

be T consecutive in equal time period observations of the process $Y(t), t \in Z$. The relation between processes $X(t)$ and $Y(t)$ is given by (1).

Using observations (2) the estimator of the covariance function of the process $X(t)$ can be constructed as the statistic

$$\widehat{R}^X(\tau) = \frac{1}{(T-\tau)C_\tau^d} \sum_{t=0}^{T-\tau-1} Y(t+\tau)Y(t), \text{for } \tau = 0, 1, ..., T-1, \tag{3}$$

where

$$C_\tau^d = \begin{cases} \alpha^2, \tau \neq 0, \\ \alpha + \alpha^2, \tau = 0, \end{cases} \tag{4}$$

$$\widehat{R}^X(\tau) = \widehat{R}^X(-\tau), \widehat{R}^X(\tau) = 0, \text{for } |\tau| > T.$$

Theorem 1. *Let $\{X(t) : t \in Z\}$ and $\{d(t) : t \in Z\}$ be defined as above. Then the estimator $\widehat{R}^X$ of the covariance function of the process $X(t)$, given in (3), is an asymptotically unbiased estimator. Moreover, if $\sum\limits_{u=-\infty}^{\infty} \left(R^X(u)\right)^2 < \infty$ and $\sum\limits_{u=-\infty}^{\infty} c_4^X(u+\tau, u, \tau) < \infty$, for all $\tau \in Z$, then it is mean-square consistent.*

Proof. Using independence between $X(t)$ and $d(t)$ it is easy to show the unbiasedness of the estimator:

$$E\widehat{R}^X(\tau) = \frac{1}{(T-\tau)C_\tau^d} \sum_{t=0}^{T-\tau-1} R^X(\tau)Ed(t+\tau)d(t) = R^X(\tau).$$

Further, taking into account the relations between moments and cumulants, it is proved that the variance of the estimator, given in (3), has the following property:

$$\lim_{T\to\infty}(T-\tau)Var\widehat{R}^X(\tau) = \left(\frac{(\alpha+\alpha^2)^2}{\alpha^4} - 1\right)$$

$$\times \left(c_4^X(\tau, 0, \tau) + 2\left(R^X(\tau)\right)^2 + \left(R^X(0)\right)^2\right) + \sum_{u=-\infty}^{\infty} J(u, \tau),$$

where

$$J(u, \tau) = c_4^X(u+\tau, u, \tau) + 2\left(R^X(u)\right)^2 + R^X(u-\tau)R^X(u+\tau).$$

Under conditions of the theorem it is clear that the variance vanishes if the number of observations tends to infinity. $\square$

Next the problem of construction of an estimator of the spectral density is investigated. It is proposed to consider the following statistic:

$$I^T(\lambda) = \frac{1}{2\pi T}\sum_{t=0}^{T-1}\sum_{s=0}^{T-1}\frac{Y(t)Y(s)}{C_{t-s}^d}e^{-i\lambda(t-s)} \text{ for } \lambda\in\Pi\,, \tag{5}$$

where C_{t-s}^d is defined in (4).

As a result the following theorem is formulated and proved.

Theorem 2. *Let the semi-invariant spectral density of the fourth order $f_4^X(\lambda_1,\lambda_2,\lambda_3)$ be continuous on Π^3 and the spectral density $f^X(\lambda)$ be continuous on Π. Then the statistic defined in (5) is an asymptotically unbiased estimator for $f^X(\lambda)$ and*

$$cov\left\{I^T(\lambda_1), I^T(\lambda_2)\right\}\underset{T\to\infty}{\longrightarrow}\left\{\begin{array}{c} 0, \lambda_1\pm\lambda_2\neq 0(mod 2\pi),\\ f^X(\lambda_1)f^X(\lambda_2), \lambda_1\pm\lambda_2 = 0(mod 2\pi).\end{array}\right.$$

Proof. Using representation of the spectral density and the semi-invariant spectral density of the fourth order through the covariance function and the cumulant of the fourth order it is obtained that

$$EI^T(\lambda) = \frac{1}{2\pi T}\left[\sum_{t=0}^{T-1}\frac{EX^2(t)Ed^2(t)}{\alpha^2+\alpha}\right.$$

$$\left.+\sum_{t=0}^{T-1}\sum_{s=0,s\neq t}^{T-1}\frac{EX(t)X(s)Ed(t)d(s)}{\alpha^2}e^{-i\lambda(t-s)}\right]$$

$$= \frac{1}{2\pi T}\int_\Pi f^X(z)\sum_{t=0}^{T-1}\sum_{s=0}^{T-1}e^{i(t-s)(z-\lambda)}dz = \int_\Pi f^X(z+\lambda)\Phi_T(z)dz,$$

where $\Phi_T(z) = \frac{1}{2\pi T}sin^2\frac{Tz}{2}sin^{-2}\frac{z}{2}$ is the Fejer kernel (Anderson (1971)). Taking into account continuity of the spectral density and properties of the Fejer kernel the asymptotic unbiasedness is proved.

Using the definition of covariance and independence between $X(t)$ and $d(t)$, the following expression is obtained:

$$cov\left\{I^T(\lambda_1), I^T(\lambda_2)\right\} = EI^T(\lambda_1)\overline{I^T(\lambda_2)} - EI^T(\lambda_1)E\overline{I^T(\lambda_2)}$$

$$= \frac{1}{(2\pi T)^2}\sum_{t,s,j,k=0}^{T-1}e^{-i\lambda_1(t-s)+i\lambda_2(j-k)}$$

$$\times\left[\frac{E(X(t)X(s)X(j)X(k))E(d(t)d(s)d(j)d(k))}{C_{t-s}^dC_{j-k}^d}\right.$$

$$\left.-E(X(t)X(s))E(X(j)X(k))\right].$$

It is easy to show that

$$E(d(t)d(s)d(j)d(k)) = \begin{cases} \alpha^4+6\alpha^3+7\alpha^2+\alpha, t=s=j=k, \\ \alpha^4, t\neq s\neq j\neq k, \\ (\alpha+\alpha^2)\alpha^2, t=s\neq j\neq k, \\ (\alpha+\alpha^2)^2, t=s\neq j=k, \\ \alpha(\alpha^3+3\alpha^2+\alpha), t\neq s=j=k. \end{cases}$$

Using relations between moments and cumulants, the representation of the spectral density and the semi-invariant spectral density of the fourth order through the covariance function and the cumulant of the fourth order and the kernel function $\Phi_T(y_1,y_2,y_3)$ (Bentkus (1972)), it is possible to write that

$$cov\left\{I^T(\lambda_1), I^T(\lambda_2)\right\}$$

$$= \frac{2\pi}{T}\iiint\limits_{\Pi^3} f_4^X(y_1+\lambda_1, y_2-\lambda_1, y_3-\lambda_2)\Phi_T(y_1,y_2,y_3)dy_1dy_2dy_3$$

$$+\int\limits_{\Pi} f^X(x_1)\Phi_T(x_1-\lambda_1,x_1-\lambda_2)dx_1\int\limits_{\Pi} f^X(x_2)\Phi_T(x_2+\lambda_1,x_2+\lambda_2)dx_2$$

$$+\int\limits_{\Pi} f^X(x_1)\Phi_T(x_1-\lambda_1,x_1+\lambda_2)dx_1\int\limits_{\Pi} f^X(x_2)\Phi_T(x_2+\lambda_1,x_2-\lambda_2)dx_2$$

$$+\frac{1}{(2\pi)^2T}\left(\frac{\alpha^4+6\alpha^3+7\alpha^2+\alpha}{(\alpha^2+\alpha)^2}-1\right)$$

$$\times\left(\iiint\limits_{\Pi^3} f_4^X(x_1,x_2,x_3)dx_1dx_2dx_3+3\Big(\int\limits_{\Pi} f^X(x_1)dx_1\Big)^2\right)$$

$$+\frac{1}{\pi T}\left(\frac{\alpha^3+3\alpha^2+\alpha}{(\alpha^2+\alpha)\alpha^2}-1\right)\left(3\int\limits_{\Pi} f^X(x_1)dx_1\int\limits_{\Pi} f^X(x_2)\Phi_T(x_2+\lambda_2)dx_2\right.$$

$$\left.+\iiint\limits_{\Pi^3} f_4^X(x_1,x_2,x_3)\Phi_T(x_1+x_2+x_3+\lambda_2)dx_1dx_2dx_3\right)$$

$$+\frac{1}{\pi T}\left(\frac{\alpha^3+3\alpha^2+\alpha}{(\alpha^2+\alpha)\alpha^2}-1)\right)\left(3\int\limits_{\Pi} f^X(x_1)dx_1\int\limits_{\Pi} f^X(x_2)\Phi_T(x_2-\lambda_1)dx_2\right.$$

$$\left.+\iiint\limits_{\Pi^3} f_4^X(x_1,x_2,x_3)\Phi_T(x_1+x_2+x_3-\lambda_1)dx_1dx_2dx_3\right)$$

$$+\frac{3}{T}\left(\frac{(\alpha^2+\alpha)\alpha^2}{\alpha^4}-1\right)\left(\int\limits_{\Pi} f^X(x_1)dx_1\int\limits_{\Pi} f^X(x_2)\Phi_T(x_2+\lambda_1;-x_2-\lambda_2)dx_2\right.$$

$$+\iiint\limits_{\Pi^3} f_4^X(x_1,x_2,x_3)\Phi_T(x_1+x_2-\lambda_1+\lambda_2;x_3+\lambda_1)dx_1dx_2dx_3$$

$$\left.+2\iint\limits_{\Pi^2} f^X(x_1)f^X(x_2)\Phi_T(x_1-\lambda_1+\lambda_2;-x_1+x_2+\lambda_1)dx_1dx_2\right).$$

Taking into account continuity of the spectral density $f^X(\lambda)$ on Π, continuity of the semi-invariant spectral density $f_4^X(\lambda_1,\lambda_2,\lambda_3)$ of the fourth order on Π^3 and properties of the kernel function $\Phi_T(y_1,y_2,y_3)$ the statement of the theorem is obtained. □

To get a consistent estimator of the spectral density of the process $X(t)$ it is necessary to smooth this estimator by using spectral windows $\varphi^T(k)$:

$$\widehat{f}^T(\lambda_s)=\sum_{k=-\left[\frac{T}{2}\right]+1}^{\left[\frac{T}{2}\right]}\varphi^T(k)I^T(\lambda_{s+k}), \tag{6}$$

$\lambda_s=\frac{2\pi s}{T}$, $-\left[\frac{T}{2}\right]+1\leq s\leq\left[\frac{T}{2}\right]$, $\left[\frac{T}{2}\right]$ is the integer part of number $\frac{T}{2}$.

Theorem 3. *If the semi-invariant spectral density of the fourth order $f_4^X(\lambda_1,\lambda_2,\lambda_3)$ is continuous on Π^3, the spectral density $f^X(\lambda)$ is continuous on Π and*

$$\sum_{k=-\left[\frac{T}{2}\right]+1}^{\left[\frac{T}{2}\right]}\left[\varphi^T(k)\right]^2\underset{T\to\infty}{\longrightarrow}0,$$

then the statistic defined as (6) is a mean-square consistent estimator.

The proof is based on the previous theorem.

Let us consider now the case when irregularities in observations $d(t)$ are defined as a stationary random process. This process has mean $m^d\neq 0$, the covariance functions $R^d(\tau),\tau\in Z$, the spectral density $f^d(\lambda),\lambda\in\Pi=[-\pi,\pi]$, the semi-invariant spectral density of the fourth order $f_4^d(\lambda_1,\lambda_2,\lambda_3)$, $\lambda_i\in\Pi$, $i=1,2,3$, the 4-th moment $m_4^d(t_1,t_2,t_3)$, $t_i\in Z$, $i=1,2,3$, and the cumulant of the fourth order $c_4^d(\tau_1,\tau_2,\tau_3)$, $\tau_i\in Z$, $i=1,2,3$.

Using observations (2) of the process $Y(t)$ the estimator of the mean of the process $X(t)$ may be constructed as a statistic defined by

$$\widehat{m}^X=\frac{1}{Tm^d}\sum_{t=0}^{T-1}Y(t). \tag{7}$$

For this estimator the theorem given below holds (Troush, Iliukevich (2003)).

Theorem 4. *In the assumptions above, the statistic given in (7) is an asymptotically unbiased estimator. The limiting variance of the estimator, given in (7), is*

$$\lim_{T\to\infty} T\,Var\,\widehat{m}^X(\tau) = 2\pi\left[\frac{1}{(m^d)^2}\int\limits_{\Pi} f^X(\mu)f^d(\mu)d\mu\right.$$
$$\left. + f^X(0) + \left(\frac{m^X}{m^d}\right)^2 f^d(0)\right]$$

if the spectral density of the process $X(t)$ is bounded in Π and continuous at $\lambda = 0$ and the spectral density of the process $d(t)$ is continuous at $\lambda = 0$.

Note that from the above theorem it follows that the statistic (7) is mean-square consistent.

The statistic

$$\widehat{R}^X(\tau) = \frac{1}{(T-\tau)(R^d(\tau)+(m^d)^2)}\sum_{t=0}^{T-\tau-1} Y(t+\tau)Y(t), \text{for } \tau = 0, 1, ..., T-1,$$
$$\widehat{R}^X(\tau) = \widehat{R}^X(-\tau), \widehat{R}^X(\tau) = 0, \text{for } |\tau| > T, \tag{8}$$

is considered as an estimator of the covariance function of the process $X(t)$ (Iliukevich (2005)).

Theorem 5. *The estimator of the covariance function of the process $X(t)$ given in (8) is asymptotically unbiased. If $\sum\limits_{u=-\infty}^{\infty}\left(R^X(u)\right)^2 < \infty$, $\sum\limits_{u=-\infty}^{\infty} c_4^X(u+\tau, u, \tau) < \infty$ and*

$$\frac{(R^d(u))^2}{(T-\tau)^{(1/2)}\left(R^d(\tau)+(m^d)^2\right)^2} \underset{T\to\infty}{\longrightarrow} 0,$$
$$\frac{c_4^d(u+\tau, u, \tau)}{(T-\tau)^{(1/2)}\left(R^d(\tau)+(m^d)^2\right)^2} \underset{T\to\infty}{\longrightarrow} 0,$$

$u = -(T-\tau-1), -(T-\tau-2), ..., (T-\tau-1)$ and $\tau = 0, 1, ..., T-1$, the estimator is mean-square consistent.

Proof. The unbiasedness of the estimator is obvious. Further it is supposed that the means of processes equal to zero. We assume that to simplify the calculations. The variance of the estimator (7) is

$$Var\,\widehat{R}^X(\tau) = \frac{1}{(T-\tau)^2\left(R^d(\tau)\right)^2}\left[\sum_{t_1=0}^{T-\tau-1}\sum_{t_2=0}^{T-\tau-1} m_4^X(t_1+\tau, t_1, t_2+\tau, \tau)\right.$$

$$\times m_4^d(t_1+\tau, t_1, t_2+\tau, \tau) - (T-\tau)^2 \left(R^d(\tau)\right)^2 \left(R^X(\tau)\right)^2\Big].$$

Using stationarity of the processes $X(t)$ and $d(t)$, known relation between moments and cumulants of the fourth order and changing variables $t = t_1$, $u = t_1 - t_2$, it is possible to write the expression for variance as

$$Var\,\widehat{R}^X(\tau) = \frac{1}{(T-\tau)^2}\left[\sum_{u=-(T-\tau-1)}^{-1} \sum_{t=0}^{u+T-\tau-1} J(u,\tau) + \sum_{t=0}^{T-\tau-1} J(0,\tau)\right.$$

$$\left. + \sum_{u=1}^{T-\tau-1} \sum_{t=u}^{T-\tau-1-u} J(u,\tau)\right],$$

where

$$J(u,\tau) = \frac{c_4^X(u+\tau,u,\tau)c_4^d(u+\tau,u,\tau)}{\left(R^d(\tau)\right)^2} + \frac{(R^X(\tau))^2 c_4^d(u+\tau,u,\tau)}{\left(R^d(\tau)\right)^2}$$

$$+\frac{(R^X(u))^2 c_4^d(u+\tau,u,\tau)}{\left(R^d(\tau)\right)^2} + \frac{R^X(u+\tau)R^X(u-\tau)c_4^d(u+\tau,u,\tau)}{\left(R^d(\tau)\right)^2}$$

$$+c_4^X(u+\tau,u,\tau) + (R^X(u))^2 + R^X(u+\tau)R^X(u-\tau)$$

$$+\frac{c_4^X(u+\tau,u,\tau)\left(R^d(u)\right)^2}{\left(R^d(\tau)\right)^2} + \frac{(R^X(\tau))^2\left(R^d(u)\right)^2}{\left(R^d(\tau)\right)^2}$$

$$+\frac{(R^X(u))^2\left(R^d(u)\right)^2}{\left(R^d(\tau)\right)^2} + \frac{R^X(u+\tau)R^X(u-\tau)\left(R^d(u)\right)^2}{\left(R^d(\tau)\right)^2}$$

$$+\frac{c_4^X(u+\tau,u,\tau)R^d(u+\tau)R^d(u-\tau)}{\left(R^d(\tau)\right)^2} + \frac{(R^X(\tau))^2 R^d(u+\tau)R^d(u-\tau)}{\left(R^d(\tau)\right)^2}$$

$$+\frac{(R^X(u))^2 R^d(u+\tau)R^d(u-\tau)}{\left(R^d(\tau)\right)^2} + \frac{R^X(u+\tau)R^X(u-\tau)R^d(u+\tau)R^d(u-\tau)}{\left(R^d(\tau)\right)^2}.$$

Thus

$$Var\,\widehat{R}^X(\tau) = \frac{1}{(T-\tau)} \sum_{u=-(T-\tau-1)}^{T-\tau-1} J(u,\tau) + \frac{1}{(T-\tau)} \sum_{u=-(T-\tau-1)}^{-1} \frac{u}{T-\tau} J(u,\tau)$$

$$+\frac{1}{(T-\tau)} \sum_{u=1}^{T-\tau-1} \frac{u}{T-\tau} J(u,\tau).$$

If the assumptions of the theorem hold, then the variance tends to zero when T tends to infinity. $\square$

References

Anderson, T. W. (1971), *The Statistical Analysis of Time Series*, John Wiley & Sons, Inc., New York–London–Sydney.

Bentkus, R. Yu. (1972), *Asymptotic normality of the estimate of a spectral function*, Lithuanian Math. J. **12**, No. 3, 5–18.

Brillinger, D. R. (1975), *Time Series: Data Analysis and Theory*, Holt, Rinehart and Winston, Inc., New York–Montreal, Que.–London.

Iliukevich (Varatnitskaya), T. I. (2005), *Investigation of the estimation of covariance function of stationary random process with non-regular observations*; In: Proceedings of the International Conference on Probability Theory, Random Processes, Mathematical Statistics and Applications, Minsk, February 21-25, pp. 103–108. (Russian)

Jiang, J. and Hui, Y. V. (2004), *Spectral density estimation with amplitude modulation and outlier detection*, Ann. Inst. Statist. Math. **56**, 611–630.

Leonov, V. P. and Širjaev, A. N.(1959), *Sur le calcul des semi-invariants*, Teor. Veroyatnost. i Primenen. **4**, 342–355. (Russian)

Lee, M. S. (2004), *Strong consistency for AR model with missing data*, J. Korean Math. Soc. **41**, , 1071–1086.

Parzen, E. (1963), *On spectral analysis with missing observations and amplitude modulation*, Sankhyā Ser. A **25**, 383–392.

Troush, N. N. and Iliukevich (Varatnitskaya), T. I. (2003), *The estimation of mathematical expectation of stationary random processes with non-regular observations*, Vestnik BSU **3**, 49–52. (Russian)

Žurbenko, I. G. (1987), *Analysis of Stationary and Homogeneous Random Systems*, MSU, Moscow. (Russian)

Belarus State University, Minsk, Belarus
E-mail address: `iliukevich@bsu.by`