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Preliminary test almost unbiased ridge estimator in a linear regression model with multivariate Student-t errors

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Abstract. In this paper, the preliminary test almost unbiased ridge estimators of the regression coefficients based on the conflicting Wald (W), Likelihood ratio (LR) and Lagrangian multiplier (LM) tests in a multiple regression model with multivariate Student-t errors are introduced when it is suspected that the regression coefficients may be restricted to a subspace. The bias and quadratic risks of the proposed estimators are derived and compared. Sufficient conditions on the departure parameter Δ and the ridge parameter k are derived for the proposed estimators to be superior to the almost unbiased ridge estimator, restricted almost unbiased ridge estimator and preliminary test estimator. Furthermore, some graphical results are provided to illustrate theoretical results.

1. Introduction

It is known that the famous ordinary least square estimator (OLSE) is the best linear unbiased estimator of the vector of unknown regression coefficients in a linear regression model. However, the OLSE will not be a good estimator under a multicollinearity situation by the mean squares error (MSE) criterion and many improved biased estimators have been proposed in the literature when the criterion of unbiasedness can be dropped, such as the principal component regression estimator (PCRE) by Massy (1965), the ordinary ridge regression estimator (ORE) by Hoerl and Kennard (1970), the r-k class estimator by Baye and Parker (1984), the Liu estimator by Liu (1993), the r-d class estimator by Kaçıranlar and Sakallıoğlu (2001) and the Liu-type estimator by Liu (2003, 2004). Singh et al. (1986) introduced the

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almost unbiased generalized ridge estimator (AUGRE) using the jack-knife procedure. As a special case of the AUGRE, Akdeniz and Erol (2003) argued that the almost unbiased ridge estimator could be regarded as a bias corrected estimator of the ORE.

As an alternative accepted technique to deal with the multicollinearity problem, many researchers have also considered the cases when some exact or stochastic linear restrictions on the unknown parameters are assumed to hold, such as the restricted ridge estimator by Sarkar (1992), the restricted Liu estimator by Kaçıranlar et al. (1999), the stochastic restricted ridge estimator by Özkale (2009), the stochastic restricted Liu estimators by Hubert and Wijekoon (2006) and Yang and Xu (2009). More recently, Yang et al. (2009) considered mean squared error matrix performances of some restricted almost unbiased estimators in linear regression.

However, when the restrictions considered are suspected, one may combine the unrestricted estimators and restricted estimators to obtain new estimators with better performances, which leads to the preliminary test estimator (PTE). The preliminary test approach estimation was firstly proposed by Bancroft (1944) and then has been studied by many researchers, such as Judge and Bock (1978), Ahmed (1992), Saleh and Kibria (1993), Billah and Saleh (1998), Ahmed and Rahbar (2000), Kim and Saleh (2003), Kibria (2004). In much theoretical research work, the error terms in linear models are assumed to be normally and independently distributed. The question how the performances of the PTE change under non-normally distributed disturbances has received attention in the literature in various contexts. Kibria and Saleh (2004) introduced the preliminary test ridge estimator and Arashi and Tabatabaey (2008) considered the Stein-type estimators in linear models with multivariate Student-t errors.

In this paper, we deal with the estimation of the regression coefficients in a multiple regression model with multivariate Student-t errors and the preliminary test almost unbiased ridge estimators based on the W, LR and LM tests are introduced by combining the preliminary test approach and the almost unbiased ridge estimator. The proposed estimators are compared with some other competitors in the literature, such as the almost unbiased ridge estimator (AURE), the restricted almost unbiased ridge estimator (RAURE) and the PTE. The rest of the paper is organized as follows. In Section 2, model specifications are given and various estimators are derived. Then, the bias and risk expressions of the proposed estimators are derived in Section 3. Relative performances of the estimators with respect to the quadratic risk analysis versus the departure parameter Δ and the ridge parameter k are discussed in Section 4 and Section 5, respectively. Finally, some concluding remarks are provided in Section 6.

2. Model specifications and the estimators

Let us consider the following multiple linear regression model:

$$y = X\beta + \varepsilon, \tag{2.1}$$

where y is an $n \times 1$ vector of observations on the study variable, X is an $n \times p$ known design matrix of rank p, β is a $p \times 1$ vector of unknown regression coefficients, $\varepsilon = (\varepsilon_1, \cdots, \varepsilon_n)'$ is the $n \times 1$ error vector. It is assumed that the error vector is distributed according to a law belonging to the class of spherically symmetric distributions (SSD) with $E(\varepsilon) = 0, E(\varepsilon\varepsilon') = \sigma_\varepsilon^2 I_n$, where I_n is the n-dimensional identity matrix and σ_ε^2 is the common variance of $\varepsilon_i, i = 1, \cdots, n$. Then, the probability density function (*pdf*) of ε can be written as

$$f(\varepsilon) = \int_0^\infty f(\varepsilon|\tau) g(\tau) d\tau, \tag{2.2}$$

where the function $f(\varepsilon|\tau) = (2\pi\tau^2)^{-n/2} e^{-\frac{\varepsilon'\varepsilon}{2\tau^2}}$ is the *pdf* of the normal distribution $N_n(0, \tau^2 I_n)$ and $g(\tau)$ is the *pdf* of τ with support $[0, \infty)$.

Following Giles (1991) and Kibria and Saleh (2004), we can get the multivariate Student-t distribution if $g(\tau)$ is assumed to be the inverted gamma density with scale parameter σ^2 and degrees of freedom v, namely, $g(\tau) = \frac{2}{\Gamma(v/2)} (\frac{v\sigma^2}{2})^{v/2} \frac{1}{\tau^{v+1}} e^{-\frac{v\sigma^2}{2\tau^2}}$, $0 < \tau, v, \sigma < \infty$. Then, we can get that

$$\begin{aligned} f(\varepsilon) &= \int_0^\infty (2\pi\tau^2)^{-n/2} \frac{2e^{-\frac{\varepsilon'\varepsilon}{2\tau^2}}}{\Gamma(\frac{v}{2})\tau^{v+1}} (\frac{v\sigma^2}{2})^{v/2} e^{-\frac{v\sigma^2}{2\tau^2}} d\tau \\ &= \frac{\Gamma(\frac{n+v}{2})}{\Gamma(\frac{v}{2})(\pi v\sigma^2)^{\frac{n}{2}}} (1 + \frac{\varepsilon'\varepsilon}{v\sigma^2})^{-\frac{n+v}{2}}, \end{aligned} \tag{2.3}$$

where $0 < v, \sigma < \infty, -\infty < \varepsilon_i < \infty$. Note that $E(\varepsilon\varepsilon') = E[E(\varepsilon\varepsilon'|\tau)] = E(\tau^2 I) = \sigma_\varepsilon^2 I$ and $E(\varepsilon) = 0$, where $\sigma_\varepsilon^2 = \frac{v}{v-2}\sigma^2, v > 2$. We can verify that for $v = 1$, the *pdf* (2.3) becomes the *pdf* of the Cauchy distribution and as $v \to \infty$, the *pdf* approaches the *pdf* of the normal distribution.

In this paper, we mainly consider the estimation of β when it is suspected that β may be restricted to the following subspace

$$R\beta = r, \tag{2.4}$$

where R is a $J \times p$ known matrix of rank $J (J < p)$ and r is a $J \times 1$ known vector. For the unrestricted model specified by Equation (2.1), the ordinary ridge regression estimator (ORE) by Hoerl and Kennard (1970) and almost unbiased ridge regression estimator (AURE) by Singh et al. (1986) are defined as

$$\hat{\beta}_{ORE}(k) = S_k^{-1} X' y = T_k \hat{\beta}_{OLSE}, \tag{2.5}$$

$$\hat{\beta}_{AURE}(k) = (I - k^2 S_k^{-2})\hat{\beta}_{OLSE} = A_k \hat{\beta}_{OLSE}, \tag{2.6}$$

where $k > 0, S = X'X, S_k = S + kI, T_k = S_k^{-1}S, A_k = I - k^2 S_k^{-2}$ and $\hat{\beta}_{OLSE} = S^{-1}X'y$ is the OLSE of β. The restricted ridge estimator (RRE) by Sarkar (1992) is given by

$$\hat{\beta}_{RRE}(k) = T_k \hat{\beta}_{RLSE}, \tag{2.7}$$

where $\hat{\beta}_{RLSE} = \hat{\beta}_{OLSE} - S^{-1}R'(RS^{-1}R')^{-1}(R\hat{\beta}_{OLSE} - r)$.

Following the idea of Sarkar (1992), we can firstly structure the following restricted almost unbiased ridge estimator (RAURE):

$$\hat{\beta}_{RAURE}(k) = A_k \hat{\beta}_{RLSE}. \tag{2.8}$$

When the validity of the restrictions $R\beta = r$ is suspected, we are to introduce the preliminary test almost unbiased ridge estimators based on the well-known W, LR and LM tests. The three test statistics for testing the null hypothesis $H_0 : \delta = R\beta - r = 0$ against the alternative hypothesis $H_1 : \delta \neq 0$ are firstly derived by Ullah and Zinde-Walsh (1984) and given by

$$\begin{aligned}
\zeta_W &= \tfrac{\lambda(R\hat{\beta}-r)'(RS^{-1}R')^{-1}(R\hat{\beta}-r)}{(y-X\hat{\beta})'(y-X\hat{\beta})/n} = \tfrac{\lambda nJF}{n-p}, \\
\zeta_{LR} &= n \ln \tfrac{(y-X\tilde{\beta})'(y-X\tilde{\beta})}{(y-X\hat{\beta})'(y-X\hat{\beta})} = n\ln(1 + \tfrac{JF}{n-p}), \\
\zeta_{LM} &= \tfrac{(R\hat{\beta}-r)'(RS^{-1}R')^{-1}(R\hat{\beta}-r)}{\lambda(y-X\tilde{\beta})'(y-X\tilde{\beta})/n} = \tfrac{\lambda^{-1}nJF}{n-p+JF},
\end{aligned}$$

where $\lambda = \frac{n+v}{n+v+2}$ and $F = \frac{(R\hat{\beta}-r)'(RS^{-1}R')^{-1}(R\hat{\beta}-r)/J}{(y-X\hat{\beta})'(y-X\hat{\beta})/(n-p)}$ is just the familiar F-statistics for testing $H_0 : \delta = 0$ under the normal theory. When the errors follow multivariate Student-t distribution with v degrees of freedom, Giles (1991) proved that the F-statistic has the following *pdf*:

$$g_{J,n-p}(F;\Delta,v) = \sum_{i=0}^{\infty} \frac{(\frac{J}{n-p})^{\frac{J}{2}+i}(\frac{\Delta}{v-2})^i(1+\frac{\Delta}{v-2})^{-(\frac{v}{2}+i)}}{B(\frac{v}{2}-1, i+1)B(\frac{n-p}{2}, \frac{J}{2}+i)} \frac{2(2-v)^{-1}F^{\frac{J}{2}+i-1}}{(1+\frac{J}{n-p}F)^{\frac{J+n-p}{2}+i}},$$

where $\Delta = \delta'(RS^{-1}R')^{-1}\delta/\sigma_\varepsilon^2$ and $B(\cdot;\cdot)$ is the usual Beta function.

From Kibria and Saleh (2004) and Yang and Xu (2009), we know that under the null hypothesis H_0, the three test statistics ζ_W, ζ_{LR} and ζ_{LM} have the same asymptotic central chi-square distribution with J degrees of freedom. Thus, the critical value for an α level test of H_0 is approximated by the central chi-square critical value $\chi_J^2(\alpha)$. Now, by combining the AURE by Singh et al. (1986) and the RAURE given by (2.8), we are to consider the following preliminary test almost unbiased ridge estimator (PTAURE) based on the W, LR and LM tests:

$$\begin{aligned}
\hat{\beta}_{PTAURE}(k,\zeta_*) &= A_k \hat{\beta}_{PTE}(\zeta_*) \\
&= \hat{\beta}_{RAURE}(k) I_{[0,\chi_J^2(\alpha))}(\zeta_*) + \hat{\beta}_{AURE}(k) I_{[\chi_J^2(\alpha),\infty)}(\zeta_*),
\end{aligned}$$

where $\hat{\beta}_{PTE}(\zeta_*) = \hat{\beta}_{RLSE} I_{[0,\chi^2_J(\alpha))}(\zeta_*) + \hat{\beta}_{OLSE} I_{[\chi^2_J(\alpha),\infty)}(\zeta_*)$ is the PTE by Billah and Saleh (1998), ζ_* stands for either of ζ_W, ζ_{LR} or ζ_{LM}. Our object in this paper is to examine the performances of the proposed estimators based on the three large sample tests.

3. Bias and quadratic risks of the estimators

In this section, we will derive the expressions for the bias and quadratic risks of the proposed estimators. It is known that for some regular weight matrix K, the expectation, bias and weighted quadratic risks of the OLSE and RLSE are given by

$$\begin{aligned}
&E(\hat{\beta}_{OLSE}) = \beta,\\
&E(\hat{\beta}_{RLSE}) = \beta - S^{-1}R'(RS^{-1}R')^{-1}\delta,\\
&Bias(\hat{\beta}_{OLSE}) = 0,\\
&Bias(\hat{\beta}_{RLSE}) = -S^{-1}R'(RS^{-1}R')^{-1}\delta = -\eta,\\
&Risk(\hat{\beta}_{OLSE}, K) = \sigma^2_\varepsilon tr(S^{-1}K),\\
&Risk(\hat{\beta}_{RLSE}, K) = \sigma^2_\varepsilon tr(S^{-1}K - AK) + \eta' K\eta,
\end{aligned}$$

where $A = S^{-1}R'(RS^{-1}R')^{-1}RS^{-1}, \eta = S^{-1}R'(RS^{-1}R')^{-1}\delta$. Therefore, we can calculate the expectation, bias and quadratic risks of the AURE and RAURE as

$$E(\hat{\beta}_{AURE}(k)) = A_k E(\hat{\beta}_{OLSE}) = A_k\beta, \tag{3.1}$$

$$E(\hat{\beta}_{RAURE}(k)) = A_k E(\hat{\beta}_{RLSE}) = A_k(\beta - \eta), \tag{3.2}$$

$$Bias(\hat{\beta}_{AURE}(k)) = (A_k - I)\beta = -k^2 S_k^{-2}\beta, \tag{3.3}$$

$$Bias(\hat{\beta}_{RAURE}(k)) = (A_k - I)\beta - A_k\eta = -k^2 S_k^{-2}\beta - A_k\eta, \tag{3.4}$$

$$\begin{aligned}
Risk(\hat{\beta}_{AURE}(k)) &= Risk(\hat{\beta}_{OLSE}, A_k^2) + \beta'(A_k - I)^2\beta\\
&\quad +2\beta'(A_k - I)A_k \cdot Bias(\hat{\beta}_{OLSE})\\
&= \sigma^2_\varepsilon tr(S^{-1}A_k^2) + \beta'(A_k - I)^2\beta,
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
Risk(\hat{\beta}_{RAURE}(k)) &= Risk(\hat{\beta}_{RLSE}, A_k^2) + \beta'(A_k - I)^2\beta\\
&\quad +2\beta'(A_k - I)A_k \cdot Bias(\hat{\beta}_{RLSE})\\
&= \sigma^2_\varepsilon tr(S^{-1}A_k^2) - \sigma^2_\varepsilon tr(AA_k^2) + \beta'(A_k - I)^2\beta\\
&\quad +\eta' A_k^2\eta - 2\beta'(A_k - I)A_k\eta.
\end{aligned} \tag{3.6}$$

Furthermore, according to Billah and Saleh (1998), the expectation, bias and weighted quadratic risk of the PTE based on W, LR and LM tests are given by

$$E(\hat{\beta}_{PTE}(\zeta_*)) = \beta - \eta\tilde{G}_{J+2,n-p,\Delta}(l^*), \tag{3.7}$$

$$Bias(\hat{\beta}_{PTE}(\zeta_*)) = -\eta\tilde{G}_{J+2,n-p,\Delta}(l^*), \tag{3.8}$$

$$Risk(\hat{\beta}_{PTE}(\zeta_*), K) = \sigma^2_\varepsilon tr(S^{-1}K) - \sigma^2_\varepsilon tr(AK)\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)$$

$$+\eta' K\eta[2\tilde{G}_{J+2,n-p,\Delta}(l^*)-\tilde{G}_{J+4,n-p,\Delta}(l^*)], \quad (3.9)$$

where $l^W = \chi^2_J(\alpha)/[n\lambda + \chi^2_J(\alpha)], l^{LR} = 1 - e^{-\chi^2_J(\alpha)/n}, l^{LM} = \lambda\chi^2_J(\alpha)/n$, l^* stands for either of l^W, l^{LR}, l^{LM} and

$$\tilde{G}_{J+2,n-p,\Delta}(l^*) = \sum_{i=0}^{\infty} \frac{\Gamma(\frac{v}{2}+i)}{\Gamma(v/2)\Gamma(i+1)} I_{l^*}(\frac{J+2}{2}+i, \frac{n-p}{2}) \frac{(\frac{\Delta}{v-2})^i}{(1+\frac{\Delta}{v-2})^{\frac{v}{2}+i}},$$

$$\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*) = \sum_{i=0}^{\infty} \frac{\Gamma(\frac{v}{2}+i-1)}{\Gamma(\frac{v}{2}-1)\Gamma(i+1)} I_{l^*}(\frac{J+2}{2}+i, \frac{n-p}{2}) \frac{(\frac{\Delta}{v-2})^i}{(1+\frac{\Delta}{v-2})^{\frac{v}{2}+i-1}},$$

$$\tilde{G}_{J+4,n-p,\Delta}(l^*) = \sum_{i=0}^{\infty} \frac{\Gamma(\frac{v}{2}+i)}{\Gamma(v/2)\Gamma(i+1)} I_{l^*}(\frac{J+4}{2}+i, \frac{n-p}{2}) \frac{(\frac{\Delta}{v-2})^i}{(1+\frac{\Delta}{v-2})^{\frac{v}{2}+i}},$$

where $I(,;,)$ is Pearson's regularized incomplete beta function. Thus, using (3.7), (3.8) and (3.9), we can calculate the expectation, bias and quadratic risk of the PTAURE as

$$\begin{aligned} E(\hat{\beta}_{PTAURE}(k,\zeta_*)) &= A_k E(\hat{\beta}_{PTE}(\zeta_*)) \\ &= A_k\beta - A_k\eta\tilde{G}_{J+2,n-p,\Delta}(l^*), \end{aligned} \quad (3.10)$$

$$Bias(\hat{\beta}_{PTAURE}(k,\zeta_*)) = -k^2S_k^{-2}\beta - (I - k^2S_k^{-2})\eta\tilde{G}_{J+2,n-p,\Delta}(l^*), \quad (3.11)$$

$$\begin{aligned} Risk(\hat{\beta}_{PTAURE}(k,\zeta_*)) &= Risk(\hat{\beta}_{PTE}(\zeta_*), A_k^2) + \beta'(A_k - I)^2\beta \\ &\quad +2\beta'(A_k - I)A_k \cdot Bias(\hat{\beta}_{PTE}(\zeta_*)) \\ &= \sigma_\varepsilon^2 tr(S^{-1}A_k^2) - \sigma_\varepsilon^2 tr(AA_k^2)\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*) \\ &\quad +\eta' A_k^2\eta[2\tilde{G}_{J+2,n-p,\Delta}(l^*) - \tilde{G}_{J+4,n-p,\Delta}(l^*)] \\ &\quad -2\beta'(A_k-I)A_k\eta\tilde{G}_{J+2,n-p,\Delta}(l^*)+\beta'(A_k-I)^2\beta. \end{aligned} \quad (3.12)$$

4. Risk analysis of the estimators as a function of Δ

In this section, we focus on the quadratic risk comparisons among the estimators as a function of the departure parameter Δ.

4.1. Risk comparison between the PTAURE and AURE. In this subsection, we will compare the PTAURE with the AURE under the quadratic risk criterion. We can firstly get from (3.5) and (3.12) that

$$\begin{aligned} &Risk(\hat{\beta}_{AURE}(k)) - Risk(\hat{\beta}_{PTAURE}(k,\zeta_*)) \\ &\quad = \sigma_\varepsilon^2 tr(AA_k^2)\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*) - 2k^2\beta' S_k^{-2}A_k\eta\tilde{G}_{J+2,n-p,\Delta}(l^*) \\ &\quad\quad -\eta' A_k^2\eta[2\tilde{G}_{J+2,n-p,\Delta}(l^*) - \tilde{G}_{J+4,n-p,\Delta}(l^*)]. \end{aligned} \quad (4.1)$$

Note that the risk difference in Equation (4.1) is non-negative if and only if

$$\eta' A_k^2\eta \le \frac{\sigma_\varepsilon^2 tr(AA_k^2)\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*) - 2k^2\beta' S_k^{-2}A_k\eta\tilde{G}_{J+2,n-p,\Delta}(l^*)}{2\tilde{G}_{J+2,n-p,\Delta}(l^*) - \tilde{G}_{J+4,n-p,\Delta}(l^*)}. \quad (4.2)$$

On the other hand, observing that

$$\lambda_p(A_k^2S^{-1})\sigma_\varepsilon^{-2}\eta' S\eta \le \sigma_\varepsilon^{-2}\eta' A_k^2\eta \le \lambda_1(A_k^2S^{-1})\sigma_\varepsilon^{-2}\eta' S\eta, \quad (4.3)$$

where $\lambda_1(A_k^2S^{-1})$ and $\lambda_p(A_k^2S^{-1})$ denote the largest and smallest eigenvalues of the matrix $A_k^2S^{-1}$, respectively, we can get that a sufficient condition for the PTAURE $\hat{\beta}_{PTAURE}(k,\zeta_*)$ to be superior to the AURE $\hat{\beta}_{AURE}(k)$ under the quadratic risk criterion is that $\sigma_\varepsilon^{-2}\eta' S\eta = \Delta \leq \Delta_1^*$, where

$$\Delta_1^* = \frac{tr(AA_k^2)\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*) - 2k^2\sigma_\varepsilon^{-2}\beta' S_k^{-2}A_k\eta\tilde{G}_{J+2,n-p,\Delta}(l^*)}{[2\tilde{G}_{J+2,n-p,\Delta}(l^*) - \tilde{G}_{J+4,n-p,\Delta}(l^*)]\lambda_1(A_k^2S^{-1})}.$$

However, a sufficient condition for $\hat{\beta}_{AURE}(k)$ to be superior to $\hat{\beta}_{PTAURE}(k,\zeta_*)$ under the quadratic risk criterion is that $\sigma_\varepsilon^{-2}\eta' S\eta = \Delta \geq \Delta_2^*$, where

$$\Delta_2^* = \frac{tr(AA_k^2)\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*) - 2k^2\sigma_\varepsilon^{-2}\beta' S_k^{-2}A_k\eta\tilde{G}_{J+2,n-p,\Delta}(l^*)}{[2\tilde{G}_{J+2,n-p,\Delta}(l^*) - \tilde{G}_{J+4,n-p,\Delta}(l^*)]\lambda_p(A_k^2S^{-1})}.$$

Under the null hypothesis, we can get that $\eta = 0$ and the risk difference (4.1) reduces to be $\sigma_\varepsilon^2 tr(AA_k^2)\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)$, which is always positive for all $k > 0$ and $\alpha \in (0,1)$. Thus, the estimator $\hat{\beta}_{PTAURE}(k,\zeta_*)$ performs better than the estimator $\hat{\beta}_{AURE}(k)$ under the null hypothesis.

Based on the above analysis, we may state the following theorem.

Theorem 1. (1) *Under the null hypothesis, the PTAURE $\hat{\beta}_{PTAURE}(k,\zeta_*)$ based on W, LR and LM tests have smaller quadratic risks than the AURE $\hat{\beta}_{AURE}(k)$;*

(2) *Under the alternative hypothesis, when $0 < \Delta \leq \Delta_1^*$, we have*

$$Risk(\hat{\beta}_{AURE}(k)) \geq Risk(\hat{\beta}_{PTAURE}(k,\zeta_*)),$$

while if $\Delta \geq \Delta_2^*$, we have $Risk(\hat{\beta}_{AURE}(k)) \leq Risk(\hat{\beta}_{PTAURE}(k,\zeta_*))$.

4.2. Risk comparison between the PTAURE and RAURE. In this subsection, we will compare the PTAURE with the RAURE according to the quadratic risk criterion. According to (3.6) and (3.12), we have

$$\begin{aligned} &Risk(\hat{\beta}_{RAURE}(k)) - Risk(\hat{\beta}_{PTAURE}(k,\zeta_*)) \\ &\quad = \eta' A_k^2\eta[1 + \tilde{G}_{J+4,n-p,\Delta}(l^*) - 2\tilde{G}_{J+2,n-p,\Delta}(l^*)] \\ &\qquad - \sigma_\varepsilon^2 tr(AA_k^2)[1 - \tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)] \\ &\qquad + 2k^2\beta' S_k^{-2}A_k\eta[1 - \tilde{G}_{J+2,n-p,\Delta}(l^*)]. \end{aligned} \tag{4.4}$$

Thus, we can similarly get that a sufficient condition for $\hat{\beta}_{PTAURE}(k,\zeta_*)$ to be superior to $\hat{\beta}_{RAURE}(k)$ under the quadratic risk criterion is that $\Delta \geq \Delta_3^*$, where

$$\Delta_3^* = \frac{\sigma_\varepsilon^2 tr(AA_k^2)[1 - \tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)] - 2k^2\beta' S_k^{-2}A_k\eta[1 - \tilde{G}_{J+2,n-p,\Delta}(l^*)]}{[1 + \tilde{G}_{J+4,n-p,\Delta}(l^*) - 2\tilde{G}_{J+2,n-p,\Delta}(l^*)]\lambda_p(A_k^2S^{-1})}.$$

Also a sufficient condition for $\hat{\beta}_{RAURE}(k)$ to be superior to $\hat{\beta}_{PTAURE}(k, \zeta_*)$ under the quadratic risk criterion is that $\Delta \leq \Delta_4^*$, where

$$\Delta_4^* = \frac{\sigma_\varepsilon^2 tr(AA_k^2)[1 - \tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)] - 2k^2\beta' S_k^{-2} A_k \eta [1 - \tilde{G}_{J+2,n-p,\Delta}(l^*)]}{[1 + \tilde{G}_{J+4,n-p,\Delta}(l^*) - 2\tilde{G}_{J+2,n-p,\Delta}(l^*)]\lambda_1(A_k^2 S^{-1})}.$$

Observing that under the null hypothesis, the risk difference (4.4) reduces to be $-\sigma_\varepsilon^2 tr(AA_k^2)[1 - \tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)]$, which is always negative for all $k > 0$ and $\alpha \in (0, 1)$. Therefore, $\hat{\beta}_{RAURE}(k)$ performs better than $\hat{\beta}_{PTAURE}(k, \zeta_*)$ under the null hypothesis.

Based on the above analysis, we may state the following theorem.

Theorem 2. (1) *Under the null hypothesis, the RAURE $\hat{\beta}_{RAURE}(k)$ outperforms the PTAURE $\hat{\beta}_{PTAURE}(k, \zeta_*)$ based on the W, LR and LM tests.*

(2) *Under the alternative hypothesis, when $\Delta \geq \Delta_3^*$, we have*

$$Risk(\hat{\beta}_{RAURE}(k)) \geq Risk(\hat{\beta}_{PTAURE}(k, \zeta_*)),$$

while if $0 \leq \Delta \leq \Delta_4^*$, we have $Risk(\hat{\beta}_{RAURE}(k)) \leq Risk(\hat{\beta}_{PTAURE}(k, \zeta_*))$.

4.3. Risk comparison between the PTAURE and PTE. In this subsection, we shall compare the PTAURE with PTE by Billah and Saleh (1998) according to the quadratic risk criterion as a function of Δ. We can firstly get from (3.9) and (3.12) that

$$\begin{aligned} &Risk(\hat{\beta}_{PTE}(\zeta_*)) - Risk(\hat{\beta}_{PTAURE}(k, \zeta_*)) \\ &\quad = \sigma_\varepsilon^2 tr[S^{-1}(I - A_k^2)] - \sigma_\varepsilon^2 tr[A(I - A_k^2)]\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*) \\ &\quad\quad + \eta'(I - A_k^2)\eta[2\tilde{G}_{J+2,n-p,\Delta}(l^*) - \tilde{G}_{J+4,n-p,\Delta}(l^*)] \\ &\quad\quad - 2k^2\beta' S_k^{-2} A_k \eta \tilde{G}_{J+2,n-p,\Delta}(l^*) - k^4 \beta' S_k^{-4}\beta. \end{aligned} \tag{4.5}$$

Note that $I - A_k^2 = (I - A_k)(I + A_k) = k^2 S_k^{-2}(I + A_k) > 0$, we can similarly get that a sufficient condition for $\hat{\beta}_{PTAURE}(k, \zeta_*)$ to be superior to $\hat{\beta}_{PTE}(\zeta_*)$ under the quadratic risk criterion is that $\Delta \geq \Delta_5^*$, where

$$\Delta_5^* = \frac{f(k, l^*)}{[2\tilde{G}_{J+2,n-p,\Delta}(l^*) - \tilde{G}_{J+4,n-p,\Delta}(l^*)]\lambda_p[S^{-1}(I - A_k^2)]},$$

and

$$\begin{aligned} f(k, l^*) = {} & \sigma_\varepsilon^2 tr[A(I - A_k^2)]\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*) + k^4\beta' S_k^{-4}\beta \\ & + 2k^2\beta' S_k^{-2} A_k \eta \tilde{G}_{J+2,n-p,\Delta}(l^*) - \sigma_\varepsilon^2 tr[S^{-1}(I - A_k^2)]. \end{aligned}$$

Also a sufficient condition for $\hat{\beta}_{PTE}(\zeta_*)$ to be superior to $\hat{\beta}_{PTAURE}(k, \zeta_*)$ under the quadratic risk criterion is that $\Delta \leq \Delta_6^*$, where

$$\Delta_6^* = \frac{f(k, l^*)}{[2\tilde{G}_{J+2,n-p,\Delta}(l^*) - \tilde{G}_{J+4,n-p,\Delta}(l^*)]\lambda_1[S^{-1}(I - A_k^2)]}.$$

Based on the above analysis, we may state the following theorem.

Theorem 3. *Under the alternative hypothesis, when* $\Delta \geq \Delta_5^*$*, we have*

$$Risk(\hat{\beta}_{PTE}(\zeta_*)) \geq Risk(\hat{\beta}_{PTAURE}(k,\zeta_*)),$$

while if $0 \leq \Delta \leq \Delta_6^*$*, we have* $Risk(\hat{\beta}_{PTE}(\zeta_*)) \leq Risk(\hat{\beta}_{PTAURE}(k,\zeta_*))$.

5. Risk analysis of the estimators as a function of k

In this section, we focus on the quadratic risk comparisons among the estimators as a function of the ridge parameter k. Let P be the orthogonal matrix such that $P'SP = \Lambda = diag(\lambda_1, \cdots, \lambda_p)$, where $\lambda_1 \geq \cdots \geq \lambda_p$ denote the ordered eigenvalues of the matrix S. Then we can get that $S_k = P(\Lambda + kI)P', A_k = I - k^2 S_k^{-2} = P[I - k^2(\Lambda + kI)^{-2}]P'$ and

$$\begin{aligned}
&tr(S^{-1}A_k^2) = tr\{\Lambda^{-1}[I - k^2(\Lambda + kI)^{-2}]^2\} = \sum_{i=1}^{p} \tfrac{\lambda_i(\lambda_i+2k)^2}{(\lambda_i+k)^4},\\
&tr(AA_k^2) = tr\{AP[I - k^2(\Lambda + kI)^{-2}]^2 P'\} = \sum_{i=1}^{p} \tfrac{\tilde{a}_{ii}\lambda_i^2(\lambda_i+2k)^2}{(\lambda_i+k)^4},\\
&\eta' A_k^2 \eta = \eta' P[I - k^2(\Lambda + kI)^{-2}]^2 P'\eta = \sum_{i=1}^{p} \tfrac{\tilde{\eta}_i^2\lambda_i^2(\lambda_i+2k)^2}{(\lambda_i+k)^4},\\
&\beta'(A_k - I)A_k\eta = \sum_{i=1}^{p} \tfrac{-k^2\gamma_i\tilde{\eta}_i}{(\lambda_i+k)^2}[1 - \tfrac{k^2}{(\lambda_i+k)^2}] = \sum_{i=1}^{p} \tfrac{-k^2\gamma_i\tilde{\eta}_i\lambda_i(\lambda_i+2k)}{(\lambda_i+k)^4},\\
&\beta'(A_k - I)^2\beta = k^4\beta' P(\Lambda + kI)^{-4}P'\beta = \sum_{i=1}^{p} \tfrac{k^4\gamma_i^2}{(\lambda_i+k)^4},
\end{aligned}$$

where $\tilde{A} = P'AP, \tilde{a}_{ii} \geq 0$ is the i-th diagonal element of the matrix $\tilde{A}$, $\gamma = P'\beta = (\gamma_1, \cdots, \gamma_p)'$ and $\tilde{\eta} = P'\eta = (\tilde{\eta}_1, \cdots, \tilde{\eta}_p)'$. Therefore, the risk differences (4.1) and (4.4) can be rewritten as

$$\begin{aligned}
&Risk(\hat{\beta}_{AURE}(k)) - Risk(\hat{\beta}_{PTAURE}(k,\zeta_*))\\
&\quad= -\sum_{i=1}^{p} \frac{\tilde{\eta}_i^2\lambda_i^2(\lambda_i + 2k)^2[2\tilde{G}_{J+2,n-p,\Delta}(l^*) - \tilde{G}_{J+4,n-p,\Delta}(l^*)]}{(\lambda_i + k)^4}\\
&\qquad+\sum_{i=1}^{p} \frac{\sigma_\varepsilon^2\tilde{a}_{ii}\lambda_i^2(\lambda_i + 2k)^2\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)}{(\lambda_i + k)^4}\\
&\qquad-\sum_{i=1}^{p} \frac{2k^2\gamma_i\tilde{\eta}_i\lambda_i(\lambda_i + 2k)\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)}{(\lambda_i + k)^4}\\
&\quad= \sum_{i=1}^{p} \frac{(\lambda_i + 2k)f_i - g_i k^2}{(\lambda_i + 2k)^{-1}\lambda_i^{-1}(\lambda_i + k)^4},
\end{aligned} \tag{5.1}$$

$$\begin{aligned}
&Risk(\hat{\beta}_{RAURE}(k)) - Risk(\hat{\beta}_{PTAURE}(k,\zeta_*))\\
&\quad= \sum_{i=1}^{p} \frac{\tilde{\eta}_i^2\lambda_i^2(\lambda_i + 2k)^2[1 + \tilde{G}_{J+4,n-p,\Delta}(l^*) - 2\tilde{G}_{J+2,n-p,\Delta}(l^*)]}{(\lambda_i + k)^4}
\end{aligned}$$

$$-\sum_{i=1}^{p}\frac{\sigma_\varepsilon^2\tilde{a}_{ii}\lambda_i^2(\lambda_i+2k)^2[1-\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)]}{(\lambda_i+k)^4}$$
$$+\sum_{i=1}^{p}\frac{2k^2\gamma_i\tilde{\eta}_i\lambda_i(\lambda_i+2k)[1-\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)]}{(\lambda_i+k)^4}$$
$$=\sum_{i=1}^{p}\frac{\tilde{g}_ik^2-(\lambda_i+2k)\tilde{f}_i}{(\lambda_i+2k)^{-1}\lambda_i^{-1}(\lambda_i+k)^4}, \tag{5.2}$$

where

$$f_i=\sigma_\varepsilon^2\tilde{a}_{ii}\lambda_i\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)$$
$$-\tilde{\eta}_i^2\lambda_i[2\tilde{G}_{J+2,n-p,\Delta}(l^*)-\tilde{G}_{J+4,n-p,\Delta}(l^*)],$$
$$\tilde{f}_i=\sigma_\varepsilon^2\tilde{a}_{ii}\lambda_i[1-\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)]$$
$$-\tilde{\eta}_i^2\lambda_i[1+\tilde{G}_{J+4,n-p,\Delta}(l^*)-2\tilde{G}_{J+2,n-p,\Delta}(l^*)],$$
$$g_i=2\gamma_i\tilde{\eta}_i\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*),\tilde{g}_i=2\gamma_i\tilde{\eta}_i[1-\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)].$$

It is easy to verify that the risk difference (5.1) is nonnegative if $0\le k\le k_1^*$, where

$$k_1^*=\min_{1\le i\le p}\{\frac{f_i+\sqrt{f_i^2+f_ig_i\lambda_i}}{g_i}\}. \tag{5.3}$$

While we can similarly get that the AURE would have smaller quadratic risk than the PTAURE if $k\ge k_2^*$, where

$$k_2^*=\max_{1\le i\le p}\{\frac{f_i+\sqrt{f_i^2+f_ig_i\lambda_i}}{g_i}\}. \tag{5.4}$$

Thus, we may state the following theorem.

Theorem 4. *The PTAURE dominates the AURE if* $0\le k\le k_1^*$*, while if* $k\ge k_2^*$*, then the AURE would outperform the PTAURE with respect to the quadratic risk criterion.*

Let us further compare the PTAURE with the RAURE. We can get from (5.2) that the PTAURE would outperform the RAURE if $k\ge k_3^*$, where

$$k_3^*=\max_{1\le i\le p}\{\frac{\tilde{f}_i+\sqrt{\tilde{f}_i^2+\tilde{f}_i\tilde{g}_i\lambda_i}}{\tilde{g}_i}\}. \tag{5.5}$$

While the RAURE has smaller quadratic risk than the PTAURE if $0\le k\le k_4^*$, where

$$k_4^*=\min_{1\le i\le p}\{\frac{\tilde{f}_i+\sqrt{\tilde{f}_i^2+\tilde{f}_i\tilde{g}_i\lambda_i}}{\tilde{g}_i}\}. \tag{5.6}$$

Thus, we have the following result.

Theorem 5. *The PTAURE dominates the RAURE if $k \geq k_3^*$, while if $0 \leq k \leq k_4^*$, then the RAURE would outperform the PTAURE with respect to the quadratic risk criterion.*

Finally, let us further compare the PTAURE with the PTE. Note that the risk function (3.12) of the PTAURE can be rewritten as

$$\begin{aligned}
Risk(\hat{\beta}_{PTAURE}(k,\zeta_*)) &= \sum_{i=1}^{p}\frac{\sigma_\varepsilon^2\lambda_i(\lambda_i+2k)^2}{(\lambda_i+k)^4} + \sum_{i=1}^{p}\frac{k^4\gamma_i^2}{(\lambda_i+k)^4} \\
&+\sum_{i=1}^{p}\frac{\tilde{\eta}_i^2\lambda_i^2(\lambda_i+2k)^2[2\tilde{G}_{J+2,n-p,\Delta}(l^*)-\tilde{G}_{J+4,n-p,\Delta}(l^*)]}{(\lambda_i+k)^4} \\
&-\sum_{i=1}^{p}\frac{\sigma_\varepsilon^2\tilde{a}_{ii}\lambda_i^2(\lambda_i+2k)^2\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)}{(\lambda_i+k)^4} \\
&+\sum_{i=1}^{p}\frac{2k^2\gamma_i\tilde{\eta}_i\lambda_i(\lambda_i+2k)\tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)}{(\lambda_i+k)^4} \\
&=\sum_{i=1}^{p}\frac{\gamma_i^2k^4+k^2(\lambda_i+2k)\cdot\lambda_i g_i+(\lambda_i+2k)^2\cdot\lambda_i(\sigma_\varepsilon^2-f_i)}{(\lambda_i+k)^4}. \qquad (5.7)
\end{aligned}$$

Differentiating the risk function of the PTAURE with respect to k, we have

$$\frac{\partial Risk(\hat{\beta}_{PTAURE}(k,\zeta_*))}{\partial k} = \sum_{i=1}^{p}\frac{h_i(k)}{(\lambda_i+k)^5}, \qquad (5.8)$$

where

$$\begin{aligned}
h_i(k) &= [\gamma_i^2k^4+(k^2\lambda_i+2k^3)\cdot\lambda_i g_i+(\lambda_i+2k)^2\cdot\lambda_i(\sigma_\varepsilon^2-f_i)]'(\lambda_i+k) \\
&\quad -4[\gamma_i^2k^4+k^2(\lambda_i+2k)\cdot\lambda_i g_i+(\lambda_i+2k)^2\cdot\lambda_i(\sigma_\varepsilon^2-f_i)] \\
&= 4\gamma_i^2\lambda_i k^3+2k\lambda_i g_i\cdot[(\lambda_i+3k)\lambda_i+(\lambda_i+3k)k-2k(\lambda_i+2k)] \\
&\quad +(\lambda_i+2k)4\lambda_i(\sigma_\varepsilon^2-f_i)\cdot[\lambda_i+k-(\lambda_i+2k)] \\
&= 4\gamma_i^2\lambda_i k^3+2k\lambda_i g_i\cdot(\lambda_i^2+2k\lambda_i-k^2)-4k(\lambda_i+2k)\lambda_i(\sigma_\varepsilon^2-f_i) \\
&= 2k\lambda_i\cdot\{(2\gamma_i^2-g_i)k^2+2[\lambda_i g_i-2(\sigma_\varepsilon^2-f_i)]k+\lambda_i[\lambda_i g_i-2(\sigma_\varepsilon^2-f_i)]\}.
\end{aligned}$$

Therefore, a sufficient condition for (5.8) to be negative is that $0<k<k_5^*$, where

$$k_5^* = \min_{1\leq i\leq p}\{\frac{\sqrt{2q_i[\lambda_i(g_i-\gamma_i^2)-(\sigma_\varepsilon^2-f_i)]}-q_i}{2\gamma_i^2-g_i}\}. \qquad (5.9)$$

and $q_i=\lambda_i g_i-2(\sigma_\varepsilon^2-f_i)$. While we can similarly get that a sufficient condition for (5.8) to be nonnegative is that $k\geq k_6^*$, where

$$k_6^* = \max_{1\leq i\leq p}\{\frac{\sqrt{2q_i[\lambda_i(g_i-\gamma_i^2)-(\sigma_\varepsilon^2-f_i)]}-q_i}{2\gamma_i^2-g_i}\}. \qquad (5.10)$$

Note that under the null hypothesis, we have $f_i = \sigma_\varepsilon^2 \tilde{a}_{ii} \lambda_i \tilde{G}^{(1)}_{J+2,n-p,\Delta}(l^*)$ and $g_i = 0$. Thus, $\frac{\partial Risk(\hat{\beta}_{PTAURE}(k,\zeta_*))}{\partial k} < 0$ if $0 < k < k_7^*$, where

$$k_7^* = \min_{1\le i\le p}\{\frac{(\sigma_\varepsilon^2 - f_i) + \sqrt{(\sigma_\varepsilon^2 - f_i)[\lambda_i\gamma_i^2 + (\sigma_\varepsilon^2 - f_i)]}}{\gamma_i^2}\}. \tag{5.11}$$

On the other hand, we can similarly get that $\frac{\partial Risk(\hat{\beta}_{PTAURE}(k,\zeta_*))}{\partial k} \geq 0$ if $k \geq k_8^*$, where

$$k_8^* = \max_{1\le i\le p}\{\frac{(\sigma_\varepsilon^2 - f_i) + \sqrt{(\sigma_\varepsilon^2 - f_i)[\lambda_i\gamma_i^2 + (\sigma_\varepsilon^2 - f_i)]}}{\gamma_i^2}\}. \tag{5.12}$$

Therefore, we have the following result.

Theorem 6. (1) *Under the null hypothesis, the PTAURE dominates the PTE if* $0 \leq k \leq k_7^*$, *while if* $k \geq k_8^*$, *then the PTE would outperform the PTAURE with respect to the quadratic risk criterion.*

(2) *Under the alternative hypothesis, the PTAURE dominates the PTE if* $0 \leq k \leq k_5^*$, *while if* $k \geq k_6^*$, *then the PTE would outperform the PTAURE with respect to the quadratic risk criterion.*

We can find from the risk analysis in Section 4 and Section 5 that for fixed ridge parameter k, the performances of the PTAURE heavily depend on the departure parameter Δ. While for fixed departure parameter Δ, the performances of the proposed estimators also depend on the ridge parameter k. For an illustrative purpose, we are to give some graphical representation of the relative risks of the estimators following Kibria and Saleh (2004) and Arashi and Tabatabaey (2008). On the one hand, for the case when $n = 20, p = 4, J = 3, v = 5, \alpha = 0.05, 0.10$ and fixed $k = 0.2$, the quadratic risks of the AURE, RAURE, PTE and PTAURE based on the W, LR and LM tests versus the departure parameter Δ are presented in Figure 1 − Figure 3, respectively. On the other hand, for the case when $n = 20, p = 4, J = 3, v = 5, \alpha = 0.05, 0.10$ and fixed $\Delta = 2$, the quadratic risks of the AURE, RAURE, PTE and PTAURE based on the W, LR and LM tests versus the ridge parameter k are presented in Figure 4 − Figure 6, respectively. In order to facilitate the numerical computation of the quadratic risks of the estimators, we consider the orthonormal regression case following Kibria and Saleh (2004) and similar results can also be obtained for other combinations of n, p, J, v and α.

As shown in Figure 1 − Figure 3, we can find that for fixed ridge parameter, the quadratic risk of the RAURE increases quickly and becomes unbounded as Δ increases, while the risk of the PTAURE converges to the risk of the AURE as Δ approaches infinity. When Δ is relatively small and near 0, the risks of the PTAURE and PTE are smaller than that of the AURE, but larger

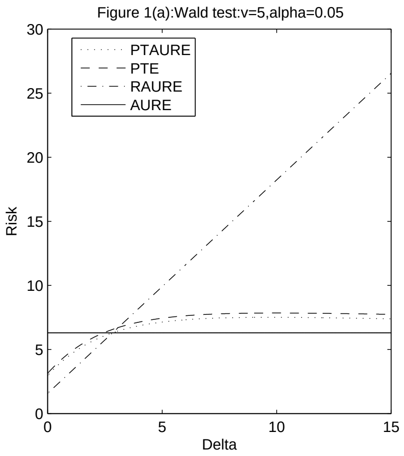

FIGURE 1. Risks of the estimators based on Wald test for fixed $k = 0.2$.

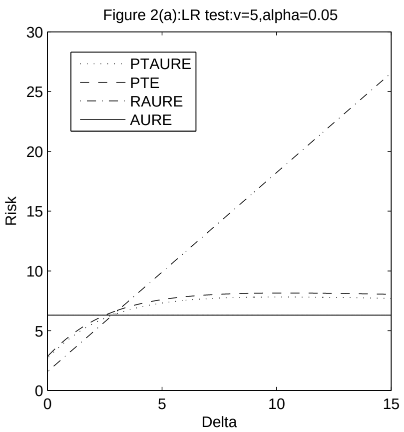

FIGURE 2. Risks of the estimators based on LR test for fixed $k = 0.2$.

than that of the RAURE. However, when Δ becomes relatively large, both the PTAURE and PTE have larger risks than the AURE but smaller risks than the RAURE. Furthermore, the theoretical findings for the case when the departure parameter is fixed are also well supported by the graphical results shown in Figure 4 – Figure 6.

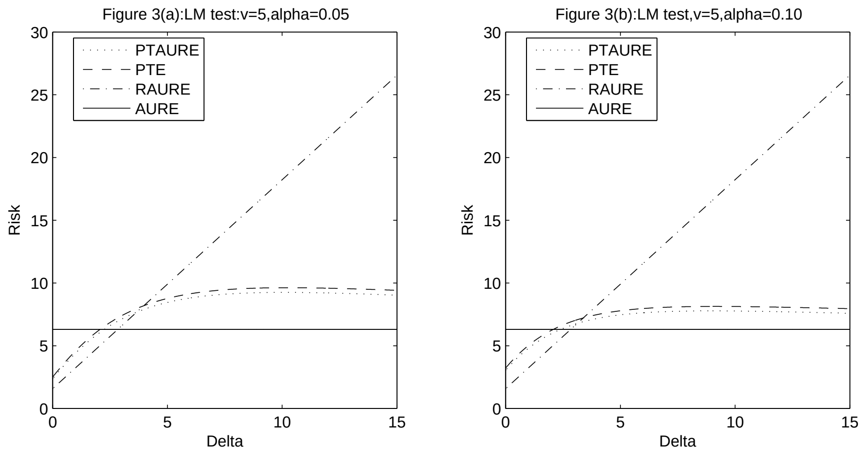

FIGURE 3. Risks of the estimators based on LM test for fixed $k = 0.2$.

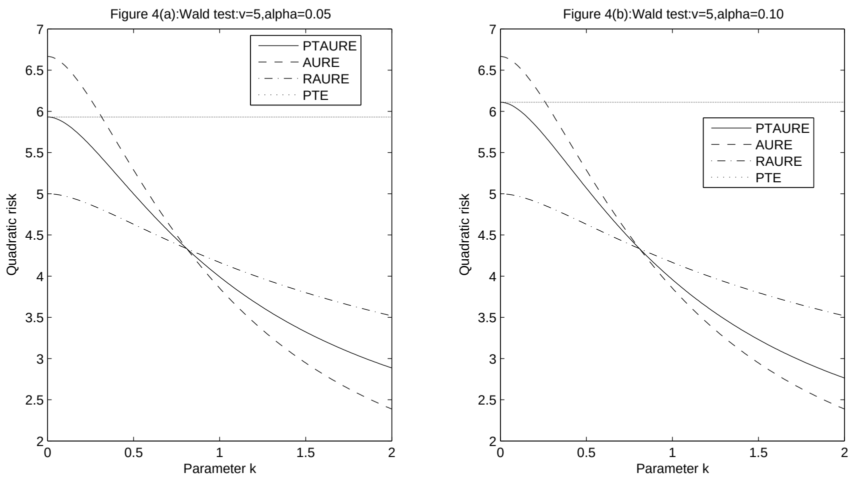

FIGURE 4. Risks of the estimators based on Wald test for fixed $\Delta = 2$.

6. Concluding remarks

In this paper, we studied the performances of the preliminary test almost unbiased estimators based on the W, LR and LM tests when it is suspected that the regression coefficients may be restricted to a subspace and the errors are assumed to follow multivariate Student-t distribution. We have effectively determined some sufficient conditions on the departure parameter Δ and the ridge parameter k for the superiority of the proposed estimators over the competitors in the literature and some graphical results are provided to illustrate the theoretical results.

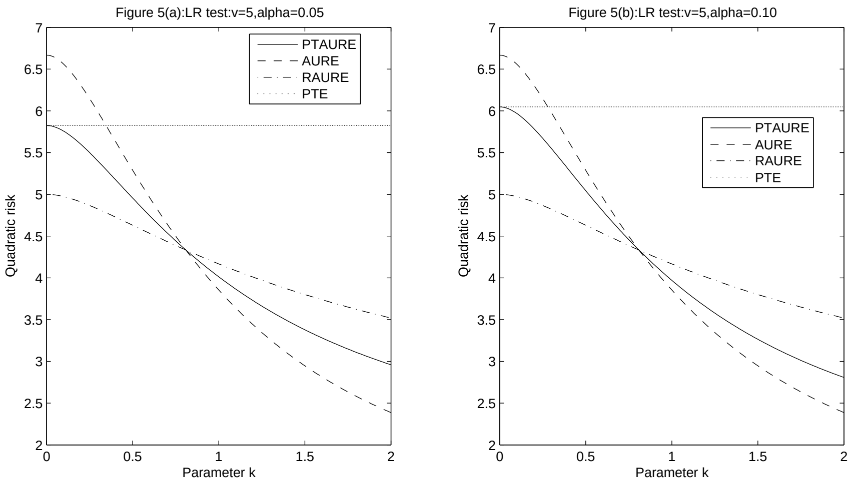

FIGURE 5. Risks of the estimators based on LR test for fixed $\Delta = 2$.

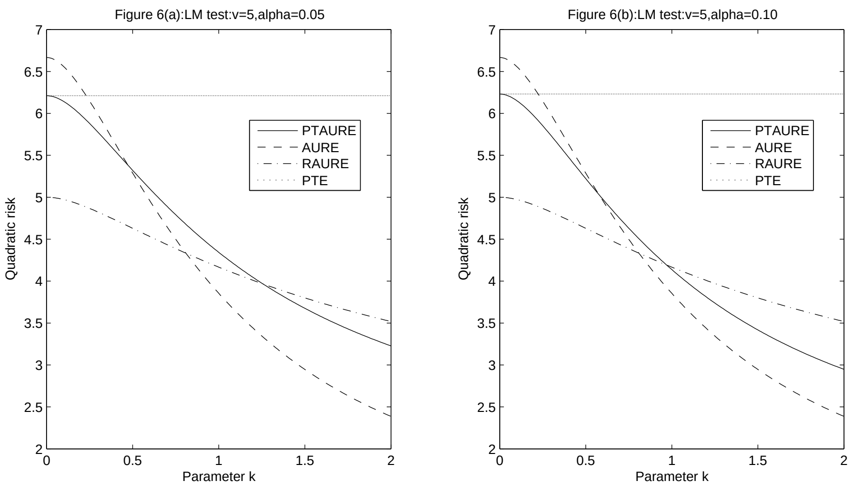

FIGURE 6. Risks of the estimators based on LM test for fixed $\Delta = 2$.

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