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Bilinear generating relations for a family of q-polynomials and generalized basic hypergeometric functions

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Abstract. In this paper, we derive a bilinear q-generating function involving basic analogue of Fox's H-function and a general class of q-hypergeometric polynomials. Applications of the main results are also illustrated.

1. Introduction and preliminaries

For a, $q \in \mathbb{C}$ the q-shifted factorial (see [2]) is defined by

$$(a\,;q)_n = \begin{cases} 1 & ; \quad n=0 \\ (1-a)(1-a\,q)\cdots(1-a\,q^{n-1}) & ; \quad n \in \mathbb{N}\,, \end{cases} \tag{1.1}$$

and its natural extension is

$$(a\,;q)_\alpha = \frac{(a\,;q)_\infty}{(aq^\alpha\,;q)_\infty},\ \ \alpha \in \mathbb{C},\ \ |q|<1. \tag{1.2}$$

The definition (1.1) remains meaningful for $n=\infty$ as a convergent infinite product

$$(a\,;q)_\infty = \prod_{j=0}^{\infty}(1-a\,q^j)\ . \tag{1.3}$$

The q-analogue of the power (binomial) function $(x \pm y)^n$ (cf. Ernst [1]) is given by

$$(x \pm y)^{(n)} \equiv (x \pm y)_n \equiv x^n(\mp y/x;q)_n = x^n \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q q^{k(k-1)/2}\,(\pm y/x)^k\ , \tag{1.4}$$

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where the q-binomial coefficient is defined as

$$\left[\begin{array}{c}\alpha\\ k\end{array}\right]_q = \frac{(q^{-\alpha};q)_k}{(q;q)_k}(-q^{\alpha})^k q^{-k(k-1)/2} \ (k\in\mathbb{N},\alpha\in\mathbb{R}). \tag{1.5}$$

It satisfies

$$\left[\begin{array}{c}n\\ k\end{array}\right]_q = \frac{(q;q)_n}{(q;q)_{n-k}(q;q)_k}. \tag{1.6}$$

For a bounded sequence A_n of real or complex numbers, let $f(x)=\sum\limits_{n=0}^{\infty} A_n x^n$ be a power series in x, (see, for instance, [1, page 502, equation (3.18)]), then we have

$$f\left[(x\pm y)\right] = \sum_{n=0}^{\infty} A_n \ x^n(\mp y/x;q)_n. \tag{1.7}$$

The q-gamma function (cf. [2]) is defined by

$$\Gamma_q(x) = \frac{(q;q)_\infty}{(q^x;q)_\infty}(1-q)^{1-x} \ (x\in\mathbb{C}, x\notin\{0,-1,-2,\cdots\}). \tag{1.8}$$

And it satisfies

$$\Gamma_q(x+1) = \frac{1-q^x}{1-q}\Gamma_q(x). \tag{1.9}$$

In terms of a bounded complex sequence $\{S_{k,q}\}_{k=0}^{\infty}$, the family of general class of basic (or q-) polynomials $f_{n,m}(x;q)$ (cf. Srivastava and Agarwal [9]) is defined as

$$f_{n,m}(x;q) = \sum_{k=0}^{[n/m]} \left[\begin{array}{c}n\\ mk\end{array}\right]_q S_{k,q} \ x^k \ (n\in\mathbb{N}), \tag{1.10}$$

where m is a positive integer.

With the appropriate choice of the sequence $\{S_{k,q}\}_{k=0}^{\infty}$, the q-polynomial family $f_{n,m}(x;q)$ yields a number of known q-polynomials as its special cases. These include, the q-Hermite polynomials, the q-Laguerre polynomials, the q-Jacobi polynomials, the Wall polynomials, the q-Konhauser polynomials and several others.

Following Saxena, Modi and Kalla [8], the basic analogue of the Fox's H-function is defined as

$$H_{P,Q}^{M,N}\left[x;q \,\middle|\, \begin{array}{c}(a,\alpha)\\ (b,\beta)\end{array}\right] = \frac{1}{2\pi i}\int_C \theta(s;q)\ x^s \ d_q s, \tag{1.11}$$

where

$$\theta(s;q) = \frac{\left\{\prod_{j=1}^{M} G(q^{b_j-\beta_j s})\right\} \left\{\prod_{j=1}^{N} G(q^{1-a_j+\alpha_j s})\right\} \pi}{\left\{\prod_{j=M+1}^{Q} G(q^{1-b_j+\beta_j s})\right\} \left\{\prod_{j=N+1}^{P} G(q^{a_j-\alpha_j s})\right\} G(q^{1-s}) \sin \pi s} \tag{1.12}$$

and

$$G(q^a) = \left\{\prod_{n=0}^{\infty}(1-q^{a+n})\right\}^{-1} = \frac{1}{(q^a;q)_\infty}. \tag{1.13}$$

Also $0 \le M \le Q$, $0 \le N \le P$, α_i's and β_j's are all positive integers. The contour C is a line parallel to $\Re(\omega s) = 0$ with indentations if necessary, in such a manner that all the poles of $G(q^{b_j-\beta_j s})$, $1 \le j \le M$ are to the right, and those of $G(q^{1-a_j+\alpha_j s})$, $1 \le j \le N$ to the left of C. For large values of $|s|$, the integral converges if $\Re[s\log(x) - \log\sin\pi s] < 0$ on the contour C, i.e. if $\left|\left\{\arg(x) - w_2 w_1^{-1}\log|x|\right\}\right| < \pi$, where $0<|q|<1$, $\log q = -w = -(w_1+iw_2)$, w_1 and w_2 being real.

Further, if we set $\alpha_i = \beta_j = 1$, $\forall$ i and j in (1.11), we obtain the basic analogue of Meijer's G-function due to Saxena, Modi and Kalla [8]:

$$G_{P,Q}^{M,N}\left[x;q \,\middle|\, \begin{matrix} a_1, a_2, \cdots, a_P \\ b_1, b_2, \cdots, b_Q \end{matrix}\right] = \frac{1}{2\pi i}\int_C \theta'(s;q)\; x^s \; d_q s, \tag{1.14}$$

where

$$\theta'(s;q) = \frac{\left\{\prod_{j=1}^{M} G(q^{b_j-s})\right\} \left\{\prod_{j=1}^{N} G(q^{1-a_j+s})\right\} \pi}{\left\{\prod_{j=M+1}^{Q} G(q^{1-b_j+s})\right\} \left\{\prod_{j=N+1}^{P} G(q^{a_j-s})\right\} G(q^{1-s}) \sin \pi s}. \tag{1.15}$$

A detailed account of Meijer's G-function, Fox's H-function and various functions expressible in terms of Fox's H-function can be found in the research monographs due to Mathai and Saxena [4, 5], Mathai, Saxena and Haubold [6] and Srivastava, Gupta and Goyal [10]. Further, the basic functions of one variable (elementary and hypergeometric) expressible in terms of the functions $G_q(.)$ can be found in the works of Yadav and Purohit [12] and [13].

2. The q-generating relations

In this section, we shall derive certain bilinear q-generating relations involving basic analogue of Fox's H-function and a general class of q-hypergeometric polynomials.

Theorem 1. *Let $\{S_{k,q}\}_{k=0}^{\infty}$ be an arbitrary bounded sequence, let M, N, P, Q be positive integers such that $0 \leq M \leq Q$, $0 \leq N \leq P$, let $h > 0$, and let m be an arbitrary positive integer. Then the following bilinear q-generating relation holds:*

$$\sum_{n=0}^{\infty} f_{n,m}(\rho\ x;q)\ H_{P+1,Q}^{M,N+1}\left[y;q \left| \begin{array}{c} (1-\lambda-n,h),\ (a,\alpha) \\ (b,\beta) \end{array} \right.\right] \frac{t^n}{(q;q)_n}$$

$$= \frac{1}{(1-t)^{(\lambda)}} \sum_{k=0}^{\infty} \frac{S_{k,q}}{(q;q)_{mk}} \frac{(\rho\ x\ t^m)^k}{(1-tq^{\lambda})^{(mk)}}$$

$$\times H_{P+1,Q}^{M,N+1}\left[\frac{y}{(1-tq^{\lambda+mk})^{(h)}};q \left| \begin{array}{c} (1-\lambda-mk,h),\ (a,\alpha) \\ (b,\beta) \end{array} \right.\right], \tag{2.1}$$

where $|t| < 1$, $0 < |q| < 1$, and ρ and λ are arbitrary numbers.

Proof. Denoting, for convenience, the left-hand side of (2.1) by L and using the contour integral representation (1.11) for the basic analogue of Fox's H-function and the definition (1.10) for the general class of q-polynomials $f_{n,m}(\rho\ x;q)$, we get

$$L = \frac{1}{2\pi\, i} \sum_{n=0}^{\infty} \left(\sum_{k=0}^{[n/m]} \left[\begin{array}{c} n \\ mk \end{array} \right]_q S_{k,q}\ (\rho\ x)^k \right)$$

$$\times \left\{ \int_C \theta(s;q) G(q^{\lambda+n+hs})\ y^s\ d_q s \right\} \frac{t^n}{(q;q)_n}.$$

Changing the order of summations and integration, we obtain

$$L = \frac{1}{2\pi\, i} \int_C \theta(s;q) \sum_{n=0}^{\infty} \sum_{k=0}^{[n/m]} \frac{G(q^{\lambda+n+hs})}{(q;q)_n} \left[\begin{array}{c} n \\ mk \end{array} \right]_q S_{k,q}\ (\rho\ x)^k\ t^n y^s\ d_q s\ , \tag{2.2}$$

where $\theta(s;q)$ is given by (1.12). Using of the relation for q-gamma function, namely

$$G(q^a) = \frac{\Gamma_q(a)\ (1-q)^{a-1}}{(q;q)_{\infty}}\ , \tag{2.3}$$

we obtain

$$L = \frac{1}{2\pi\, i} \int_C \theta(s;q) \frac{\Gamma_q(\lambda+hs)\ (1-q)^{\lambda+hs-1}}{(q;q)_{\infty}}$$

$$\times \sum_{n=0}^{\infty} \sum_{k=0}^{[n/m]} \frac{(q^{\lambda+hs};q)_n}{(q;q)_n} \begin{bmatrix} n \\ mk \end{bmatrix}_q S_{k,q}\ (\rho\ x)^k\ t^n\ y^s\ d_q s\ .$$

Again, changing the order of summations and making use of the series rearrangement relation (cf. Srivastawa and Manocha [11])

$$\sum_{n=0}^{\infty} \sum_{k=0}^{[n/m]} B(k,n) = \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} B(k, n+mk), \tag{2.4}$$

we obtain

$$L = \frac{1}{2\pi\, i} \int_C \theta(s;q) \frac{\Gamma_q(\lambda+hs)\ (1-q)^{\lambda+hs-1}}{(q;q)_\infty}$$
$$\times \sum_{k=0}^{\infty} S_{k,q} \frac{(\rho\ x\ t^m)^k}{(q;q)_{mk}} \sum_{n=0}^{\infty} \frac{(q^{\lambda+hs};q)_{n+mk}}{(q;q)_n}\ t^n\ y^s\ d_q s\ . \tag{2.5}$$

Summing the inner series with the help of the q-binomial theorem (see [2]), namely

$${}_1\Phi_0(a;-;q;z) = \frac{(az;q)_\infty}{(z;q)_\infty},\ |z|<1,\ 0<|q|<1, \tag{2.6}$$

we find that

$$L = \frac{1}{2\pi\, i} \int_C \theta(s;q) \frac{\Gamma_q(\lambda+hs)\ (1-q)^{\lambda+hs-1}}{(q;q)_\infty}$$
$$\times \sum_{k=0}^{\infty} \frac{(q^{\lambda+hs};q)_{mk}\ (\rho\ x\ t^m)^k}{(t;q)_{\lambda+hs+mk}(q;q)_{mk}}\ S_{k,q}\ y^s\ d_q s\ . \tag{2.7}$$

Now by interchanging the order of contour integral and summation, and using the q-identities (see [2]), namely

$$(a;q)_{n+k} = (a;q)_n (aq^n;q)_k \tag{2.8}$$

and

$$(a;q)_n = \frac{\Gamma(a+n)(1-q)^n}{\Gamma(a)}\ (n>0), \tag{2.9}$$

we obtain

$$L = \frac{1}{(t;q)_\lambda} \sum_{k=0}^{\infty} \frac{(\rho\ x\ t^m)^k}{(tq^\lambda;q)_{mk}(q;q)_{mk}}\ S_{k,q}$$
$$\times \frac{1}{2\pi\, i} \int_C \theta(s;q) \frac{\Gamma_q(\lambda+hs+mk)\ (1-q)^{\lambda+hs+mk-1}}{(tq^{\lambda+mk};q)_{hs}(q;q)_\infty}\ y^s\ d_q s\ . \tag{2.10}$$

The desired result follows by interpreting the contour integral of (2.10) in light of the definition (1.11) and the notation (2.3). This completes the proof of Theorem 1. □

Observe that, if we set the bounded sequence $S_{k,q} = 1$ and take $\rho = 0$, then for the family of q-polynomials one has

$$f_{n,m}(\rho\ x; q) = 1,$$

and thus in view of the right-hand side of (2.1) for $k = 0$, we obtain the following theorem.

Theorem 2. *Let M, N, P, Q be positive integers satisfying $0 \leq M \leq Q$, $0 \leq N \leq P$. Let $h > 0$, let λ be an arbitrary number, and let m be an arbitrary positive integer. Then the q-generating relation for the basic analogue of Fox's H-function is given by*

$$\sum_{n=0}^{\infty} H^{M,N+1}_{P+1,Q}\left[y; q \,\middle|\, \begin{matrix} (1-\lambda-n, h),\ (a,\alpha) \\ (b,\beta) \end{matrix}\right] \frac{t^n}{(q;q)_n} = \frac{1}{(1-t)^{(\lambda)}} \times H^{M,N+1}_{P+1,Q}\left[\frac{y}{(1-tq^{\lambda})^{(h)}}; q \,\middle|\, \begin{matrix} (1-\lambda, h),\ (a,\alpha) \\ (b,\beta) \end{matrix}\right], \tag{2.11}$$

where $|t| < 1$ and $0 < |q| < 1$.

3. Concluding observations and remarks

In this section, we consider some consequences of the results derived in previous section.

If we set $\alpha_i = \beta_j = 1$ for all i and j, $m = h = 1$, and take (1.14) into account, then Theorems 1 and 2 yield Corollaries 1 and 2 below, respectively.

Corollary 1. *Let $\{S_{k,q}\}_{k=0}^{\infty}$ be an arbitrary bounded sequence and let M, N, P, Q be positive integers satisfying $0 \leq M \leq Q$, $0 \leq N \leq P$. Then the following bilinear generating relation for the function $G_q(.)$ holds:*

$$\sum_{n=0}^{\infty} f_{n,1}(\rho\ x; q)\ G^{M,N+1}_{P+1,Q}\left[y; q \,\middle|\, \begin{matrix} 1-\lambda-n, a_1, \cdots, a_P \\ b_1, \cdots, b_Q \end{matrix}\right] \frac{t^n}{(q;q)_n} = \frac{1}{(1-t)^{(\lambda)}} \times \sum_{k=0}^{\infty} \frac{S_{k,q}}{(q;q)_k} \frac{(\rho\ x\ t)^k}{(1-tq^{\lambda})^{(k)}} G^{M,N+1}_{P+1,Q}\left[\frac{y}{(1-tq^{\lambda+k})}; q \,\middle|\, \begin{matrix} 1-\lambda-k, a_1, \cdots, a_P \\ b_1, \cdots, b_Q \end{matrix}\right], \tag{3.1}$$

where $|t| < 1$, $0 < |q| < 1$ and λ is an arbitrary number.

Corollary 2. *Let M, N, P, Q be positive integers satisfying $0 \leq M \leq Q$, $0 \leq N \leq P$ and let λ be an arbitrary number. Then the q-generating relation for the basic analogue of Meijer's G-function is given by*

$$\sum_{n=0}^{\infty} G^{M,N+1}_{P+1,Q}\left[y; q \,\middle|\, \begin{matrix} 1-\lambda-n, a_1, \cdots, a_P \\ b_1, \cdots, b_Q \end{matrix}\right] \frac{t^n}{(q;q)_n} = \frac{1}{(1-t)^{(\lambda)}}$$

$$\times\, G^{M,N+1}_{P+1,Q}\left[\frac{y}{(1-tq^{\lambda})};q\left|\begin{array}{c}1-\lambda,a_1,\cdots,a_P\\ b_1,\cdots,b_Q\end{array}\right.\right], \tag{3.2}$$

where $|t|<1$ *and* $0<|q|<1$.

Further, it is interesting to observe that in view of the following limiting cases:

$$\lim_{q\to 1^-}\Gamma_q(a) \;=\; \Gamma(a) \text{ and } \lim_{q\to 1^-}\frac{(q^a;q)_n}{(1-q)^n} \;=\; (a)_n\ , \tag{3.3}$$

where

$$(a)_n = a(a+1)\cdots(a+n-1), \tag{3.4}$$

the q-generating relation (2.1) of Theorem 1 provides the q-extension of the known result due to Raina [7, page 301, equation (2.1)].

By assigning suitable special values to the sequence $\{S_{k,q}\}_{k=0}^{\infty}$, our main result (Theorem 1) can be applied to derive certain bilinear q-generating relations for the product of orthogonal q-polynomials and the basic analogue of Fox's H-function. To illustrate this, we consider the following example.

Setting $m=1$ and

$$S_{k,q} = \frac{(-1)^k q^{k(k-1)}(\alpha q;q)_n}{(\alpha q;q)_k (q;q)_n}, \tag{3.5}$$

we find from (1.10) that

$$f_{n,1}(x;q) = L_n^{(\alpha)}(x;q),$$

where $L_n^{(\alpha)}(x;q)$ denotes the q-Laguerre polynomial defined by (cf. [9])

$$L_n^{(\alpha)}(x;q) = \frac{(\alpha q;q)_n}{(q;q)_n}\ {}_1\Phi_1\left[\begin{array}{ccc} q^{-n} & ; & \\ & & -xq^n \\ \alpha q & ; & \end{array}\right]. \tag{3.6}$$

Thus in view of the above relations, Theorem 1 yeilds the q-generating relation involving q-Laguerre polynomial and the basic Fox's H-function as below:

$$\sum_{n=0}^{\infty} L_n^{(\alpha)}(\rho\ x;q)\ H^{M,N+1}_{P+1,Q}\left[y;q\left|\begin{array}{c}(1-\lambda-n,h),\,(a,\alpha)\\ (b,\beta)\end{array}\right.\right]\frac{t^n}{(q;q)_n}$$

$$= \frac{(\alpha q;q)_n}{(1-t)^{(\lambda)}(q;q)_n}\sum_{k=0}^{\infty}\frac{(-1)^k q^{k(k-1)}}{(q;q)_k(\alpha q;q)_k}\,\frac{(\rho\ x\ t)^k}{(1-tq^{\lambda})^{(k)}}$$

$$\times\, H^{M,N+1}_{P+1,Q}\left[\frac{y}{(1-tq^{\lambda+k})^{(h)}};q\left|\begin{array}{c}(1-\lambda-k,h),\,(a,\alpha)\\ (b,\beta)\end{array}\right.\right]. \tag{3.7}$$

Again, if we set $m=1$ and

$$S_{k,q} = \frac{(\alpha q;q)_n(\alpha\beta q^{n+1};q)_k(-1)^k q^{k(k+1)/2-nk}}{(\alpha q;q)_k(q;q)_n}, \tag{3.8}$$

we find from (1.10) that

$$f_{n,1}(x;q) = P_n^{(\alpha,\beta)}(x;q),$$

where $P_n^{(\alpha,\beta)}(x;q)$ denotes the q-Jacobi polynomial defined by (cf. [9])

$$P_n^{(\alpha,\beta)}(x;q) = \frac{(\alpha q;q)_n}{(q;q)_n}\ {}_2\Phi_1\left[\begin{array}{ll} q^{-n}, \alpha\beta q^{n+1} & ; \\ & xq \\ \alpha q & ; \end{array}\right]. \tag{3.9}$$

Then, Theorem 1 provides the q-generating relation involving q-Jacobi polynomial and the basic Fox's H-function, namely

$$\sum_{n=0}^{\infty} P_n^{(\alpha,\beta)}(\rho\ x;q)\ H_{P+1,Q}^{M,N+1}\left[y;q \left| \begin{array}{c} (1-\lambda-n,h),\ (a,\alpha) \\ (b,\beta) \end{array}\right.\right] \frac{t^n}{(q;q)_n}$$

$$= \frac{(\alpha q;q)_n}{(1-t)^{(\lambda)}(q;q)_n} \sum_{k=0}^{\infty} \frac{(\alpha\beta q^{n+1};q)_k(-1)^k q^{k(k+1)/2-nk}}{(q;q)_k(\alpha q;q)_k} \frac{(\rho\ x\ t)^k}{(1-tq^{\lambda})^{(k)}}$$

$$\times H_{P+1,Q}^{M,N+1}\left[\frac{y}{(1-tq^{\lambda+k})^{(h)}};q \left| \begin{array}{c} (1-\lambda-k,h),\ (a,\alpha) \\ (b,\beta) \end{array}\right.\right]. \tag{3.10}$$

A detailed account of various hypergeometric orthogonal q-polynomials can be found in the research monograph by Koekoek, Lesky and Swarttouw [3] and in [9]. It is worth mentioning that the definitions of q-Laguerre and q-Jacobi polynomials given by the equations (3.6) and (3.9), respectively, are slightly different from those given in the seminal work [3]. Therefore, one can derive similar type of results by taking into consideration the definitions of the q-polynomials given in [3].

We conclude with the remark that by suitably assigning values to the sequence $\{S_{k,q}\}_{k=0}^{\infty}$, the q-generating relation (2.1) being of general nature, will lead to several generating relations for the product of orthogonal q-polynomials and the basic analogue of the Fox's H-functions.

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