ACTA ET COMMENTATIONES UNIVERSITATIS TARTUENSIS DE MATHEMATICA
Volume 17, Number 2, December 2013
Available online at http://acutm.math.ut.ee

Common fixed point theorems for ψ-weakly commuting maps in fuzzy metric space

Bhagwati Prasad and Ritu Sahni

Abstract. In this paper we obtain some fixed point and common fixed point theorems for mappings satisfying general contractivity condition in the setting of fuzzy metric spaces. Some recent results are also derived as special cases.

1. Introduction and preliminaries

The notion of a fuzzy set was first introduced by Zadeh [12] in 1965. After that a number of extensions of this idea enriched the literature and the concept of fuzziness is supplied in almost every direction of mathematics such as arithmetic, topology, probability theory, logic etc. Fixed points and common fixed points of maps with varying structures are widely studied by a number of authors. Kramosil and Michalek [4] and many others have introduced the concept of fuzzy metric spaces in various ways. George and Veeramani [2] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek. Grabiec [3] obtained the fuzzy version of Banach contraction principle. Grabiec's results were further generalized by Subramanyam [9] for pair of commuting mappings. Mishra et al. [6] introduced the concept of compatible mappings in fuzzy metric spaces which is more general than the commutativity. In the sequel, several authors proved fixed point and common fixed point theorems for compatible maps in fuzzy metric spaces. To study fixed-point theorems in fuzzy metric spaces for the mappings which are discontinuous at their common fixed point, noncompatible mappings are generally taken into consideration. Vasuki [10] introduced the concept of R-weakly commutativity in fuzzy metric spaces for studying common fixed points of noncompatible maps. Recently the notion of ψ-weakly

Received February 8, 2012.
2010 *Mathematics Subject Classification.* 47H10, 54H25.
Key words and phrases. Fuzzy metric spaces, ψ-weakly commuting maps, R-weakly commuting maps, property (C), property (Φ).
http://dx.doi.org/10.12697/ACUTM.2013.17.10

commutativity, more general than that of the R-weakly commutativity was introduced by Saadati et al. [8]. In this paper we prove some common fixed point theorems for ψ-weakly commuting mappings which are not necessarily continuous.

The following are useful definitions to prove our theorems.

Definition 1.1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a t-norm if $*$ satisfies the following conditions for all $a, b, c, d \in [0,1]$:

(i) $a * b = b * a$,
(ii) $(a * b) * c = a * (b * c)$,
(iii) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$,
(iv) $a * 1 = a,\ a * 0 = 0$.

Definition 1.2 ([2])**.** The triplet $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions:

(i) $M(x, y, t) > 0$,
(ii) $M(x, y, t) = 1$ if and only if $x = y$,
(iii) $M(x, y, t) = M(y, x, t)$,
(iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s),\ t, s > 0,\ \forall x, y, z \in X$
(v) $M(x, y, .) : (0, \infty) \to [0,\ 1]$ is continuous.

Definition 1.3. Let $(X, M, *)$ be a fuzzy metric space. A sequence $\{x_n\}$ in X is said to be

(i) convergent to a point $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$, for all $t > 0$,
(ii) a Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$, for all $t > 0$, $p > 0$.

Definition 1.4. A fuzzy metric space $(X, M, *)$ is called a complete fuzzy metric space if every Cauchy sequence of it converges to a point in it.

Definition 1.5 ([11])**.** Two mappings A and S of a fuzzy metric space $(X, M, *)$ into itself are R-weakly commuting provided there exists some positive real number R such that

$$M(ASx, SAx, t) \geq M(Ax, Sx, t/R) \text{ for each } x \in X \text{ and } t > 0.$$

Definition 1.6 ([8])**.** Two mappings A and S of a fuzzy metric space $(X, M, *)$ into itself are ψ-weakly commuting provided there exists some real function $\psi : (0, \infty) \to (0, \infty)$ such that

$$M(ASx, SAx, t) \geq M(Ax, Sx, \psi(t)) \text{ for each } x \in X \text{ and } t > 0.$$

Definition 1.7 ([7])**.** A fuzzy metric space $(X, M, *)$ is said to have the property (C) if it satisfies the following condition: $M(x, y, t) = C$, for all $t > 0$, implies $C = 1$.

The following lemmas are required for our results.

Lemma 1.1 ([6]). *Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, t)$ is non-decreasing with respect to t, for all x, y in X.*

Lemma 1.2 ([1]). *Let a function $\phi : [0, \infty) \to [0, \infty)$ satisfy the condition*

(Φ) *$\phi(t)$ is non-decreasing and $\sum_{n=1}^{\infty} \phi^n(t) < \infty \quad (t > 0)$, where $\phi^n(t)$ denotes the n-th iterative function of $\phi(t)$.*

Then $\phi(t) < t$, for all $t > 0$.

Lemma 1.3 ([7]). *Let $(X, M, *)$ be a fuzzy metric space and define $E_{\lambda,M} : X^2 \to [0, \infty)$ by*

$$E_{\lambda,M}(x, y) = inf\{t > 0 : M(x, y, t) > 1 - \lambda\},$$

for each $\lambda \in (0, 1)$ and $x, y \in X$. Then

(i) *for any $\mu \in (0, 1)$ there exists $\lambda \in (0, 1)$ such that*

$$E_{\mu,M}(x_1, x_n) \le E_{\lambda,M}(x_1, x_2) + \cdots + E_{\lambda,M}(x_{n-1}, x_n),$$

for any $x_1, ..., x_n \in X$,

(ii) *the sequence $\{x_n\}_{n \in N}$ is convergent with respect to a fuzzy metric M if and only if $E_{\lambda,M}(x_n, x) \to 0$. Also the sequence $\{x_n\}$ is a Cauchy sequence with respect to a fuzzy metric M if and only if it is a Cauchy sequence with $E_{\lambda,M}$, i.e., $E_{\lambda,M}(x_n, x_m) \to 0$ as n, $m \to \infty$, $n < m$.*

2. Main results

We first present the following lemma.

Lemma 2.1. *Let A, B, S, T, I and J be mappings from a fuzzy metric space $(X, M, *)$ into itself satisfying*

$$AI(X) \subset T(X), BJ(X) \subset S(X), \tag{2.1}$$

$$M(AIx, BJy, \phi(t)) \ge r(M(Sx, Ty, t)), \tag{2.2}$$

where the function ϕ maps $[0, \infty)$ continuously onto itself and satisfies condition (Φ) and $r : [0,\ 1] \to [0,\ 1]$ is a continuous function such that $r(t) > t$, for each $0 < t < 1$ and for all x, $y \in X$. Then the sequence $\{y_n\}$ defined by

$$y_{2n} = Tx_{2n+1} = AIx_{2n},\ y_{2n+1} = Sx_{2n+2} = BJx_{2n+1},\ n = 0, 1, 2, \ldots \tag{2.3}$$

is a Cauchy sequence in X.

Proof. From condition (2.1), we may construct a sequence $\{x_n\}$ as follows. Pick an arbitrary point x_0 in X and choose x_1 in X such that $Tx_1 = AIx_0$. For this point x_1 we fix a point x_2 in X such that $Sx_2 = BJx_1$, and so on. Thus we get the sequence $\{x_n\}$ such that $Tx_{2n+1} = AIx_{2n}$ and $Sx_{2n} = BJx_{2n-1}$ for any $n = 0, 1, 2, \ldots$. Now, using the sequence $\{x_n\}$, we define an another sequence $\{y_n\}$ in X by the equalities (2.3).

We first prove the inequalities

$$M(y_n, y_{n+1}, \phi^n(t)) \geq M(y_0, y_1, t), \quad n = 1, 2, \dots. \tag{2.4}$$

Consider $M(y_{2n}, y_{2n+1}, \phi^{2n}(t))$. For $n = 1$ we have

$$\begin{aligned} M(y_2, y_3, \phi^2(t)) &= M(AIx_2, BJx_3, \phi^2(t)) \\ &\geq r(M(Sx_2, Tx_3, \phi(t))) = r(M(y_1, y_2, \phi(t))) \\ &> M(y_1, y_2, \phi(t)) \geq M(y_0, y_1, t) \end{aligned}$$

in view of Lemmas 1.1 and 1.2. Further, for $n = 2$ we get

$$\begin{aligned} M(y_4, y_5, \phi^4(t)) &= M(AIx_4, BJx_5, \phi^4(t)) \\ &\geq r(M(Sx_4, Tx_5, \phi^3(t))) = r(M(y_3, y_4, \phi^3(t))) \\ &> M(y_3, y_4, \phi^3(t)) \geq ... \geq M(y_0, y_1, t). \end{aligned}$$

Thus, by induction, we have

$$M(y_{2n}, y_{2n+1}, \phi^{2n}(t)) \geq M(y_0, y_1, t), \ \ n = 1, 2, \dots. \tag{2.5}$$

Similarly we show that

$$M(y_{2n+1}, y_{2n+2}, \phi^{2n+1}(t)) \geq M(y_0, y_1, t), \quad n = 1, 2, \dots$$

which together with (2.5) gives (2.4).

Now, because ϕ maps $[0, \infty)$ onto $[0, \infty)$ and is continuous, from Lemma 1.3 and equation (2.4) we have

$$\begin{aligned} E_{\lambda,M}(y_n, y_{n+1}) &= \inf\{\phi^n(t) > 0 : M(y_n, y_{n+1}, \phi^n(t)) > 1 - \lambda\} \\ &\leq \inf\{\phi^n(t) > 0 : M(y_0, y_1, t) > 1 - \lambda\} \\ &= \phi^n(\inf\{t > 0 : M(y_0, y_1, t) > 1 - \lambda\}) \\ &= \phi^n(E_{\lambda,M}(y_0, y_1)), \end{aligned}$$

for every $\lambda \in (0, 1)$. Again, by Lemma 1.3, for every $\mu \in (0, 1)$ there exists $\lambda \in (0, 1)$ such that

$$\begin{aligned} E_{\mu,M}(y_n, y_m) &\leq E_{\lambda,M}(y_n, y_{n+1}) \\ &\quad + E_{\lambda,M}(y_{n+1}, y_{n+2}) + ... + E_{\lambda,M}(y_{m-1}, y_m) \\ &\leq \sum_{j=n}^{m-1} \phi^j(E_{\lambda,M}(y_0, y_1)) \to 0 \ \text{ as } \ m, n \to \infty, \ \ m > n, \end{aligned}$$

because the function ϕ satisfies condition (Φ). Thus $\{y_n\}$ is a Cauchy sequence in X. □

Theorem 2.2. *Let $(X, M, *)$ be a complete fuzzy metric space with the property* (C) *and let A, B, S, T, I, J, ϕ and r be the same as in Lemma 2.1. Suppose that one of A, B, S, T, I or J is continuous and the pairs (AI, S), (BJ, T) are ψ-weakly commuting on X. Then A, B, S, T, I and J have a unique common fixed point.*

Proof. By Lemma 2.1, $\{y_n\}$ is a Cauchy sequence and by the completeness of X, $\{y_n\}$ converges to a point $z \in X$. Consequently, the subsequences $\{AIx_{2n}\}$, $\{Sx_{2n+2}\}$, $\{BJx_{2n+1}\}$ and $\{Tx_{2n+1}\}$ of $\{y_n\}$ also converge to z.

Assume that S is continuous. Since the pair (AI, S) is ψ-weakly commuting, it follows that

$$M(AISx_n, SAIx_n, \phi^n(t)) \geq M(AIx_n, Sx_n, \psi(t)).$$

Letting here $n \to \infty$, we get $AISx_n \to Sz$.

By (2.2), we obtain

$$(AISx_{2n}, BJx_{2n+1}, \phi^{2n}(t)) \geq r(M(SSx_{2n}, Tx_{2n+1}, t))$$

and taking $n \to \infty$, we have

$$M(Sz,\ z,\ t)\ \geq\ r(M(Sz,\ z,\ t\,))\ >\ M(Sz,\ z,\ t),$$

which is a contradiction. Therefore $Sz = z$.

By (2.2), we also obtain

$$M(AIz, BJx_{2n+1}, \phi^{2n}(t)) \geq r(M(Sz, Tx_{2n+1}, t)).$$

Taking here $n \to \infty$, we have

$$M(AIz,\ z,\ t)\ \geq\ r(M(z,\ z,\ t))\ \geq\ M(AIz,\ z,\ \mathrm{t}),$$

which is a contradiction. Therefore $AIz = z$.

Since $AI(X) \subset T(X)$, for any u in X there exists a point z in X such that $AIz = Tu$. Hence $z = AIz = Tu$ and so

$$\begin{aligned} M(z, BJu, \phi^n(t)) &= M(AIz, BJu, \phi^n(t)) \geq rM(Sz, Tu, t)) \\ &= rM(z, z, t)) = rM(BJu, z, t) > M(BJu, z, t). \end{aligned}$$

On the other hand, by Lemmas 1.1 and 1.2, this implies that

$$M(z, BJu, \phi^n(\mathrm{t}))\ \leq\ M(z, BJu,\ t).$$

Hence $M(z, BJu,\ t) = C$ for all $t > 0$. Since X has the property (C), it follows that $C = 1$, i.e., $BJu = z$.

Since the pair *(BJ, T)* is ψ-weakly commuting on X, it follows that

$$M(BJTu,\ TBJu, \phi^n(\mathrm{t}))\ \geq\ M(BJu,\ Tu, \psi(\mathrm{t})).$$

Also $Tu = BJu = z$. This implies that $Tz = TBJu = BJTu = BJz$. Moreover, by (2.2), we obtain

$$\begin{aligned} M(z, Tz, \phi^n(t)) &= M(AIz, BJz, \phi^n(t)) \geq r(M(Sz, Tz, t)) \\ &= r(M(z, Tz, t)) > M(z, Tz, t). \end{aligned}$$

Again, by Lemmas 1.1 and 1.2, this implies that

$$M(z,\ Tz, \phi^n(\mathrm{t}))\ \leq\ M(z,\ Tz,\ t).$$

Hence $M(z, Tz, t) = C$ for all $t > 0$. Since X has the property (C), it follows that $C = 1$, i.e., $z = Tz$. Consequently, z is a common fixed point of A, B, S, T, I and J.

Similarly, we can also complete the proof by assuming any one of the mappings A, B, T, I and J is continuous.

Now, to prove the uniqueness, let if possible $z' \neq z$ be an another common fixed point of A, B, S, T, I and J. Then there exists $t > 0$ such that $M(z, z', \phi^n(t)) < 1$ and

$$\begin{aligned} M(z, z', \phi^n(t)) &= M(AIz, BJz', \phi^n(t)) \geq r(M(Sz, Tz', \phi^{n-1}(t)) \\ &\geq ... \geq r(M(Sz, Tz', t)) = r(M(z, z', t)) \\ &> M(z, z', t) \end{aligned}$$

By Lemmas 1.1 and 1.2, $M(z, z', \phi^n(t)) \leq M(z, z', t)$. Hence, $M(z, z', t) = C$ for all $t > 0$. Since X has the property (C), it follows that $C = 1$. Therefore $z = z'$, i.e., z is a unique common fixed point of A, B, S, T, I and J. □

If we put $I = J = id$, i.e., the identity map, $\phi(t) = t$ and $\psi(t) = t/R$, $R > 0$, in Theorem 2.2, then we obtain a result of Kumar [5].

Corollary 2.3 ([5], Theorem 3.2). *Let A, B, S and T be mappings from a complete fuzzy metric space $(X, M, *)$ into itself satisfying*

$$A(X) \subset T(X), B(X) \subset S(X)$$

and

$$M(Ax, By, t) \geq r(M(Sx, Ty, t)).$$

Suppose that one of A, B, S and T is continuous and the pairs (A,S), (B,T) are R-weakly commuting on X. Then the sequences $\{x_n\}$ and $\{y_n\}$ in X are such that

$$x_n \to x, \ y_n \to y, \ t > 0 \ \textit{implies} \ M(x_n, y_n, t) \to M(x, y, t).$$

Now we establish some results for the maps satisfying a different contractive condition.

Lemma 2.4. *Let A, B, S, T, I and J be mappings from fuzzy metric space $(X, M, *)$ into itself satisfying* (2.1) *and*

$$\begin{aligned} M(AIx, BJy, \phi(t)) \geq r(\min\{&M(Sx, Ty, t), M(AIx, Sx, t), \\ &M(BJy, Ty, t), M(AIx, Ty, t)\}), \end{aligned} \tag{2.6}$$

where ϕ and r are the same as in Lemma 2.1. Then the sequence $\{y_n\}$ defined by (2.3) *is a Cauchy sequence in X.*

Proof. We have for $n = 1$,

$$\begin{aligned} M(y_2, y_3, \phi^2(t)) &= M(AIx_2, BJx_3, \phi^2(t)) \\ &\geq r(\min\{M(Sx_2, Tx_3, \phi(t)), M(AIx_2, Sx_2, \phi(t)), \\ &\quad M(BJx_3, Tx_3, \phi(t)), M(AIx_2, Tx_3, \phi(t))\} \\ &= r(\min\{M(y_1, y_2, \phi(t)), M(y_2, y_1, \phi(t)), \\ &\quad M(y_3, y_2, \phi(t)), M(y_2, y_2, \phi(t))\} \\ &= r(M(y_1, y_2, \phi(t))) > M(y_1, y_2, \phi(t)) \\ &\geq M(y_0, y_1, t). \end{aligned}$$

For $n = 2$, we get

$$\begin{aligned} M(y_4, y_5, \phi^4(t)) &= M(AIx_4, BJx_5, \phi^4(t)) \\ &\geq r(\min\big\{M(Sx_4, Tx_5, \phi^3(t)), M(AIx_4, Sx_4, \phi^3(t)), \\ &M(BJx_5, Tx_5, \phi^3(t), M(AIx_4, Tx_5, \phi^3(t))\big\} \\ &= r(\min\big\{M(y_3, y_4, \phi^3(t)), M(y_4, y_3, \phi^3(t)), \\ &\quad M(y_5, y_4, \phi^3(t)), M(y_4, y_4, \phi^3(t))\big\} \\ &= r(M(y_3, y_4, \phi^3(t))) > M(y_2, y_3, \phi^2(t)) \geq \dots \\ &\geq M(y_0, y_1, t). \end{aligned}$$

Thus, by induction we have

$$M(y_{2n}, y_{2n+1}, \phi^{2n}(t)) \geq M(y_0, y_1, t).$$

Similarly,

$$M(y_{2n+1}, y_{2n+2}, \phi^{2n+1}(t)) \geq M(y_0, y_1, t).$$

Thus, in general,

$$M(y_n, y_{n+1}, \phi^n(t)) \geq M(y_0, y_1, t).$$

Now the proof follows from Lemma 2.1. □

Theorem 2.5. *Let A, B, S, T, I and J be mappings from a complete fuzzy metric space into itself satisfying conditions* (2.1), (2.6) *and property* (C). *Suppose that one of A, B, S, T, I and J is continuous and the pairs (AI, S) and (BJ, T) are ψ-weakly commuting on X. Then A, B, S, T, I and J have a unique common fixed point.*

Proof. By Lemma 2.4, the sequence $\{y_n\}$ is a Cauchy sequence and by the completeness of X, $\{y_n\}$ converges to some point $z \in X$. Consequently, the subsequences $\{AIx_{2n}\}, \{Sx_{2n+2}\}$, $\{BJx_{2n+1}\}$ and $\{Tx_{2n+1}\}$ of $\{y_n\}$ also converge to z.

Assume that S is continuous. By the ψ-weakly commutativity of the pair *(AI, S)* we get $AISx_{2n} \to Sz$ as $n \to \infty$. By (2.6), we obtain

$$M(AISx_{2n}, BJx_{2n+1}, \phi^{2n}(t)) \geq r(\min\{M(SSx_{2n}, Tx_{2n+1}, t), M(AISx_{2n}, SSx_{2n}, t), M(BJx_{2n+1}, Tx_{2n+1}, t), M(AISx_{2n}, Tx_{2n+1}, t)\}).$$

Taking here $n \to \infty$, we arrive at a contradiction, thus $Sz = z$.

By (2.6), we also obtain

$$M(AIz, BJx_{2n+1}, \phi^{2n}(t)) \geq r(\min\{M(Sz, Tx_{2n+1}, t), M(AIz, Sz, t), M(BJx_{2n+1}, Tx_{2n+1}, t), M(AIz, Tx_{2n+1}, t))\}).$$

Taking here $n \to \infty$, we again have a contradiction. Therefore $AIz = z$. Since $AI(X) \subset T(X)$, for u in X there exists a point z in X such that $AIz = Tu$. Hence $z = AIz = Tu$. Now

$$\begin{aligned} M(z, BJu, \phi^n(t)) = M(AIz, BJu, \phi^n(t) &\geq r(\min\{M(Sz, Tu, t), M(AIz, Sz, t), M(BJu, Tu, t), M(AIz, Tu, t)\}\}) \\ &= r(\min\{M(z, z, t), M(z, z, t), M(BJu, z, t), M(z, z, t)\}) \\ &= r(M(BJu, z, t)) > M(BJu, z, t). \end{aligned}$$

Lemmas 1.1 and 1.2 imply that

$$M(z, BJu, \phi^n(t)) \leq M(z, BJu, t).$$

Hence $M(z, BJu, t) = C$ for all $t > 0$. Since X has the property (C), it follows that $C = 1$. Therefore $BJu = z$.

Since the pair *(BJ, T)* is ψ-weakly commuting on X, it follows that

$$M(BJTu, TBJu, \phi^n(t)) \geq M(BJu, Tu, \psi(t)).$$

Also $Tu = BJu = z$. This implies

$$Tz = TBJu = BJTu = BJz.$$

Moreover, by (2.6), we obtain

$$\begin{aligned} M(z, Tz, \phi^n(t)) &= M(AIz, BJz, \phi^n(t)) \geq r(\min\{M(Sz, Tz, t), M(AIz, Sz, t), M(BJz, Tz, t), M(AIz, Tz, t))\}) \\ &= r(\min\{M(z, Tz, t), M(z, z, t), M(Tz, Tz, t), M(z, Tz, t))\}) \\ &= r(M(z, Tz, t)) > M(z, Tz, t). \end{aligned}$$

By Lemmas 1.1 and 1.2,

$$M(z, Tz, \phi^n(t)) \leq M(z, Tz, t).$$

Hence $M(z, Tz, t) = C$ for all $t > 0$. Since X has the property (C), it follows that $C = 1$, i.e., $z = Tz$. Therefore, z is a common fixed point of A, B, S, T, I and J.

The proof follows on a similar manner if any one of the mappings *A, B, T, I* and *J* is continuous.

Uniqueness is obvious if we proceed on the pattern of Theorem 2.2 with condition (2.6). □

Corollary 2.6. *Let A, B, S and T be mappings from a complete fuzzy metric space* $(X, M, *)$ *into itself satisfying*

$$A(X) \subset T(X), B(X) \subset S(X)$$

and

$$M(Ax, By, t) \geq r\left(\min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, t)\}\right).$$

Suppose that one of A, B, S or T is continuous and the pairs (A, S) and (B, T) are R-weakly commuting on X. Then A, B, S, T, I and J have a unique common fixed point.

Proof. The result follows at once when we set $I = J = id,$ the identity map, and ψ (t) $= t/R$, $R > 0$, in Theorem 2.5. Then the proof follows similarly to the proof of Theorem 2.2.

□

Acknowledgements

The authors would like to thank Dr. Enno Kolk and the anonymous referees for their valuable comments and suggestions for the improvement of this manuscript.

References

[1] J. X. Fang, *A note on fixed point theorem of Hadžić*, Fuzzy Sets and Systems **48** (1992), 391–395.

[2] A. George and P. Veeramani, *On some results in fuzzy metric spaces*, Fuzzy Sets and Systems **64** (1994), 395–399.

[3] M. Grabiec, *Fixed points in fuzzy metric spaces*, Fuzzy Sets and Systems **27** (1988), 385–389.

[4] O. Kramosil and J. Michalek, *Fuzzy metric and statistical metric spaces*, Kybernetika (Prague) **11** (1975), 326–344.

[5] S. Kumar, *Common fixed point theorem for minimal commutativity type mappings in fuzzy metric spaces*, Thai J. Math. **6** (2008), 239–270.

[6] S. N. Mishra, Nilima Sharma, and S. L. Singh, *Common fixed points of self maps on fuzzy metric spaces*, Internat. J. Math. Math. Sci. **17** (1994), 253–258.

[7] M. Mohamadi, R. Saadati, A. Shahmari, and S. M. Vaezpour, *Nonlinear contraction theorems in fuzzy spaces*, J. Appl. Sci. **9** (2009), 1397–1400.

[8] R. Saadati, S. Sedghi, and H. Zhou, *A common fixed point theorem for ψ-weakly commuting maps in $\mathfrak{L}$-fuzzy metric spaces*, Iran. J. Fuzzy Syst. **5** (2008), 47–53.

[9] P. V. Subrahmanyam, *Common fixed point theorems in fuzzy metric spaces*, Inform. Sci. **83** (1995), 109–112.

[10] R. Vasuki, *A common fixed point theorem in a fuzzy metric space*, Fuzzy Sets and Systems **97** (1998), 395–397.
[11] R. Vasuki, *Common fixed points of R-weakly commuting mappings in fuzzy metric spaces*, Indian J. Pure. Appl. Math. **30** (1999), 419–423.
[12] L. A. Zadeh, *Fuzzy sets*, Inform. and Control **8** (1965), 338–353.

Department of Mathematics, Jaypee Institute of Information Technology A-10, Sector-62, Noida-201307, India
E-mail address: b_prasad10@yahoo.com; bhagwati.prasad@jiit.ac.in