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Supplementary notes on Lewis Carroll, Graeco–Latin squares and magic squares with an annexe on Maria Theresa thalers and British banknotes

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Abstract. Following a brief critique of the wording of Lewis Carroll's *Alice's Adventures in Wonderland*, this article offers some supplementary remarks relating to the solution of a 3×3 pseudo-magic square problem posed by Lewis Carroll, a generalisation of the 3×3 *Lo-Su* or *luoshu* magic square, the problem of installing $m^2 = 25$ or $m^2 = 36$ officers of m different ranks and m different regiments in a $m \times m$ Graeco–Latin square (with apposite remarks on the military career of Leonhard Euler's third son and the recent coinage of the Empress Maria Theresa) and a list of fourteen portraits of mathematicians on banknotes paying particular attention to those of Sir Isaac Newton and Florence Nightingale.

1. Introduction

This article is offered by way of a tribute to George Styan on his seventy-sixth birthday (10 September 2013). I first met George Styan at a conference in Tampere, Finland, in August 1983. Since then we have met on numerous occasions and cooperated on several articles, notably [15, 16] and [17]. Sections 2 and 3 of the present article relate to certain aspects of the work of Lewis Carroll, first drawn to my attention by George Styan. In Sections 4 and 5 we pass on to a discussion of aspects of Graeco–Latin squares and their associated magic squares. In particular, we present some new historical information relating to the so-called 36 officers problem. Section 6

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relates to a familiar portrait of the Empress Maria Theresa, the mother of the Emperor Joseph II that modern tradition identifies as the Emperor of the 36 officers problem. Finally, Sections 7 to 10 relate to different aspects of two aborted joint articles on linear algebra and statistics that George Styan and I had intended to illustrate with the portraits of mathematicians featured on banknotes.

2. Charles Lutwidge Dodgson and symbolic algebra

On 24 January 2010 George Styan kindly sent me a copy of Melanie Bayley's article [1] analyzing the mathematical symbolism of Lewis Carroll's *Alice's Adventures in Wonderland* [4]. Building on Helena Pycior's earlier analysis of the trial of the Knave of Hearts, Bayley [1] develops the thesis that Alice's variations in size, the transformation of the Duchess's baby into a pig, and the Mad Hatter's tea party were all prompted by Carroll's adverse reaction to such recent developments in symbolic algebra as Poncelet's modern projective geometry and Hamilton's quaternions. [For a short account of the life of Charles Lutwidge Dodgson (better known as Lewis Carroll) (1832–1898), see Farebrother, Jensen and Styan [15].] Implicitly citing Dodgson's book on Euclidean geometry [9], Bayley [1] argues that

> Dodgson took his mathematics to his fiction. Using a technique familiar from Euclid's proofs, *reductio ad absurdum*, he picked apart the "semi-logic" of the new abstract mathematics, mocking its weakness by taking these premises to their logical conclusions, with mad results. The outcome is *Alice's Adventures in Wonderland.*

A brief examination of Carroll's text [4] suggests that Bayley's thesis does not hold much mathematical water. If the Mad Hatter's tea party were truly intended as a parody of Hamilton's quaternions then Carroll would surely have chosen a table without "corners" and "ends". Even then, an application of the Mad Hatter's "I want a clean cup" transformation would only serve to move the participants into the next seats round the table as if they were points on the circumference of a circle in a Wessel–Argand diagram, see Farebrother [12]. By contrast, six of the eight features protecting the Philosopher's Stone in the last two chapters of J. K. Rowling's *Harry Potter and the Philosopher's Stone* [25] have far more immediate parallels in the *Alice* books of Lewis Carroll: an enormous puppy-dog, a long fall on to a bed of leaves and a locked door with an inaccessible key all feature in *Alice's Adventures in Wonderland* [4]; a game of chess, a magical mirror and mirror writing all feature in *Through the Looking Glass and What Alice Found There* [5]; and, if the riddle posed by the White Queen at the end of [5] does not offer a sufficient parallel for the logical conundrum solved by Hermione

Granger in [25], then there are always the books of logic [6, 7] which Carroll wrote for the Hermione Grangers of his day. Moreover, Carroll's conventional use of the word "temper" on numerous occasions elsewhere in his *Alice* books [4, 5] strongly suggests that Bayley's [1] novel interpretation of the phrase "keep your temper" in such a way that "the Caterpillar could well be telling Alice to keep her body in proportion" is most unlikely to be valid. My preferred alternative explanation of this phrase relates it to the same words engraved on various types of brass gaming tokens produced in Britain during the Victorian and Edwardian periods (1837–1910). These tokens were employed by governesses and mothers as late as the 1930s to remind their charges of proper decorum when playing indoor games. A set of brass tokens with "keep your temper" on both sides came into my mother's possession in the late 1950s. Presumably they were employed by her mother as a governess to three girls and later as the mother of three girls. I do not know whether such tokens were used for this purpose in the Liddell household but a splendid example dating from 1862 with Queen Victoria's head on the obverse was recently offered for sale on the World Wide Web.

3. Lewis Carroll's magic squares

On 11 February 2013 George Styan drew my attention to a note in Wakeling [31, p. 8] which outlines a simple problem posed by Lewis Carroll (1832–1898) This problem was a special case of the following general problem: Given the positive integers $m, n \leq m^2$ and r, we are asked to insert multiple copies of the numerals $1, 2, ..., n$ into an $m \times m$ array in such a way that the m elements in each row, each column and possibly one or both principal diagonals sum to the "magic sum" r. If $n = m^2$, then $r = m(n+1)/2$ and this problem defines the traditional magic square. In this section we shall restrict our attention to the special case $m = 3$, $n = 5$ actually addressed by Carroll. In this context we find that five of the $m^2 = 9$ cells of the 3×3 array must contain the numerals $1, 2, 3, 4, 5$ whilst the remaining four cells are free to take any value in the range $1 \leq x \leq 5$. Thus, the sum of all nine elements must lie between $15 + 4 = 19$ and $15 + 20 = 35$. Further, $19 \leq 3r \leq 35$ implies that the magic sum r must satisfy the condition $7 \leq r \leq 11$. And we would seem to have five distinct values of r to consider. However, on replacing each element x of the 3×3 array by $6 - x$ we find that the case $r = 7$ may be twinned with $r = 11$ and $r = 8$ with $r = 10$.

Case $r = 7$. First we consider the case $r = 7$: the single element 5 must have two units in the same row and two units in the same column for them each to sum to 7. Four cells then remain for the elements $2, 4, 3, 3$ which only ensures that three rows and one column (or three columns and one row) sum to $r = 7$ and thus falls well short of the requirements of the above problem. The case for $r = 11$ runs parallel to that for $r = 7$ with x replaced by $6 - x$ throughout the argument.

Case $r = 9$. Moving to the case $r = 9$, we find by inspection that the nonsymmetric two-diagonal magic square $\mathcal{A}$ given below satisfies the conditions of the basic problem stated above. But Carroll's original problem [31, p. 8] requires the array to have two unit elements (one for the penny stamp and one for the two halfpenny stamps). Adding and subtracting unity to the second and third entries in the first and third rows of $\mathcal{A}$ followed by the interchange of the last two rows yields the symmetric no-diagonal magic square $\mathcal{B}$ which, with its row and column permuted variants, satisfies all the conditions of Carroll's original problem for $r = 9$.

$$\mathcal{A} = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 3 & 5 \\ 4 & 3 & 2 \end{bmatrix}, \qquad \mathcal{B} = \begin{bmatrix} 4 & 4 & 1 \\ 4 & 2 & 3 \\ 1 & 3 & 5 \end{bmatrix}.$$

Case $r = 8$. On subtracting unity from each element on the principal secondary diagonal of $\mathcal{A}$, we have $\mathcal{C}$. And, on moving the last row to first position and interchanging the first two columns, we have the symmetric one-diagonal magic square $\mathcal{D}$ which satisfies the conditions of the original problem for $r = 8$.

$$\mathcal{C} = \begin{bmatrix} 4 & 3 & 1 \\ 1 & 2 & 5 \\ 3 & 3 & 2 \end{bmatrix}, \qquad \mathcal{D} = \begin{bmatrix} 3 & 3 & 2 \\ 3 & 4 & 1 \\ 2 & 1 & 5 \end{bmatrix}.$$

Case $r = 10$. Replacing x by $6-x$ in $\mathcal{D}$ before interchanging the last two rows and the last two columns, we have $\mathcal{F}$. Then, on adding and subtracting unity in the four corners of $\mathcal{F}$, we have the symmetric no-diagonal magic square $\mathcal{G}$ which again solves the basic problem for $r = 10$.

$$\mathcal{F} = \begin{bmatrix} 3 & 4 & 3 \\ 4 & 1 & 5 \\ 3 & 5 & 2 \end{bmatrix}, \qquad \mathcal{G} = \begin{bmatrix} 4 & 4 & 2 \\ 4 & 1 & 5 \\ 2 & 5 & 3 \end{bmatrix}.$$

Finally, it should be noted that the ready appearance of symmetric solutions to both variants of Carroll's magic square problem suggests that, where feasible, the imposition of this additional constraint may help to speed the discovery of a solution. For an Integer Programming formulation of the traditional 4×4 magic square problem, see Farebrother [10].

4. Generalised Lo-Su magic squares

On adding unity to elements on the principal primary diagonal of array $\mathcal{D}$ of Section 3, and six to the elements on its first wrap-around primary diagonal, we have an instance of the familiar 3×3 *Lo-Su* [20] or *Luoshu* [28] magic square given below as array $\mathcal{L}$. More generally, let $m \geq 3$ be an odd prime number and let $n = m^2$ and $h = (n+1)/2$, then we may define a $m \times m$ generalised *Lo-Su* magic square as an $m \times m$ array containing the numerals $1, 2, ..., n$ in its $n = m^2$ cells in such a way that the m entries in its m rows, its m columns and its $m-1$ pseudo-diagonals each sum to $r = mh$ where, for $p = 1, 2, ..., m-1$, the pth pseudo-diagonal passes through the cell containing h (the "focal cell") and is constructed in such a way that each time the row index is increased by unity $(\,mod\; m)$ the column index is increased by $p\,(\,mod\; m)$. [And, hence the row index is increased by $q\,(\,mod\; m)$ each time the column index is increased by unity $(\,mod\; m)$ where q is the integer reciprocal to p satisfying $pq = 1 \pmod m$]. Augmenting this set of $m-1$ pseudo-diagonals by the row and the column passing through the focal cell (corresponding to the choices $p = 0$ and $p = m$ respectively), we find that these $m+1$ sets cannot intersect each other except at the focal cell as m is prime. So that $sp_1 = sp_2\,(\,mod\; m)$ for some $1 \leq s < m$ if and only if $p_1 = p_2\,(\,mod\; m)$. And we may therefore confirm that the n numerals in the $m \times m$ array sum to $(m+1)(m-1)h + h = nh$, as required. We shall refer to any such arrangement in which the focal cell appears at the centre of the array as the standard form of the generalised *Lo-Su* magic square. Setting $m = 3$ we have the standard 3×3 *Lo-Su* magic square $\mathcal{L}$ given below with $h = 5$ in the centre and magic sum $mh = 15$. The corresponding 3×3 Graeco–Latin square $\mathcal{K}$ may readily be obtained from this array by replacing each entry in $\mathcal{L}$ by the pair (i, j) where the values of i and j have been chosen in such a way that the corresponding entry in $\mathcal{L}$ takes the value $3(i-1) + j$:

$$\mathcal{K} = \begin{bmatrix} (2,1) & (3,3) & (1,2) \\ (1,3) & (2,2) & (3,1) \\ (3,2) & (1,1) & (2,3) \end{bmatrix}, \qquad \mathcal{L} = \begin{bmatrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{bmatrix}.$$

Noting that the 3×3 array $\mathcal{K}$ is a "bishop's move" Graeco–Latin square of the type described by Farebrother [11], we initiate our discussion of the case $m = 5$ by selecting Farebrother's 5×5 "bishop's move" Graeco–Latin square from [11, p. 33]. Unfortunately, this matrix is not yet in the required form, and, as a minor adjustment of the argument employed in [13, p. 24], we have to interleave the first two rows of Farebrother's 5×5 matrix [11, p. 33] into the last three rows in the order $3, 1, 4, 2, 5$ to obtain the Knut Vik or "knight's move" Graeco–Latin square $\mathcal{M}$ noted below. Then, given the matrix $\mathcal{M}$, we may identify the typical pair (i, j) with the value $5(i-1) + j$ to obtain the required 5×5 generalised *Lo-Su* magic square $\mathcal{N}$ with $h = 13$

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in the centre and magic sum $mh = 65$.

$$\mathcal{M} = \begin{bmatrix} (5,2) & (3,4) & (1,1) \\ (4,3) & (2,5) & \\ (1,3) & (4,5) & (2,2) \\ (5,4) & (3,1) & \\ (2,4) & (5,1) & (3,3) \\ (1,5) & (4,2) & \\ (3,5) & (1,2) & (4,4) \\ (2,1) & (5,3) & \\ (4,1) & (2,3) & (5,5) \\ (3,2) & (1,4) & \end{bmatrix}, \qquad \mathcal{N} = \begin{bmatrix} 22 & 14 & 1 & 18 & 10 \\ 3 & 20 & 7 & 24 & 11 \\ 9 & 21 & 13 & 5 & 17 \\ 15 & 2 & 19 & 6 & 23 \\ 16 & 8 & 25 & 12 & 4 \end{bmatrix}.$$

Of course, as in [17], the standard *Lo-Su* structure of both array $\mathcal{L}$ and array $\mathcal{N}$ is preserved under transpositions about the central row, the central column, the principal primary diagonal or the principal secondary diagonal and under rotations through one, two or three right angles about the central cell.

5. Leonhard Euler and the 36 officers problem

As noted by Puntanen and Styan [24], Styan [27] and Styan, Boyer and Chu [28] amongst others, Clifford Pearce [23] has suggested a pleasant, if entirely fictitious, account of a possible origin of the 6×6 Graeco–Latin square. The so-called 36 officers problem, was allegedly posed in the 1770s in an attempt to impress an unnamed Emperor (or Empress). The details of this problem are as follows: given m, we are required to arrange m officers of different rank from each of m regiments in the m^2 cells of a $m \times m$ array in such a way that each row and each column contains one officer of each rank and one officer from each regiment. Nowadays, it is well-known that it is not possible to construct the 6×6 Graeco–Latin square implied by the solution of this problem for $m = 6$. However, the 5×5 Knut Vik Graeco–Latin square $\mathcal{M}$ of Section 4 may be used to solve the corresponding problem for $m = 5$. Associating a distinct military rank (say, Ensign, Lieutenant, Captain, Major, Colonel) with each of the five numerals in first position and the names of five regiments (say, Grenadier, Coldstream, Scots, Irish, Welsh Guards) with each numeral in second position, we can arrange the five officers from each of five regiments in a 5×5 square in such a way that each row, each column, each wrap-around primary diagonal and each wrap-around secondary diagonal contains one officer of each rank and one officer from each regiment. In 1987 George Styan, as Editor of the *IMS Bulletin*, asked his readers to identify the Emperor in question. Later he awarded the prize to Clifford Pearce, the source of the problem, who gave an argument favouring the Emperor Joseph II (1741–1790) of the Holy Roman Empire. At this late stage, I adduce an additional item of information, apparently not available at the time, which tends to support the candidacy of the Empress Catherine II (1729–1796) of Russia. As noted by Oscar Sheynin (personal communication 7 August 2008):

> [Leonhard] Euler's third son, Christoff, became lieutenant colonel [...] in the Russian artillery not later than 1769. See N. Fuss, Eloge of Euler in Euler's *Opera omnia*, ser. 1, t. 1, p. LXXXVI. Christoff Euler finally became major General (Youshkevich, Euler, *Dictionary of Scientific biography* vol. 4, 1971, p. 472). C. E. (= Christoff Euler) began his military career as an officer in Prussia.

6. Maria Theresa thaler

In view of the decision recorded in Section 5, Puntanen and Styan [24] chose to illustrate their article by a postage stamp featuring a portrait of the Emperor Joseph II. But they should perhaps also have mentioned his formidible mother, the Holy Roman Empress Maria Theresa (1717–1780), who continued to hold the reins of power after the death of her husband Francis Stephen (1709–1765). [In passing, I note that Maria Theresa was also the mother of the ill-fated Queen Marie-Antoinette (1755–1794) of France.] Thus, Puntanen and Styan [24] might also have chosen to illustrate their article with a reproduction of a postage stamp or, better a facsimile of a silver coin, featuring a portrait of the Empress Maria Theresa. In the same way as the British gold sovereign is still minted in Llantrisant, Wales, the 1780 Maria Theresa thaler (doller) is still minted in Carson City, Denver, and San Francisco USA for use by traders who do not trust token or paper money. Indeed, Agatha Christie records in her autobiography [22] that she and her second husband Max Mallowan had illegally bought Maria Theresa thalers in Persia (now Iran) for their visit to the neighbouring states of the southern Soviet Union in 1929-30. Readers wishing to do so will be able to purchase modern Maria Theresa thalers from their local coin dealers; the only apparent difference between the modern coin and the original is that the place of origin is indicated on the reverse as "CC", "D", or "SF". Further, as Stigler [26, p.426n] remarks:

> For a description of some of the other ways Maria Theresa affected the intellectual history of statistics, including statistics in national administration, education generally, and population counts, see Zarkovich (1990, pp. 22–24).

7. Mathematicians on banknotes

As indicated above, George Styan has written several papers with various coauthors illustrated by the portraits of mathematicians on postage stamps.

In February 2008 he and I were in the preliminary stages of writing two similar articles illustrated by the portraits of linear algebraists and statisticians featured on banknotes. Unfortunately, neither of these articles came to fruition as we were not able to resolve the copyright issues involved. In the meantime, George had searched the web site *Banknotes featuring Scientists and Mathematicians* (maintained by Jacob Lewis Bourjaily of the University of Michigan): http://www-personal.umich.edu/%7Ejbourj/money.htm and the web site *Mathematics and Physics on Banknotes* (maintained by Hung Manh Bui of the University of Bristol): http://www.maths.bris.ac.uk /hb0262/Collection/Notes/banknotes.htm and increased my original list of four statisticians (Boscovich, Gauss, Newton, Nightingale) to the following list of fourteen scientists featured on banknotes who had made some contributions to linear algebra, magic squares, projective geometry, statistical mechanics or mathematical statistics:

> Rogerius Josephus Boscovich (1711–1787), René Descartes (1596–1650), Albrecht Dürer (1471–1528), Albert Einstein (1879–1955), Leonhard Euler (1707–1783), Benjamin Franklin (1706–1790), Carl Friedrich Gauss (1777–1855), Christiaan Huygens (1629–1695), Urbain Jean Joseph Le Verrier (1811–1877), Isaac Newton (1642–1727),Florence Nightingale (1820–1910), Pedro Nunes (Nonius Petrus) (1492–1577), Blaise Pascal (1623–1662), Erwin Schrödinger (1987–1961).

Readers may readily recover representations of the relevant banknotes either from these web sites or by submitting the scientists name and “banknote” to their preferred search engine. If the latter course is followed with the names of Laplace or Fourier, no such portrait will be identified. Instead, we find that the transforms named for these scientists are employed in electronic techniques for the detection of counterfeit banknotes

8. Isaac Newton and Dan Brown

I shall refrain from discussing the mathematical contributions of each of these scientists in turn. However, I have some particular information on the banknotes featuring portraits of Newton and Nightingale: The last English one Pound Note issued by the Bank of England features a portrait of Sir Isaac Newton and a diagram from his *Principia Mathematica Philosophiae Naturalis* of 1687 surrounded by apple blossom. Readers will note the embarassing error in the placement of a representation of the Sun at the centre of the ellipse marked C rather than at the focus marked S. Also note that Sir Leigh Teabing in Dan Brown’s *The da Vinci Code* [2] could easily have identified “the orb that ought be on his tomb” if only he had kept an old one pound note in his wallet. Similarly, in Dan Brown’s *The Lost Symbol* [3, p. 265 and 388], Robert Langdon employs Dürer’s 4×4 magic square and

Franklin's 8×8 magic square respectively as paradigms for decoding a 4×4 and an 8×8 array of symbols.

9. Florence Nightingale and Charles Darwin

Florence Nightingale is usually regarded as the founder of modern nursing. However, she also made significant contributions to hospital administration and medical statistics. Indeed, she is described as a "passionate statistician" in Kopf's [21] obituary for her work on medical statistics. It is intriguing to note that the bottom edge of the reverse of the five pound note issued by the Bank of England featuring a portrait of Florence Nightingale has an explicit © mark to discourage counterfeiters together with a small solid triangle to help visually impaired persons distinguish it from notes of higher denomimation identified by a square, a pentagon, and a circle. Both symbols failed to achieve their purpose and were dropped from subsequent issues of Bank of England notes. In particular, they are absent from the Bank of England ten pound note featuring a portrait of the biologist Charles Robert Darwin (1809–1882). In this connection, it is pertinent to remark that Florence Nightingale and Charles Robert Darwin are probably the most closely related of persons (other than heads of state) to be featured on distinct banknotes as one of Florence Nightingale's cousins married a cousin of Sir Francis Galton (1822–1911) who was himself a half-cousin of Charles Darwin; so that they have a total of just ten marriages separating them, see Farebrother [14] for details.

10. Other British coins and banknotes

The portraits of Newton and Nightingale mentioned above appear on notes issued by the Bank of England. Several Scottish banks and the Bank of Northern Ireland issue their own notes. Although Scotland has produced many well-known mathematicians and statisticians, it does not seem to have honoured any of them on its banknotes. The best that George Styan and I could offer are several portraits of the economist Adam Smith (1723–1790), the historical novelist Sir Walter Scott (1771–1832) and the physicist William Thomson, Lord Kelvin (1824–1907), who was actually born in Belfast, Northern Ireland. At the end of the Eighteenth Century the British economy suffered from a severe shortage of legal tender coins of low denomination; and numerous manufacturers attempted to remedy the situation by issuing their own low value token coins. Uglow [32] has described token coins featuring a portrait of the ironmaster John Wilkinson (1728–1808) whilst the referee of the present article has mentioned token coins featuring the portraits of Adam Smith and Sir Isaac Newton. Finally, I should like to take this opportunity to advise readers of Farebrother and Styan [16] that: Tranter [29] features a lengthy account of John Spottiswoode (1565–1639)

Archbishop of St Andrews and Lord Chancellor of Scotland whilst Tranter [30] contains brief mentions of Sir Robert Spottiswoode (1596–1646), secretary of State for Scotland. Moreover, Cornwell [8] mentions Alexander Spotswood (1676–1740), Lieutenant-Governor of Virginia, and the notorious pirate Edward Teach, better known as Blackbeard. The second part (1767–1805) of Alex Haley's *Roots* [19] is set in the county town of Spotsylvania, Virginia, and on the three properties owned by the Waller family in Spotsylvania County.

References

[1] Melanie Bayley, *Alice's adventures in algebra: Wonderland solved*, New Scientist, Issue 2739 (2009). Online open access.

[2] Dan Brown, *The da Vinci Code*, Bantam, London, 2003.

[3] Dan Brown, *The Lost Symbol*, Bantam, London, 2009.

[4] Lewis Carroll, *Alice's Adventures in Wonderland*, Clarendon Press, Oxford, 1865. Annotated reprint in [18].

[5] Lewis Carroll, *Through the Looking Glass and What Alice Found There*, Macmillan, London, 1870. Annotated reprint in [18].

[6] Lewis Carroll, The Game of Logic, Macmillan, London, 1887. Reprinted in Lewis Carroll, *Symbolic Logic and the Game of Logic.*, Dover, New York, 1972.

[7] Lewis Carroll, *Symbolic Logic*, Macmillan, London, 1896. Reprinted as *Lewis Carroll's Symbolic Logic*, edited (with annotations and an introduction by William Warren Bartley), Clarkson N. Potter (Crown Publishers), New York, 1986.

[8] Patricia D. Cornwell, *Isle of Dogs*, Little, Brown and Co, Boston, 2001.

[9] Charles Lutwidge Dodgson, *Euclid and his Modern Rivals*, Macmillan, London, 1885. Reprinted (with a new introduction by H. S. M. Coxeter) by Dover, New York, 1973.

[10] Richard William Farebrother, *Simple integer programming problems: problem and solution*, ILAS Image **36** (2006), 31–32, 36–37. Online open access.

[11] Richard William Farebrother, *A simple procedure for a general matrix transformation: problem and solution*, ILAS Image **39** (2007-2008), 32–34, 41. Online open access.

[12] Richard William Farebrother, *The Argand brothers of Geneva*, ILAS Image **41** (2008), p. 27. Correction **48**, p. 4. Online open access.

[13] Richard William Farebrother, *Symmetric Sudoku matrices and magic squares*, ILAS Image **43** (2009), 23–24. Online open access.

[14] Richard William Farebrother, *A genealogy of Florence Nightingale, Charles Darwin, Francis Galton and Francis Ysidro Edgeworth with special reference to their Italian connections and an annexe on Beatrice Webb and Charles Booth*, Stat. Methods Appl. **22** (2013), 391–402. Online at SpringerLink.

[15] Richard William Farebrother, Shane Tyler Jensen, and George P. H. Styan. *Charles Lutwidge Dodgson: A biographical and philatelic note*, ILAS Image **25** (2000), 22–23. Correction **48**, p. 4. Online open access.

[16] Richard William Farebrother and George P. H. Styan, *A genealogy of the Spottiswoode Family (1510–1900)*, ILAS Image **25** (2000), 19–21. Correction **27**, p. 2. Online open access.

[17] Richard William Farebrother and George P. H. Styan, *Double sudoku Graeco–Latin squares*, ILAS Image **41** (2008), 22–24. Online open access.

[18] Martin Gardner, *The Annotated Alice*, Clarkson N. Potter, New York, 1960. Reprinted by Penguin, Harmondsworth, 1965. Also see his *More Annotated Alice*, 1990.

[19] Alex Haley, *Roots*, Doubleday, New York, and Hutchinson, London, 1976.

[20] George Gheverghese Joseph, *The Crest of the Peacock: Non-European Roots of Mathematics*, Penguin books, Harmondsworth, 1990. Second edition Princeton University press, Princeton, N.J., 2000. Third edition, Princeton, 2010.

[21] E. W. Kopf, *Florence Nightingale as a statistician*, J. Amer. Statist. Assoc. **15** (1916), 388–404. Online at JSTOR.

[22] Agatha Christie Mallowan, *Come Tell Me How You Live*, Collins, London, 1946.

[23] S. Clifford Pearce, *The Agricultural Field Experiment: A Statistical Examination of Theory and Practice*, Wiley, Chichester, 1983.

[24] Simo Puntanen and George P. H. Styan, *A philatelic introduction to chance*, ASA Chance **21** (3) (2008), 36–41. Online at JSTOR.

[25] Joanne K. Rowling, *Harry Potter and the Philosopher's Stone*, Bloomsbury, London, 1997.

[26] Stephen M. Stigler, *Statistics on the Table: The History of Statistical Concepts and Methods*, Harvard University Press, Cambridge, Massachusetts, 1999.

[27] George P. H. Styan, *A philatelic introduction to magic squares and Latin squares for Euler's 300th birthyear*, Proceedings of the Canadian Society for History and Philosophy of Mathematics **20** (2007), 306–319.

[28] George P. H. Styan, Christian Boyer, and Ka Lok Chu, *Some comments on Latin squares and on Graeco–Latin squares, illustrated with postage stamps and old playing cards*, Statist. Papers **50** (2009), 917–941. Online at SpringerLink.

[29] Nigel Tranter, *The Young Montrose*, Hodder and Stouton, London, 1972.

[30] Nigel Tranter, *Montrose, The Captain General*, Hodder and Stouton, London, 1972.

[31] Edward Wakeling, *Lewis Carroll's Games and Puzzles*, Dover Publications, New York, in association with the Lewis Carroll Birthplace Trust, Daresbury, Cheshire, England, 1992.

[32] Jennifer Uglow, *The Lunar Men: The Friends who made the Future 1730–1810*, Faber and Faber, London, 2002.

[33] Slobodan Zarkovich, *The beginning of statistics in Yugoslavia*, International Statistical Review **58** (1990), 19–28. Online at JSTOR.

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