Estimating probability of fatigue failure of steel structures

Z. Kala

Abstract. The article deals with the analysis of failure probability of the effect of random factors influencing fatigue crack propagation in a steel element under bending moment. The theoretical model of fatigue crack progression is based on linear fracture mechanics. When determining the required degree of failure probability, it is possible to specify the time of the first inspection of the construction, which will focus on the fatigue damage. Using a conditional probability, subsequent inspection times are specified. The failure probability is examined using a fairly new sensitivity analysis subordinated to a contrast. The importance ranking of the input random variables to the failure probability is investigated. Fatigue properties of steel are taken from recent experimental research. Numerical results are obtained using the Monte Carlo simulation.

1. Introduction

Globally, there are a number of steel bridges, which are subjected to repeated and increasing load from vehicle axles. One of the main factors affecting the life of steel bridges is the fatigue phenomenon, which results from the accumulation of live load stress over a long time period. Considerable increase in the total weight load from vehicle axles and crossing frequencies results in higher fatigue damage of load bearing structures than was presumed during the design of bridges.

State-of-the-art reviews on methods for predicting the fatigue life of metal structures have been performed in [5, 23, 37]. A number of methods have been developed for the estimation of the remaining fatigue life of steel bridges and load bearing steel structures [7, 8], some of which are based on probabilistic methods [4, 9, 21, 27, 28]. Articles [22, 34] present an overview of the

current state-of-the-art of life cycle analysis of steel bridges including fatigue reliability assessment.

The subject of this article is time-dependent analysis of the reliability of existing steel bridges and global sensitivity analysis (GSA) of failure probability. The application of a fairly new type of global probability-oriented sensitivity analysis (called PSA in the article) is investigated. PSA measures sensitivity using contrast functions, which are useful objects in Statistical Learning Theory [29]. PSA is part of Goal oriented sensitivity analysis methods, which estimate the importance ranking of input variables to the failure probability [25, 30, 31, 32, 33].

The limit state of a load bearing steel member is described using linear fracture mechanics, see for, e.g., [1, 36]. The article builds on earlier studies of the reliability of steel structures focused on statistical [14, 15, 26] and global sensitivity analysis [12, 13] of ultimate limit states, probabilistic analysis of the fatigue limit state [11, 17, 18] and decision problems added to probabilistic structural analysis [2, 35].

2. Fatigue life assessment of steel bridges

Linear elastic fracture mechanics analyses the propagation of a crack of magnitude a in dependence on the number of fatigue cycles N. Fatigue crack growth is generally described by Paris's rule, which is expressed by Paris and Erdogan [19]:

$$\frac{da}{dN} = (\Delta K)^m \cdot B \,, \tag{1}$$

where m and B are Paris constant and exponent. Parameter B can be expressed as

$$log(B) = b_1 + m \cdot b_2 \,,$$

where b_1, b_2 can be considered for steel of grade S235 as $b_1 = -11.141$, $b_2 = -0.507$ [16]. The range of stress intensity factor ΔK can be determined by (see [3])

$$\Delta K = \Delta \sigma_E \cdot \sqrt{\pi a} \cdot f(a) \,,$$

where $\Delta \sigma_E$ is the quasi-constant stress range and $f(a)$ is the calibration function obtained from experimental research [24] for pure bending in the form

$$f(a) = 1.114 \left[1 - 0.806 \left(\frac{a}{W} \right) + 2.4704 \left(\frac{a}{W} \right)^2 + 1.01643 \left(\frac{a}{W} \right)^3 \right] \,, \tag{2}$$

where W is the specimen width in the direction of crack propagation. The domain of (2) is [0.01, 0.5]. Rearrangement and integration of the Paris–Erdogan equation (1) and consideration of crack propagation from a_0 (initial

crack size) to a_{cr} (critical crack size), and corresponding number of cycles $N_1 = 0$ (at the time of a_0) and N (at the time of a_{cr}) provides the relation

$$\int_{a_0}^{a_{cr}} \frac{da}{\left[f(a) \cdot \sqrt{\pi \cdot a} \right]^m} = \int_0^N \Delta\sigma_E^m \cdot B dN \,. \tag{3}$$

The accumulation of damage related to crack propagation from a_0 into a_{cr} is the resistance

$$R_{cr} = \int_{a_0}^{a_{cr}} \frac{da}{\left[f(a) \cdot \sqrt{\pi \cdot a} \right]^m} \,.$$

The calculation of the conditional failure probability is based on the resistance calculated for the detectable crack size a_d. Resistance for crack propagation from initial crack a_0 into a detectable crack size a_d can be written analogously as

$$R_d = \int_{a_0}^{a_d} \frac{da}{\left[f(a)\sqrt{\pi \cdot a} \right]^m} \,.$$

The failure (fatigue limit state) occurs when $R_{cr} \leq A(t)$, where $A(t)$ (action) is the right part of (3),

$$A(t) = \Delta\sigma_E^m \cdot N \cdot B \,.$$

$A(t)$ is a function of time, because the number of cycles N is a function in time. The reliability function can be expressed as

$$Y = R_{cr} - A(t) \,. \tag{4}$$

In the relations listed above, variables a_0, W, m, $\Delta\sigma_E$, N, and a_d can be introduced as random variables [10, 11, 17, 18]. Probabilistic analysis of reliability is based on the probabilistic approach to the reliability condition (4):

$$P_f = P(Y \leq 0) \,. \tag{5}$$

Equation (5) expresses the probability of failure due to brittle fracture. This is the probability with which the crack reaches size a_{cr} after N load cycles. The reliability function (4) is a function of the random variables listed in Table 1.

TABLE 1. Input random variables.

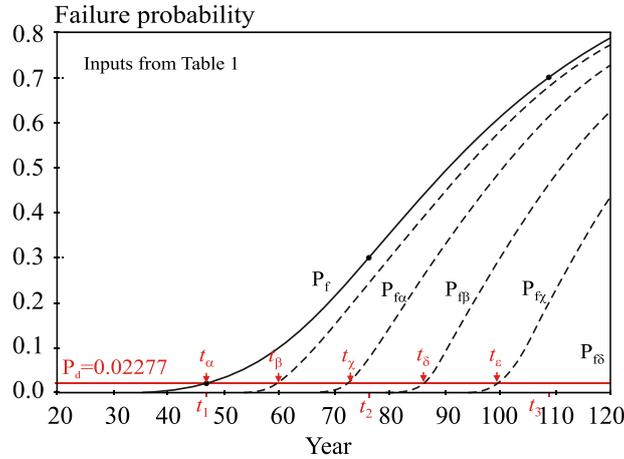| Symbol	Characteristic	Density	Mean	Stand. deviation
a_0	Initial crack size	lognormal	0.2 mm	0.06 mm
W	Specimen width W	Gauss	326 mm	15 mm
m	Parameter m	Gauss	3	0.03
$\Delta\sigma_E$	Stress peaks range	Gauss	31 MPa	3 MPa
N	Stress peaks per year	Gauss	1E6	1E5
a_d	Detectable crack size	Gauss	6 mm	0.6 mm

Figure 1. The failure probability analysis for inputs from
Table 1.

P_f is calculated with a time step of one tenth of the year, see full line in
Figure 1. P_f was evaluated at each time step using one million runs of the
Monte Carlo method (MC). The red line represents the required reliability
$P_d = 0.02277$, which corresponds to target reliability index $\beta_d = 2$ (see
[11, 17, 18]) or standard EN1990. Inspection of the bridge aimed at the
detection of cracks is performed at time $t_\alpha = 47$, $P_f = P_d$ years. If no crack
is detected, this information can be used to update the probability of failure.

$P_{f\alpha}$, $P_{f\beta}$, $P_{f\chi}$, $P_{f\delta}$ are conditional probabilities of failure, see dashed lines
in Figure 1. $P_{f\alpha}$ is the probability of failure calculated under the assumption
that no failure (detectable crack) was detected during previous inspection
at time t_α. Let us denote P_f from (5) as the probability of phenomenon C
(probability that $a \geq a_{cr}$). Then $P_{f\alpha}$ can be written as

$$P_{f\alpha} = P(C|\alpha) = \frac{P(C \cap \alpha)}{P(\alpha)},$$

where α is the random phenomenon where no crack of detectable size was
detected at time t_α. $P_{f\alpha}$ represents the random phenomenon of the oc-
currence of failure (phenomenon C), provided that no failure was detected
during previous inspection at time t_α (phenomenon α). $P(C \cap \alpha)$ is the
intersection of phenomena C and α. Conditional probabilities $P_{f\beta}$, $P_{f\chi}$, $P_{f\delta}$
are calculated analogously.

Important output from Figure 1 are the intervals of bridge inspections
determined by times t_α, t_β, t_χ, t_δ, t_ε. It is interesting that the inspection
intervals are constant $(t_\beta - t_\alpha) \approx (t_\chi - t_\beta) \approx (t_\delta - t_\chi) \approx (t_\varepsilon - t_\delta)$, which is
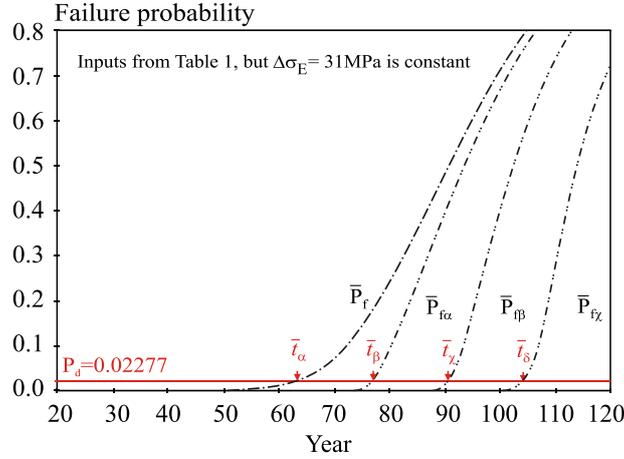in contrast with the conclusions of [17, 18].

FIGURE 2. The failure probability analysis for inputs from Table 1, but $\Delta\sigma_E$ is deterministic.

The results of the probabilistic analysis in Figure 1 are decisively influenced by the probabilistic models of input random variables a_0, a_d, m, $\Delta\sigma_E$, W, N. All input variables in Table 1 can be considered with aleatoric uncertainties, with the exception of the equivalent stress range $\Delta\sigma_E$, where epistemic uncertainties can also be discussed. If the histogram of stress range $\Delta\sigma$ is known, then the equivalent stress range $\Delta\sigma_E$ is a deterministic variable (constant value) [10, 11]. The introduction of standard deviation $\Delta\sigma_E$ is an admission that knowledge of the histogram of stress range $\Delta\sigma_E$ is incomplete. Eliminating stochastic uncertainty $\Delta\sigma_E$ from the probabilistic analysis by introducing $\Delta\sigma_E$ as a deterministic variable may significantly influence the results of reliability analysis, see Figure 2. Introduction of $\Delta\sigma_E$ as a fuzzy-random variable has been studied in [10], however, the validity of aleatoric and epistemic uncertainties in real life applications is still open to debate.

Figure 2 shows approximately constant inspection intervals and the inspection times occur later $t_\alpha < \bar{t}_\alpha$, $t_\beta < \bar{t}_\beta$, $t_\chi < \bar{t}_\chi$, $t_\delta < \bar{t}_\delta$ (compared to Figure 1). The question is which other input random variables can significantly influence the results of probabilistic analysis. The answer to this question can be obtained using sensitivity analysis.

3. Goal oriented sensitivity analysis

Sobol sensitivity analysis (SSA) is often applied in sensitivity measurements [20]. Sobol sensitivity indices are based on variance [20]. The variance is the expectation of the squared deviation of a random variable from

its mean. However, measuring the distance from the mean (central parameter) is not suitable for measuring the effect of input random variables on the probability of failure. A more general approach is offered by GSA based on contrast functions [6].

Let us consider a model in the form $Y = f(X_1, X_2, \ldots, X_M)$, with Y a scalar. The input factors $(X_1, X_2, \ldots, X_M)$ are supposed to be random variables described using identified probability distributions, which reflect the uncertain knowledge of the system under analysis.

Let Θ be some generic set and let Q be some probability measure on a space $\mathcal{Y}$. A (Θ, Q) contrast function, is defined as any function ψ.

$$\psi : \Theta \longrightarrow L_1(Q)$$
$$\theta \longmapsto \psi(\cdot, \theta) : y \in \mathcal{Y} \longmapsto \Psi(\rho, y) \,,$$

such that

$$\theta^* = \underset{\theta \in \Theta}{\text{Argmin}} \, \mathbb{E}_{Y \sim Q} \psi(Y; \theta) \tag{6}$$

is unique. The function $\Psi : \theta \mapsto \mathbb{E}_{Y \sim Q} \psi(Y; \theta)$ is the average contrast function, or abusively contrast function if there is no ambiguity [6].

Contrast functions permit the estimation of various inferences associated to probability distributions of random parameters in (4). Basic examples of contrast functions [6] used in the analysis of structural reliability are as follows.

Central parameters:

– The mean : $\Psi(\theta) = \mathbb{E}(Y - \theta)^2 \,.$ \hfill (7)

– The median (in $\mathbb{R}$) : $\Psi(\theta) = \frac{1}{2}\mathbb{E}|Y - \theta| \,.$ \hfill (8)

An excess probability : $\Psi(\theta) = \mathbb{E}|1_{Y \geq t} - \theta|^2 \,.$ \hfill (9)

All the probability tail : $\Psi(\theta) = \int_{t_0}^{\infty} \mathbb{E}|1_{Y \geq t} - \theta(t)|^2 dt \,.$ \hfill (10)

The α-quantile : $\Psi(\theta) = \mathbb{E}(Y - \theta)(\alpha - 1_{Y \leq \theta}) \,.$ \hfill (11)

All the quantile "tail" : $\Psi(\theta) = \int_{\alpha_0}^{1} \mathbb{E}(Y - \theta(\alpha))(\alpha - 1_{Y \leq \theta(\alpha)}) d\alpha \,.$ \hfill (12)

Formula (6) gives a characterization of a feature θ^* of Y by a contrast. For example, (7) gives a minimum value of $\psi(\theta)$ if θ is the mean value of random variable Y:

$$\theta^* = \underset{\theta}{\text{Argmin}} \, \Psi(\theta) = \underset{\theta}{\text{Argmin}} \, \mathbb{E}(Y - \theta)^2 = \mathbb{E}Y$$

The global probability-oriented sensitivity analysis (PSA) can be defined, using (4) and contrast function (9), by

$$\psi(\theta) = \mathbb{E}(\psi(Y, \theta)) = \mathbb{E}(1_{Y < 0} - \theta)^2.$$
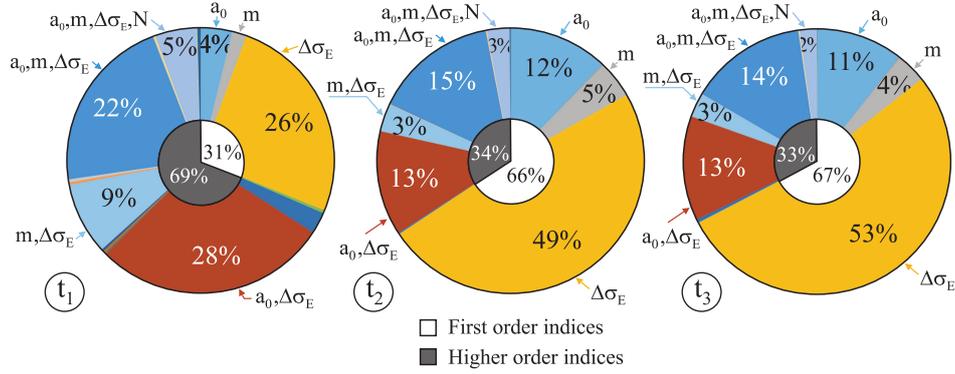
FIGURE 3. The results of sensitivity analysis PSA.

The main or the first order probability contrast index P_i (sensitivity index) can be written as

$$P_i = \frac{\Psi(\theta^*) - \mathbb{E}(\min_\theta \mathbb{E}(\psi(Y,\theta)|X_i))}{\Psi(\theta^*)}.$$

The contrast function $\Psi(\theta^*)$ is calculated based on (5) using five million MC runs. $\mathbb{E}(\min_\theta \mathbb{E}(\psi(Y,\theta)|X_i))$ is calculated using one thousand MC runs, where $\min_\theta \mathbb{E}(\psi(Y,\theta)|X_i)$ is calculated using three million MC runs. The second order sensitivity index P_{ij} is defined as

$$P_{ij} = \frac{\Psi(\theta^*) - \mathbb{E}(\min_\theta \mathbb{E}(\psi(Y,\theta)|X_i,X_j))}{\Psi(\theta^*)} - P_i - P_j.$$

The higher order sensitivity indices can be expressed analogously. The sum of all sensitivity indices must be equal to one,

$$\sum_i P_i + \sum_i \sum_{j>i} P_{ij} + \sum_i \sum_{j>i} \sum_{k>j} P_{ijk} + \cdots + P_{123\ldots M} = 1. \qquad (13)$$

The sensitivity analysis results measure the effect of the variability of input random variables a_0, W, m, $\Delta\sigma_E$, N (Table 1) on the unconditional probability of failure P_f (5). All input variables in Table 1 are considered as statistically independent, which is a necessary condition for evaluating the sensitivity indices in (13). A total of $2^5 - 1 = 31$ sensitivity indices were evaluated. Results are shown for times $t_1 = 47$ years, $t_2 = 76$ years ($P_f = 0.3$), $t_3 = 109$ years ($P_f = 0.7$), see Figure 3.

The PSA results depicted in Figure 3 show that the influence of $\Delta\sigma_E$ increases with increasing operation time of the bridge with dominance for t_3. The crucial interaction effect for t_1 is the pair $(a_0, \Delta\sigma_E)$ (28%). Significant difference is between PSA for t_1 in comparison with times t_2, t_3. The sum of interaction effects for t_1 is given as the difference $1 - \sum_i P_i = 0.69$, which is in contrast with t_2 where $1 - \sum_i P_i \approx 0.34$ and t_3 where $1 - \sum_i P_i \approx 0.33$.

It can be noted that PSA in the presented concept cannot be applied for the conditional probability of failure, because by excluding samples $a > a_d$ the remaining (non-excluded) samples used in the calculation of $P_{f\alpha}$, $P_{f\beta}$, $P_{f\chi}$, $P_{f\delta}$ have non-zero statistical correlation.

4. Conclusion

The paper presents the mathematical analysis of unconditional and conditional probability of failure, which can be used to plan bridge inspections. Bridges may remain in operation provided that a crack of detectable size is not detected during inspection. Otherwise, repairs are required.

PSA of the unconditional probability of failure showed that P_f is most influenced by the stress peaks range $\Delta\sigma_E$, initial crack size a_0, parameter m and the interaction effects of these random variables. The main effect of parameter m is relatively small, but m is significantly involved in interactions with other variables. The stress peaks range $\Delta\sigma_E$ is a dominant input variable and the modelling of its uncertainty should therefore be paid a great deal of attention. It is shown in the article that the introduction of zero values of standard deviations of $\Delta\sigma_E$ leads to significantly later inspections. The contributions of aleatoric and epistemic uncertainty and the size of standard deviation of $\Delta\sigma_E$ in real life applications are still under discussion.

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Department of Structural Mechanics, Brno University of Technology, Faculty of Civil Engineering, Veveří Str. 95, 602 00, Brno, Czech Republic
E-mail address: kala.z@fce.vutbr.cz