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Comparison between finite element analysis and rheological models for chip formation

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Abstract. In the process of cutting, often the selection of cutting parameters is done considering empirical methods. This approach is more expensive and does not usually lead to the best solutions. Numerical methods for simulating the chip formation have been under development over the last thirty years. The aim of the present research is to compare models based on rheological properties of metals with 2D Finite Element Models of chip formation process.

1. Introduction

Metal cutting is one of the most commonly used production processes in the industry, which means that a better analysis of this process is necessary in order to select cutting tools and process parameters. The mathematical models would have great importance in increasing the understanding of the cutting process and reducing the number of experimental tests which are frequently used for a selection of cutting parameters, tool design, machinability evaluation, etc. Usually, the study of cutting process is very complicated, because the material removal process is conducted in an extremely adverse environment with high temperature and pressure in the zone of cutting. Therefore, the main goal of these studies is to establish a theory that will predict chip formation dynamics, cutting forces, temperature, tool wear, which allows to solve practical problems.

The chip formation mechanism in the orthogonal metal cutting process, is the focus in the present research. To simulate this process we consider two approaches: using discrete complex mechanical (rheological) models and

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finite element models. The modeling and examination of rheological properties of materials are widely used by engineers, physicists, chemists and mathematicians. The physical properties of deformable workpieces can be also described in the form of an adequate mechanical model, constructed as a complex of mechanical elements such as elasticity, viscosity and plasticity [17]. Considering combinations of linear elastic springs, linear viscous dashpots and elements of plasticity one can build a model which gives an observable perspective reflecting the nature of material removal process, changes in deformation and stresses of material and their dependence on the structure [8, 12, 21]. The rheological models in the form of non-periodic diagrams allow to describe new properties that reflect the process of deformation and destruction of a workable material in the process of cutting. Increasing number of elements will allow to construct more accurate model describing the response of real materials. But in this case there are more material parameters that need to be evaluated by experiments, usually, it might be very difficult, if not an impossible, task.

Finite element method (FEM) is most frequently used in metal cutting analysis. In recent years, FEM has become the main tool for simulating metal cutting processes. It provides appropriate approach to visualize cutting process, because it allows to predict various characteristics of the metal cutting such as cutting forces, temperatures, stresses, chip types and shapes, etc [1, 6, 15, 22].

2. Analysis of the stability of the chip formation process using rheological models

Metal cutting is a complicated mechanical process, which includes plastic deformation and destruction of metal in the local zone, friction in interaction of the cutting tool with workable material. In accordance with the results of previous studies [21], this interaction accompanied with flat chip formation was examined in two stages up to the moment of the formation of shaving, i.e., the deformation to the shear plane and the shift of the element of small thickness on the plane of shift. As a result of these processes, a fine lamellar type structure is created in the cut off layer consisting of wide plates and narrow shear planes with uniform cyclic recurrence, which are practically invariant relative to the conditions of cutting; i.e., the formation of a thin basic periodic structure occurs, reflecting the properties inherent only in the workable material (Figure 1). Here, V_s — the cutting speed; γ — the back rake angle; β — the angle of displacement of the localized strips; Δx — the thickness of the deformation shift; Δy — the shift of the basic periodic structure; α — the thickness of the cut off layer; α_1 — the thickness of the chip.

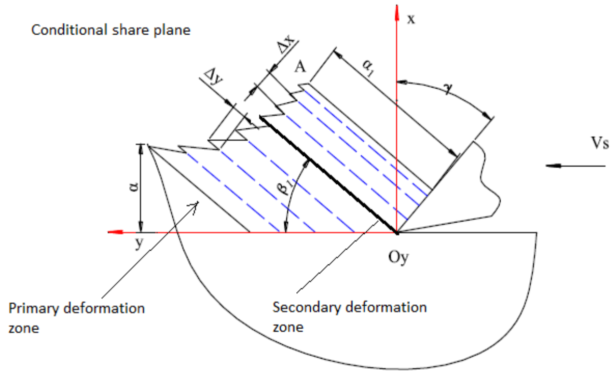

FIGURE 1. The conditional model reflecting the shaping of localized strips that shift in the region of the primary plastic deformation in the cut off layer.

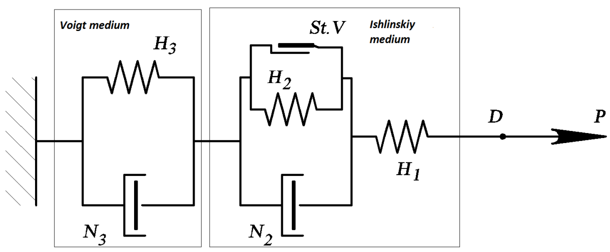

FIGURE 2. The rheological model of chip formation in the process of cutting [12].

We analyse the chip formation process by means of the rheological model proposed by Veitz and Maksarov [21]. This model is represented as a combination of two rheological bodies connected in series: the elastic-ductile-plastic medium of Ishlinskiy type [8, 12] and viscoelastic Voigt medium with the delay of deformation (Figure 2). The four-element rheological model of Ishlinskiy most fully reflects the dynamics of plastic deformation and destruction of a solid body in the cut off layer of metal. In this model the force **P** applied to the point D imitates by the linear elastic spring element H_1 the elastic instantaneous deformation, formed in front of the area of plastic deformation in the zone of chip formation. The next part of model imitates the prolonged viscoelastic deformation with the help of the spring H_2 and the dash-pot N_2 connected in parallel. After reaching the yield stress σ_y, which imitates St. Venant element $St.V$, the instantaneous residual deformation appears. This leads to the shaping of local segments of shift in the primary deformation zone of the cut off layer. At this stage, permanent, irreversible deformation begins [20]. The Voigt model, which consists of the spring H_3 and the dash-pot N_3 in parallel, describes movement of the shift of material foreword in the direction of tool movement and prolonged viscoelastic deformation.

The process describing relation of brittle and ductile fracture of material is presented in Figure 2. The brittle fracture corresponds to break of elastic external element H_1, and viscous fracture corresponds to the break of elastic internal element H_2, which is preceded by the relative shift of viscous element N_2 [12]. Elements of mechanical system can be considered in relation to the direction of deformation with the input and output, the direction of the pointer in the Figure 2 indicates the direction of deformation of the system.

The differential constitutive relations between stress σ and strain ε for the model describing the process of chip formation can be presented in the form of the following rheological equation [11, 20]:

$$a_1\ddot{\sigma} + a_2\dot{\sigma} + a_3(\sigma \pm \sigma_y) = a_4\ddot{\varepsilon} + a_5\dot{\varepsilon} + a_6\varepsilon, \tag{2.1}$$

where coefficients a_i, $(i = 1, 2, \ldots, 6)$ depend on stiffness coefficients and coefficients of linear resistance, σ_y is a yield stress. The solutions and analysis of this rheological equation were studied in [20]. We write the equation (2.1) in the short-hand notation using integral formulation and the Laplace transform. For the Ishlinskiy model we have

$$z_I(p) = \frac{p}{c_1} + \frac{p}{\beta_2 p + c_2},$$

where the properties of elastic elements H_1 and H_2 of the Ishlinskiy model are described by stiffness coefficients c_1 and c_2; the dash-pot element N_1 is characterized by the coefficient of linear resistance β_2 and $p^i = \frac{\partial^i}{\partial t^i}$ are the linear differential operators.

For the model of Voigt the operator resistance takes the form

$$z_V(p) = \frac{p}{\beta_3 p + c_3},$$

where H_3 and N_3 are described by the stiffness coefficient c_3 and the coefficient of linear resistance β_3, respectively.

The entire model describing the process of chip formation can be presented in the following form

$$\begin{aligned} z(p) &= \frac{\beta_2 p^2 + c_2 p + c_1 p}{\beta_2 c_1 p + c_1 c_2} + \frac{p}{\beta_3 p + c_3} \\ &= p \cdot \frac{\beta_2\beta_3 p^2 + (\beta_2 c_1 + \beta_2 c_3 + \beta_3 c_1 + \beta_3 c_2)p + (c_1 c_2 + c_2 c_3 + c_1 c_3)}{\beta_2\beta_3 c_1 p^2 + (\beta_2 c_1 c_3 + \beta_3 c_1 c_2)p + c_1 c_2 c_3} \end{aligned} \tag{2.2}$$

In order to simplify the calculations, we introduce the following notations for the coefficients of the differential operator p and free terms in the numerator and denominator of equation (2.2):

$$\begin{aligned} a_1 &= \beta_2\beta_3, \\ a_2 &= \beta_2 c_1 + \beta_2 c_3 + \beta_3 c_1 + \beta_3 c_2, \\ a_3 &= c_1 c_2 + c_2 c_3 + c_1 c_3, \\ a_4 &= \beta_2\beta_3 c_1, \\ a_5 &= \beta_2 c_1 c_3 + \beta_3 c_1 c_2, \\ a_6 &= c_1 c_2 c_3. \end{aligned}$$

Then the operator resistance can be expressed in the form [12]

$$z(p) = p \cdot \frac{a_1 p^2 + a_2 p + a_3}{a_4 p^2 + a_5 p + a_6},$$

it means that returning to the space of originals we obtain the rheological equation (2.1), which defines the relation between stress and deformation of the mechanical system.

To motivate the adequacy of the rheological model (2.1) we assume that the deformation in time is supposed to be known and solve this equation for the stress. We substitute σ in the equation (2.1) by a new variable $x = a_3(\sigma \pm \sigma_y)$ and get

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = f(t), \tag{2.3}$$

where $\omega_0 = \sqrt{a_3/a_1} = \sqrt{(c_1 c_2 + c_2 c_3 + c_1 c_3)/(\beta_2\beta_3)}$ is a frequency, $\gamma = a_2/(2a_1) = (\beta_2 c_1 + \beta_2 c_3 + \beta_3 c_1 + \beta_3 c_2)/(2\beta_2\beta_3)$ is a damping constant, and $f(t) = f(\varepsilon, \dot{\varepsilon}, \ddot{\varepsilon}) = (a_3 a_4 \ddot{\varepsilon} + a_3 a_5 \dot{\varepsilon} + a_3 a_6 \varepsilon)/a_1$.

Let us denote the vibration frequency by $\omega = \sqrt{\gamma^2 - \omega_0^2}$. The solution of homogeneous part of equation (2.3) for oscillation mode is

$$x(t) = e^{-\gamma t}(A\cos\omega t + B\sin\omega t), \quad \text{for } \gamma^2 - \omega_0^2 < 0,$$

and for relaxation mode

$$x(t) = Ae^{(-\gamma+\omega)t} + Be^{(-\gamma-\omega)t}, \quad \text{for } \gamma^2 - \omega_0^2 > 0.$$

The general solution of equation (2.3) for relaxation mode is

$$x(t) = e^{-\gamma t}\left[\frac{1}{2}e^{\omega t}\cdot\int\frac{f(t)e^{(\gamma-\omega)t}}{\omega}dt\right.$$
$$\left.-\frac{1}{2}e^{-\omega t}\cdot\int\frac{f(t)e^{(\gamma+\omega)t}}{\omega}dt + Ae^{\omega t} + Be^{-\omega t}\right].$$

With a fixed value of deformation $\varepsilon = \varepsilon_0$ the function $f(t)$ can be expressed as

$$f(t) = \frac{a_3 a_6 \varepsilon_0}{a_1}.$$

After transition from variable x to variable σ and taking into account initial conditions $\sigma(0) = \sigma_0$, and $\dot{\sigma}(0) = const$, an equation describing the stress relaxation takes the form

$$\begin{aligned}\sigma(t) = \sigma_y + \frac{a_6\varepsilon_0}{a_1(\gamma^2-\omega^2)} + \frac{1}{2\omega}\big[(\sigma_0-\sigma_y)((\gamma+\omega)e^{\omega t} \\ +(\omega-\gamma)e^{-\omega t}) - \frac{a_6\varepsilon_0}{a_1}\cdot\left(\frac{1}{\gamma-\omega}e^{\omega t} - \frac{1}{\gamma+\omega}e^{-\omega t}\right) \\ +\dot{\sigma}(0)(e^{\omega t}-e^{-\omega t})\big]\cdot e^{-\gamma t}.\end{aligned} \tag{2.4}$$

From equation (2.4) we can conclude that for our rheological model stress relaxes to its equilibrium with two relaxation times $\tau_1 = \frac{1}{\gamma-\omega}$ and $\tau_2 = \frac{1}{\gamma+\omega}$. We get the following expression for the rates of relaxation $\gamma-\omega$ and $\gamma+\omega$ in terms of stiffness coefficients c_i and coefficients of linear resistance β_i:

$$\begin{aligned}\gamma\pm\omega &= \frac{a_2}{2a_1} \pm \sqrt{\frac{a_2^2-4a_1a_3}{4a_1^2}} = \frac{1}{2a_1}\left(a_2 \pm \sqrt{a_2^2-4a_1a_3}\right) \\ &= \frac{1}{2\beta_2\beta_3}\big[(\beta_2c_1+\beta_2c_3+\beta_3c_1+\beta_3c_2) \\ &\pm\sqrt{(\beta_2c_1+\beta_2c_3+\beta_3c_1+\beta_3c_2)^2-4\beta_2\beta_3(c_1c_2+c_2c_3+c_1c_3)}\big].\end{aligned}$$

It follows from the proposed rheological model that in the zone of primary plastic deformation occur processes which generate instability of chip formation. The analysis of the developed plastic deformation of metal shows that this process has wavelike nature. At the moment which corresponds to the implementation of plastic deformation in the zone of plastic contact of chip with the tool, which reflects the elastic reaction of the workable material, the cut off layer undergoes the secondary deformation.

Our theoretical studies were confirmed by experimental data which were conducted in the broader band for back rake angle γ from $-5°$ to $25°$ and for different values of cutting speed. It was determined in [8] that the form of chip by processing steel AISI 45 is the following:

- discontinuous chip for $\gamma = -5°$ and cutting speed 10 m/min;
- built-up edge chip for $\gamma = 25°$ and cutting speed 25 m/min;
- continuous chip for for $\gamma = 10°$ and cutting speed 75 m/min.

It confirms our studies on the origin of the primary relaxation phenomena, which induce the origination of the self-oscillating process by turning materials.

3. Finite element analysis of chip formation process and description of simulation model

The basis of all finite elements methods involves dividing the body into an equivalent system of small finite elements for which the relevant variables and quantities are determined only at the nodes of the elements. This procedure is called discretization or meshing. The accuracy of the solution depends on the total number of elements used and their variation in size and type. The elements must be made small enough to give relevant results and large enough to reduce the time required for the simulation [7].

In recent years, numerous special-purpose and general-purpose finite element packages have been used for metal cutting analysis: Deform 2D/3D, MSC.Marc., Thirdwave AdvantEdge, Abaqus, Ls-Dyna, and etc.

In modeling the plastic material flow there are three main formulations for simulation of metal cutting: Eulerian, Lagrangian, and Arbitrary Lagrangian-Eulerian (ALE).

In the Eulerian formulation, the elements are fixed in space and material is allowed to flow through them [19]. In this method the mesh is spatially fixed and the material flows through the control volume which eliminates element distortion during the process. The main problem of the Eulerian formulation is the fact that the placement of fixed elements is not obvious. It is necessary to determine the boundaries and the shape of the chip prior to the simulation. Another obstacle of the Eulerian formulation is that the description of material property changes with strain, strain rate, and temperature as the material flows trough the element mesh. The present technique is more suitable for analysis of steady flow conditions. Therefore, the Eulerian formulation does not correspond to the real deformation process during orthogonal metal cutting.

In the Lagrangian formulation, material is divided into elements that move with the flow [2]. In this technique the elements change shape and orientation as they flow trough the deformation zone. This formulation is broadly used in solid mechanical problems and metal cutting simulations, because of the

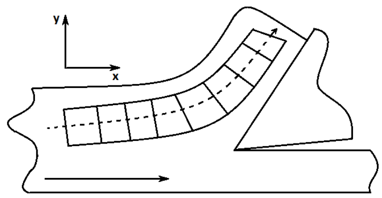

Figure 3. Eulerian formulation, material flows through a fixed element mesh.

ability to determine geometry of the chip from initial stage to the steady state, it means that the shape and boundaries of the chip do not have to be known. The geometry of the chip is a function depending on cutting parameters, plastic deformation process and material properties. In this approach analysis can be made more accurate by using smaller elements, but computation times are increased dramatically as the number of elements is increased. For cutting process models a finer mesh is needed in the primary and secondary deformation zones. The main problem of the Lagrangian technique is the need of a mesh regeneration, because metal being cut is exposed serve plastic deformation and it causes distortion of the elements.

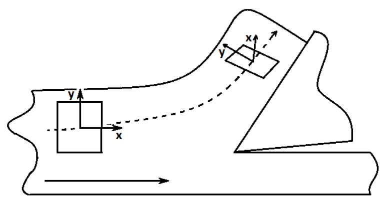

Figure 4. Lagrangian formulation, elements move with material and rotate with position.

An alternative method is to use ALE formulation, it combines the best features of Lagrangian and Eulerian formulations [4, 16]. In this case, the mesh is not attached to the material and it can move to avoid distortion and update the free chip geometry. The mesh follows the material flow and the problem is solved for displacements in Lagrangian step, while the mesh is regenerated and the problem for velocities is solved in Eulerian step.

The thermo-mechanical modeling of flow stress of workpiece material plays a very important role in metal cutting simulation. In finite element models flow stress is an instantaneous yield stress which depends on strain, strain

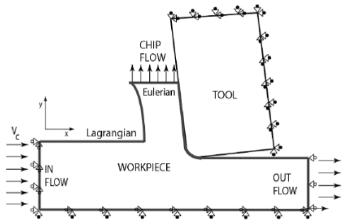

FIGURE 5. ALE formulation with Eulerian and Lagrangian boundary conditions [14].

rate and temperature. To describe the material flow during the cutting process an elasto-viscoplastic description of the material behavior can be written in the form of constitutive models

$$\sigma = f(\varepsilon, \dot{\varepsilon}, T), \tag{3.1}$$

where σ is the von-Mises stress, ε is the plastic strain, $\dot{\varepsilon}$ is the generalized strain rate, and T is the temperature. The most widely used one is the Johnson–Cook material constitutive model. This model is based on torsion and dynamic Hopkinson bar test over a wide range of strain rates and temperatures [5]. This constitutive equation has the form

$$\sigma = (A + B\varepsilon^n)\left(1 + C\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon_0}}\right)\left(1 - \left(\frac{T - T_r}{T_m - T_r}\right)^m\right). \tag{3.2}$$

The elastic-plastic term representing strain hardening is in the first parentheses. The coefficient A is the yield strength value when the stressing rate is low, B is the hardening modulus and n is the hardening coefficient. The viscosity term, where C is the strain rate sensitivity coefficient, is in the parentheses, it shows that flow stress of material increases when material is exposed to high strain rates. The last one is temperature softening term. T is the instantaneous temperature, T_r is the room temperature, T_m is the melting temperature of a given material and m is the thermal softening coefficient. Johnson–Cook constitutive model assumes that flow stress is affected by strain, strain rate and temperature independently.

Simulation of friction plays an important role in metal cutting models. Simulation results must be estimated with all process variables and friction parameter should be determined according to the shear angle results. The Coulomb friction model governs tool-chip interface contact:

$$\tau = \mu\sigma_n, \tag{3.3}$$

where τ is the frictional stress, μ is the coefficient of friction, and σ_n is the normal stress. Heat transfer is allowed at the tool chip contact area and at the backside of the tool [1].

In the present research we consider the finite element model used for the plane-strain orthogonal metal cutting simulation, which is based on the Lagrangian formulation in Third Wave Systems AdvantEdge simulation software (Figure 6).

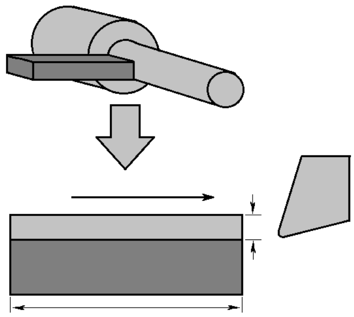

FIGURE 6. Turning operation and finite element model of AdvantEdge.

AdvantEdge integrates finite element numerics and material modeling appropriate for metal cutting. The software has comprehensive library of materials, input interface allows to set cutting parameters (feed rate, depth of cut, length of cut, cutting speed and initial temperature of the workpiece) as well as tool and workpiece geometries. Other parameters of cutting are fixed automatically, the user control on material parameters and solver are not allowed. The material model considers elastic-plastic strains, strain rate and has an isotropic power law for strain hardening [6]. In AdvantEdge a staggered method is utilized for the purpose of coupling the mechanical and heat transfer equations. Geometrically identical meshes for the mechanical and thermal models are used. An isothermal mechanical step is taken first, it is followed by a rigid transient thermal step with constant heating from plastic work and friction. The finite deformation formulations uses six-noded quadratic triangle elements. As the cutting layer transforms into chip, it separates from the workpiece splitting the nodes [3]. The separation of nodes is achieved by continuous remeshing, the software enables element distortion around the cutting edge by updating the mesh periodically both refining large elements and coarsening small elements.

4. Conclusions

(1) The rheological model in the form of a series connection of elastic-ductile-plastic relaxing medium of Ishlinskiy and the medium of Voigt describes the mechanical properties of chip formation process, but does not contain the evaluation of cutting temperature. The

analysis of the rheological model shows that instability of the process of cutting is caused by the wavelike nature of the plastic deformation and by the destruction of metal. Within the generalized rheological model of the chip formation process, material deformed in the cut off layer reflects the relaxation effect inherent in stress. Stress relaxes to its equilibrium value not according to the simple exponential law, but with a significantly more complicated law, with two relaxation times $\tau_1 = \frac{1}{\gamma - \omega}$ and $\tau_2 = \frac{1}{\gamma + \omega}$. At the moment which corresponds to the completion of plastic deformation in the zone of plastic contact of chip with the tool and which also reflects the elastic reaction of the workable material in the process of cutting, the cut off layer undergoes a second deformation. The instability of chip formation in the primary zone of the plastic deformation process is the basic reason for the appearance of instability within the process of cutting in the closed dynamic system of a machine tool. The analysis of the rheological equation (2.1) proves that the model enables an adequate description of the deformation process of chip formation and determination of the criteria to its transition to a non-stable mode, provoking self-excited oscillation.

The morphology of the chip is one of the most important areas in the study of cutting process. The present rheological model focuses on mechanical properties of chip formation. The process variables such as cutting force, friction between tool-chip interface and surface finish are affected by the chip morphology. Our theoretical studies were confirmed by experimental data which were conducted in the broader band for back rake angle γ from $-5°$ to $25°$ and for different values of cutting speed. It was determined in [8] that the form of chip by processing steel AISI 1045 is the following:

- discontinuous chip for $\gamma = -5°$ and cutting speed 10 m/min;
- built-up edge chip for $\gamma = 25°$ and cutting speed 25 m/min;
- continuous chip for for $\gamma = 10°$ and cutting speed 75 m/min.

It confirms our studies on the origin of the primary relaxation phenomena, which induce the origination of the self-oscillating process by turning materials.

High temperature in the cutting zone hostilely affects the strength, hardness and wear resistance of the cutting tool. Therefore, estimation of cutting temperature is a decisive aspect in the study of metal cutting. The future work on the present rheological model will focus on adding the temperature parameters into the rheological equation (2.1) describing the process of chip formation. To this end, we will use the rheological model proposed by Malyshev [10]. For this model it is assumed that the plastic deformation of the metal in the shear zone can be considered as the result of compression of the metal layer, taking into account the friction of the chips on the front surface of the tool and the cutting temperature.

Furthermore, it is necessary to apply general-purpose computational algorithms implemented in the form of a computer system with elements of visual design to study the constitutive relationships for chip formation whose rheological models involve a large number of elements. In [18] such an algorithm for studying the properties of granular materials was implemented in the general form in the Delphy 5 object programming environment, where the values of phenomenological parameters for elastic, viscous, and plastic elements are input variables for the computer system.

(2) With the advent of efficient software packages, FEM has become one of the most powerful tools for the simulation and analysis of cutting process. This allows studying the cutting process in greater detail than is possible in experiments. FEM simulation takes into account the material properties better than analytical models. FEM models allow to simulate the interaction of chip and tool in different forms.

In spite of many advantages of FEM models, the morphology of cutting chips is difficult to simulate. The continuous chip is often considered as more preferable, because it generates stable cutting forces. Moreover, it is easiest for simulation codes, but during experiments the sharp edged continuous chip that comes out at a high speed may be dangerous to the operator and may lead to unpredictable damage of the cutting tool. The majority of finite element codes simulates continuous chips [22]. The rheological behavior of machined materials in finite element models is usually described by the Johnson-Cook material model, which allows to consider the properties of workable material as a function depending not only on strain and strain-rate, but also on temperature. However, studies have shown [15] that for the modeling of different forms of chip in the process of cutting, it is necessary to use the Johnson-Cook progressive damage model in conjunction with the Johnson-Cook constitutive material model by using fracture energy as a damage evolution criterion.

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