ACTA ET COMMENTATIONES UNIVERSITATIS TARTUENSIS DE MATHEMATICA
Volume 25, Number 1, June 2021
Available online at http://acutm.math.ut.ee

On generalized fractional integral inequalities of Ostrowski type

Hüseyin Yildirim and Seda Kılınç Yildirim

Abstract. We obtain new generalizations of Ostrowski inequality by using generalized Riemann–Liouville fractional integrals. Some special cases are also discussed.

1. Introduction

Let $f : [a, b] \to R$ be a function continuous on $[a, b]$ and differentiable in (a, b). If $|f'(x)| \le M$ for all $x \in (a, b)$, then (see [14])

$$|f(x) - M(f; a, b)| \le \frac{M}{b-a} \frac{(b-x)^2 + (x-a)^2}{2} \tag{1}$$

for all $x \in [a, b]$, where $M(f; a, b) = \frac{1}{b-a} \int\limits_a^b f(x)dx$.

The inequality (1) is well known in the literature as Ostrowski inequality. Over the years, numerous studies have focused on generalizing this inequality, see, for example, [2–4, 6, 12, 14, 21] and the references cited therein.

Grüss [5] proved the inequality

$$|M(fg; a, b) - M(f; a, b)M(g; a, b)| \le \frac{1}{4} (M_1 - m_1)(M_2 - m_2), \tag{2}$$

where f and g are two integrable function on $[a, b]$ satisfying the conditions $m_1 \le f(x) \le M_1$ and $m_2 \le g(x) \le M_2$ for all $x \in [a, b]$. The constant $\frac{1}{4}$ is the best possible.

Received January 30, 2021.
2020 *Mathematics Subject Classification.* 26A33, 26D10, 26D15, 41A55.
Key words and phrases. Fractional integral, Ostrowski inequality, Korkine's identity, Riemann–Liouville fractional integral.
https://doi.org/10.12697/ACUTM.2021.25.09

Korkine's identity [11] states that if f and g are two integrable function on $[a,b]$, then

$$\begin{aligned} &M(fg;a,b)-M(f;a,b)M(g;a,b) \\ &= \frac{1}{2(b-a)^2}\int_a^b\int_a^b (f(t)-f(s))(g(t)-g(s))\,dsdt. \end{aligned} \tag{3}$$

Many researches have studied various types of integral inequalities for Riemann–Liouville integrals (see [7–9, 15, 18, 19, 20, 22] and references therein).

Definition 1 (see [17])**.** Let $f\in L^1[a,b]$. The Riemann–Liouville fractional integrals $J_{a^+}^{\alpha}f(x)$ and $J_{b^-}^{\alpha}f(x)$ of order $\alpha\ge 0$ are defined, respectively, by

$$J_{a^+}^{\alpha}f(x)=\tfrac{1}{\Gamma(\alpha)}\int_a^x (x-t)^{\alpha-1}f(t)\,dt,\quad x>a,$$

and

$$J_{b^-}^{\alpha}f(x)=\tfrac{1}{\Gamma(\alpha)}\int_x^b (t-x)^{\alpha-1}f(t)\,dt,\quad x<b.$$

Here $\Gamma(\alpha)$ is Gamma function and $J_{a^+}^{0}f(x)=J_{b^-}^{0}f(x)=f(x)$.

Hu [7] obtains the following generalizations for (1) by using (2), (3), and Riemann–Liouville fractional integrals.

Theorem 1 (see [7])**.** *Let f be differentiable function on $[a,b]$ and let $|f'(x)|\le M$ for any $x\in[a.b]$. Then the fractional inequality*

$$\left|\frac{(x-a)^{\alpha}+(b-x)^{\alpha}}{\Gamma(\alpha+1)}f(x)-J_{x^+}^{\alpha}f(a)-J_{x^+}^{\alpha}f(b)\right|\le M\frac{(x-a)^{\alpha+1}+(b-x)^{\alpha+1}}{\Gamma(\alpha+2)} \tag{4}$$

holds for any $x\in[a,b]$ and $\alpha\ge 0$.

Theorem 2 (see [7])**.** *Let $f:[a,b]\to R$ be a differentiable mapping and let $f'\in L^2[a,b]$. If f' is bounded on $[a,b]$ with $m\le f'(x)\le M$, then we have*

$$\begin{aligned} &\left|\frac{\alpha f(x)+f(a)}{\Gamma(\alpha)(\alpha+1)}(x-a)^{\alpha-1}-\frac{\alpha}{x-a}J_{x^-}^{\alpha}f(a)\right. \\ &\quad\left.+\frac{\alpha f(x)+f(b)}{\Gamma(\alpha)(\alpha+1)}(b-x)^{\alpha-1}-\frac{\alpha}{b-x}J_{x^+}^{\alpha}f(b)\right| \\ &\le\sqrt{\frac{1}{2\alpha+1}-\frac{1}{(\alpha+1)^2}}\,\frac{(x-a)^{\alpha}K_1+(b-x)^{\alpha}K_2}{\Gamma(\alpha)} \\ &\le\sqrt{\frac{1}{2\alpha+1}-\frac{1}{(\alpha+1)^2}}\,\frac{(x-a)^{\alpha}+(b-x)^{\alpha}}{2\Gamma(\alpha)}(M-m) \end{aligned} \tag{5}$$

for all $x\in[a,b]$ and $\alpha\ge 0$. Here

$$K_1^2=M(f^{'^2};a,x)-M^2(f^{'};a,x),\quad K_2^2=M(f^{'^2};x,b)-M^2(f^{'};x,b).$$

Now we will give definitions of generalized fractional integrals.

Definition 2 (see [9]). The space $L_{p,k}[a,b]$ is defined as

$$L_{p,k}[a,b]=\left\{f:\|f\|_{p,k}=\left(\int_a^b |f(t)|^p\, t^k dt\right)^{\frac{1}{p}}<\infty\right\},\ 1\le p<\infty,\ k\ge 0.$$

For $k=0$, the space $L_{p,k}[a,b]$ reduces to the classical space $L^p[a,b]$.

Definition 3 (see [1, 9, 11]). Let $h(x)$ be an increasing positive monotone function on $[a,b]$ such that $h'(x)$ is continuous on (a,b). The space $X_h^p(a,b)$ $(1\le p<\infty)$ is defined as the set of those real-valued Lebesque measurable functions f on $[a,b]$ for which

$$\|f\|_{X_h^p}=\left(\int_a^b |f(t)|^p\, h'(t)dt\right)^{\frac{1}{p}}<\infty.$$

In particular, if we take $h(x)=\frac{x^{k+1}}{k+1}$ $(k\ge 0)$, then the space $X_h^p(a,b)$ coincides with the space $L_{p,k}[a,b]$. For $h(x)=x$, the space $X_h^p(a,b)$ coincides with the classical space $L^p[a,b]$.

Definition 4 (see [1, 9, 11, 23]). Let $f\in X_h^p(a,b)$. The left and right generalized fractional integrals of function f of order $\alpha\ge 0$ are defined, respectively, by

$$J^{\alpha}_{a^+,h}f(x)=\tfrac{1}{\Gamma(\alpha)}\int_a^x (h(x)-h(t))^{\alpha-1}\, h'(t)\, f(t)\, dt,\quad x>a, \tag{6}$$

and

$$J^{\alpha}_{b^-,h}f(x)=\tfrac{1}{\Gamma(\alpha)}\int_x^b (h(t)-h(x))^{\alpha-1}\, h'(t)\, f(t)\, dt,\quad b>x. \tag{7}$$

Here $\Gamma(\alpha)$ is Gamma function and $J^0_{a^+,h}f(x)=J^0_{b^-,h}f(x)=f(x)$.

Remark 1. Letting $h(x)=x$ in (6) and (7), we obtain the equalities in Definition 1.

In this paper we will generalize expressions (1), (3), (4), and (5) by using generalized Riemann–Liouville fractional integrals.

2. Main results

Theorem 3. *If $f,g\in X_h^p(a,b)$, then we have, for $\alpha\ge 0$, the identity*

$$\begin{aligned}
&J^{\alpha}_{a^+,h}[f(b)\,g(b)]-\frac{\Gamma(\alpha+1)}{(h(b)-h(a))^{\alpha}}J^{\alpha}_{a^+,h}[f(b)]J^{\alpha}_{a^+,h}[g(b)]\\
&\quad=\frac{\alpha}{2\,(h(b)-h(a))^{\alpha}\,\Gamma(\alpha)}\int_a^b\int_a^b (f(t)-f(s))\,(g(t)-g(s))\\
&\qquad\times(h(b)-h(s))^{\alpha-1}\,(h(b)-h(t))^{\alpha-1}\,h'(t)h'(s)dsdt.
\end{aligned} \tag{8}$$

Proof. By $(f(t)-f(s))(g(t)-g(s)) = f(t)g(t)-f(t)g(s)-f(s)g(t)+f(s)g(s)$ we have

$$\begin{aligned}
&\int_a^b\int_a^b (f(t)-f(s))(g(t)-g(s))(h(b)-h(s))^{\alpha-1}\\
&\quad\times(h(b)-h(t))^{\alpha-1}h'(t)h'(s)dsdt\\
&= 2\left[\int_a^b (h(b)-h(s))^{\alpha-1}h'(s)ds\int_a^b f(t)g(t)(h(b)-h(t))^{\alpha-1}h'(t)dt\right]\\
&\quad-2\left[\int_a^b g(s)(h(b)-h(s))^{\alpha-1}h'(s)ds\int_a^b f(t)(h(b)-h(t))^{\alpha-1}h'(t)dt\right]\\
&= \frac{2(h(b)-h(a))^{\alpha}}{\alpha}\Gamma(\alpha)J^{\alpha}_{a^+,h}[f(b)g(b)]-2\Gamma^2(\alpha)J^{\alpha}_{a^+,h}[f(b)]J^{\alpha}_{a^+,h}[g(b)].
\end{aligned}$$

This completes the proof. □

Remark 2. Taking $\alpha=1$ in (8), we obtain the identity

$$\begin{aligned}
&M_h(fg;a,b)-M_h(f;a,b)M_h(g;a,b)\\
&\quad=\frac{1}{2(h(b)-h(a))^2}\int_a^b\int_a^b(f(t)-f(s))(g(t)-g(s))h'(t)h'(s)dsdt,
\end{aligned}\tag{9}$$

where $M_h(f;a,b)=\frac{1}{h(b)-h(a)}\int_a^b f(t)h'(t)dt$.

Remark 3. For $h(x)=x$ in (9), we obtain the Korkine's identity (3).

Theorem 4. *Let f be a differentiable function on $[a,b]$ and let $|f'(x)|\le M$ for any $x\in[a,b]$. Then we have, for $\alpha\ge 0$, the generalized fractional inequality*

$$|D(f;a,b)|\le\frac{1}{\Gamma(\alpha+2)}M[(h(x)-h(a))^{\alpha+1}+(h(b)-h(x))^{\alpha+1}],\tag{10}$$

where

$$\begin{aligned}
D(f;a,b)={}&\frac{1}{\Gamma(\alpha+1)}[(h(x)-h(a))^{\alpha}+(h(b)-h(x))^{\alpha}]f(x)\\
&-J^{\alpha}_{x^-,h}[f(a)]-J^{\alpha}_{x^+,h}[f(b)]-\left[J^{\alpha+1}_{x^-,h}\left[\frac{f(a)}{h(a)}\right]+J^{\alpha+1}_{x^+,h}\left[\frac{f(b)}{h(b)}\right]\right].
\end{aligned}$$

Proof. Using integration by parts for fractional integrals in Definition 5, we have

$$\begin{aligned}
J^{\alpha+1}_{x^-,h}f'(a)&=\frac{1}{\Gamma(\alpha+1)}(h(x)-h(a))^{\alpha}f(x)-J^{\alpha}_{x^-,h}[f(a)]-J^{\alpha+1}_{x^-,h}\left[\frac{f(a)}{h(a)}\right],\\
J^{\alpha+1}_{x^+,h}f'(b)&=\frac{-1}{\Gamma(\alpha+1)}(h(b)-h(x))^{\alpha}f(x)+J^{\alpha}_{x^+,h}[f(b)]+J^{\alpha+1}_{x^+,h}\left[\frac{f(b)}{h(b)}\right].
\end{aligned}$$

These equalities show that

$$J_{x^-,h}^{\alpha+1}f^{'}(a) - J_{x^+,h}^{\alpha+1}f^{'}(b) = D(f;a,b).$$

Therefore, using (6) and (7), by $|f'(x)| \leq M$, $x \in [a,b]$, we have

$$\begin{aligned} |D(f;a,b)| &\leq \frac{1}{\Gamma(\alpha+1)} M \left[\int_a^x (h(t)-h(a))^\alpha h'(t)dt + \int_x^b (h(b)-h(t))^\alpha h'(t)dt \right] \\ &\leq \frac{1}{\Gamma(\alpha+2)} M \left[(h(x)-h(a))^{\alpha+1} + (h(b)-h(x))^{\alpha+1} \right], \end{aligned}$$

which completes the proof. □

Remark 4. If we take $h(x) = x$ in (10), then we obtain the inequality (4) in Theorem 1.

Remark 5. If $h(x) = 0$ and $\alpha = 1$, then the inequality (10) reduces to Ostrowski inequality (1).

Theorem 5. *Let $f : [a,b] \to R$ be a differentiable mapping. If $f' \in X_h^p(a,b)$ is bounded on $[a,b]$ with $m \leq f'(x) \leq M$, then*

$$\begin{aligned} &|B_1(f;a,b) + B_2(f;a,b)| \\ &\leq \sqrt{\frac{1}{2\alpha+1} - \frac{1}{(\alpha+1)^2}} \frac{1}{\Gamma(\alpha)} (h(x)-h(a))^\alpha K_1 + (h(b)-h(x))^\alpha K_2 \\ &\leq \sqrt{\frac{1}{2\alpha+1} - \frac{1}{(\alpha+1)^2}} \frac{1}{\Gamma(\alpha)} \frac{(h(x)-h(a))^\alpha + (h(b)-h(x))^\alpha}{2} (M-m) \end{aligned} \tag{11}$$

for all $x \in [a,b]$ and $\alpha \geq 0$, where

$$\begin{aligned} B_1(f;a,b) &= \frac{1}{\Gamma(\alpha)} (h(x)-h(a))^{\alpha-1} [f(x) - \frac{1}{(\alpha+1)} \int_a^x f^{'}(t) h'(t) dt] \\ &\quad - \frac{\alpha}{h(x)-h(a)} \left(J_{x^-,h}^{\alpha}[f(a)] + J_{x^+,h}^{\alpha+1}\left[\frac{f(a)}{h(a)}\right] \right), \\ B_2(f;a,b) &= \frac{1}{\Gamma(\alpha)} (h(b)-h(x))^{\alpha-1} [f(x) + \frac{1}{(\alpha+1)} \int_x^b f^{'}(t) h'(t) dt] \\ &\quad - \frac{\alpha}{h(b)-h(x)} \left(J_{x^+,h}^{\alpha}[f(b)] + J_{x^-,h}^{\alpha+1}\left[\frac{f(b)}{h(b)}\right] \right), \end{aligned}$$

$$K_1^2 = M_h(f^{'^2};a,x) - M_h^2(f^{'};a,x), \quad K_2^2 = M_h(f^{'^2};x,b) - M_h^2(f^{'};x,b).$$

Proof. From(6) and (7) we have

$$\frac{1}{\Gamma(\alpha)(h(x)-h(a))} \int_a^x (h(t)-h(a))^\alpha h'(t) f^{'}(t)\, dt = B_1(f;a,b),$$

$$\frac{-1}{\Gamma(\alpha)(h(b)-h(x))} \int_x^b (h(b)-h(t))^\alpha h'(t) f^{'}(t)\, dt = B_2(f;a,b).$$

Then

$$\begin{aligned}
B_1(f;a,b)+B_2(f;a,b) &= \frac{1}{\Gamma(\alpha)(h(x)-h(a))}\int_a^x (h(t)-h(a))^{\alpha} h^{'}(t) f^{'}(t)\,dt \\
&- \frac{1}{\Gamma(\alpha)(\alpha+1)}(h(x)-h(a))^{\alpha-1}\int_a^x f^{'}(t)\,h'(t)dt \\
&- \frac{1}{\Gamma(\alpha)(h(b)-h(x))}\int_x^b (h(b)-h(t))^{\alpha} h'(t) f^{'}(t)\,dt \\
&+ \frac{1}{\Gamma(\alpha)(\alpha+1)}(h(b)-h(x))^{\alpha-1}\int_x^b f^{'}(t)\,h'(t)dt,
\end{aligned}$$

and by the identity (9) we get

$$\begin{aligned}
&B_1(f;a,b)+B_2(f;a,b) \\
&= \frac{1}{2\Gamma(\alpha)(h(x)-h(a))^2}\int_a^x\int_a^x [(h(t)-h(a))^{\alpha} \\
&\quad -(h(s)-h(a))^{\alpha}][f^{'}(t)-f^{'}(s)]h'(s)h'(t)dsdt \qquad (12)\\
&\quad + \frac{1}{2\Gamma(\alpha)(h(b)-h(x))^2}\int_x^b\int_x^b [(h(b)-h(s))^{\alpha} \\
&\quad -(h(b)-h(t))^{\alpha}][f^{'}(t)-f^{'}(s)]h'(s)h'(t)dsdt.
\end{aligned}$$

Using the Cauchy–Schwarz inequality for double integrals in (12), we obtain that

$$\begin{aligned}
&\left|\int_a^x\int_a^x [(h(t)-h(a))^{\alpha}-(h(s)-h(a))^{\alpha}][f^{'}(t)\right. \\
&\quad \left. -f^{'}(s)]h'(s)^{\frac{1}{2}}h'(t)^{\frac{1}{2}}h'(s)^{\frac{1}{2}}h'(t)^{\frac{1}{2}}dsdt\right| \\
&\le \left(\int_a^x\int_a^x [(h(t)-h(a))^{\alpha}-(h(s)-h(a))^{\alpha}]^2 h'(s)h'(t)dsdt\right)^{\frac{1}{2}} \qquad (13)\\
&\quad \times\left(\int_a^x\int_a^x [f^{'}(t)-f^{'}(s)]^2 h'(s)h'(t)dsdt\right)^{\frac{1}{2}}.
\end{aligned}$$

Since

$$\begin{aligned}
&\int_a^x\int_a^x [(h(t)-h(a))^{\alpha}-(h(s)-h(a))^{\alpha}]^2 h'(s)h'(t)dsdt \\
&= 2(h(x)-h(a))^{2\alpha+2}\left(\frac{1}{2\alpha+1}-\frac{1}{(\alpha+1)^2}\right)
\end{aligned}$$

and

$$\int_a^x\int_a^x [f^{'}(t)-f^{'}(s)]^2 h'(s)h'(t)dsdt$$

$$= 2\left(h(x)-h(a)\right)^2\left[M_h(f^{'^2};a,x)-M_h^2(f^{'};a,x)\right],$$

by (13) we have

$$\begin{aligned}&\left|\frac{1}{2\Gamma(\alpha)\left(h(x)-h(a)\right)}\int_a^x\int_a^x[(h(t)-h(a))^\alpha-(h(s)\right.\\&\qquad\left.-h(a))^\alpha][f^{'}(t)-f^{'}(s)]h'(s)h'(t)dsdt\right|\\&\qquad\le\frac{(h(x)-h(a))^\alpha}{\Gamma(\alpha)}\sqrt{\frac{1}{2\alpha+1}-\frac{1}{(\alpha+1)^2}}\left[M_h(f^{'^2};a,x)-M_h^2(f^{'};a,x)\right]^{\frac{1}{2}}.\end{aligned}\tag{14}$$

Similarly we find that

$$\begin{aligned}&\left|\frac{1}{2\Gamma(\alpha)\left(h(b)-h(x)\right)}\int_x^b\int_x^b[(h(b)-h(s))^\alpha-(h(b)\right.\\&\qquad\left.-h(t))^\alpha][f^{'}(t)-f^{'}(s)]h'(s)h'(t)dsdt\right|\\&\qquad\le\frac{(h(b)-h(x))^\alpha}{\Gamma(\alpha)}\sqrt{\frac{1}{2\alpha+1}-\frac{1}{(\alpha+1)^2}}\left[M_h(f^{'^2};x,b)-M_h^2(f^{'};x,b)\right]^{\frac{1}{2}}.\end{aligned}\tag{15}$$

Using (12), (14), and (15), we obtain the first inequality of (11). Moreover, if $m\le f'(x)\le M$ on $[a,b]$, then by Grüss inequality we get

$$\begin{aligned}0&\le\frac{1}{h(x)-h(a)}\left\|f^{'}\right\|^2_{X_h^2(a,x)}-(M_h(f^{'};a,x))^2\le\frac{1}{2}(M-m)^2,\\0&\le\frac{1}{h(b)-h(x)}\left\|f^{'}\right\|^2_{X_h^2(x,b)}-(M_h(f^{'};x,b))^2\le\frac{1}{2}(M-m)^2\end{aligned}$$

which proves the last inequality of (11). □

Remark 6. If we set $h(x)=x$ in (11), then we obtain the inequality (5) in Theorem 2.

Corollary 1. *Under the assumptions of Theorem 5 with $\alpha=1$ the following inequality holds:*

$$\begin{aligned}&2f(x)+\frac{1}{2}\left[\int_x^b f'(t)h'(t)dt-\int_a^x f'(t)h'(t)dt\right]\\&\quad-\frac{1}{h(x)-h(a)}\left[J^1_{x^-,h}[f(a)]+J^2_{x^+,h}\left[\frac{f(a)}{h(a)}\right]\right]\left[J^1_{x^+,h}[f(b)]+J^2_{x^-,h}\left[\frac{f(b)}{h(b)}\right]\right]\\&\quad\le\frac{1}{4\sqrt{3}}(h(b)-h(a))(M-m).\end{aligned}\tag{16}$$

Remark 7. If we set $h(x) = x$ and $\alpha = 1$ in (16), then we obtain the inequality (2.31) of [23].

3. Concluding remarks

In this study, we presented Ostrowski type generalized inequalities via generalized fractional integrals. It is also shown that the results proved here are a strong generalization of some already published results.

References

[1] A. Akkurt, M. Esra Yıldırım, and H.Yıldırım, *On some integral inequalities for* (k, h)*-Riemann-Lioville fractional integral*, New Trends Math. Sci. **4**(2) (2016), 138–146.

[2] M. Alomari and M. Darus, *Some Ostrowski type inequalities for convex functions with applications*, RGMIA **13**(1) (2010), Article 3, 13 pp.

[3] S. Belarbi and Z. Dahmani, *On some new fractional integral inequalities*, J. Inequal. Pure Appl. Math. **10**(3) (2009), Article 86, 5 pp.

[4] Z. Dahmani, L. Tabharit, and S. Taf, *Some fractional integral inequalities*, Nonlinear. Sci. Lett. A **1**(2) (2010), 155–160.

[5] S. S. Dragomir, *On the Ostrowski's integral inequality for mappings with bounded variation and applications*, Math. Inequal. Appl. **1**(2) (2001), 59–66.

[6] G. Grüss, *Über das Maximum des absoluten Betrages von* $\frac{1}{b-a}\int_a^b f(x)g(x)dx - \frac{1}{(b-a)^2}\int_a^b f(x)dx \int_a^b g(x)dx$, Math. Z. **39** (1935), 215–226.

[7] Y. Hu, Ostrowski inequality for fractional integrals and related fractional inequalities, TJMM **5** (2013), 85–89.

[8] H. Kalsoom, M. Idrees, A. Kashuri, M. Uzair Awan, and Y.-M. Chu, *Some new* (p_1p_2, q_1q_2)*-estimates of Ostrowski-type integral inequalities via n-polynomials s-type convexity*, AIMS Math. **5**(6) (2020), 7122–7144.

[9] U. N. Katugampola, *New approach to a generalized fractional integral*, Appl. Math. Comput. **218**(3) (2011), 860–865.

[10] M. A. Khan, S. Begum, Y. Khurshid, and Y.-M. Chu, *Ostrowski type inequalities involving conformable fractional integrals*, J. Inequal. Appl. **2018**, Paper No. 70, 14 pp.

[11] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, *Theory and Applications of Fractional Diferential Equations*, Elsevier B.V., Amsterdam, 2006.

[12] A. N. Korkine, *Sur une therome de M. Tchebychef*, C.R. Acad. Sci. Paris **96** (1883), 316–327.

[13] Z. Liu, *Some companions of an Ostrowski type inequality and application*, J. Inequal. Pure Appl. Math. **10**(2) (2009), Article 52, 12 pp.

[14] A. M. Ostrowski, *Über die Absolutabweichung einer differentiebaren Function von ihrem integral Mittelwert*, Comment. Math. Helv. **10** (1938), 226–227.

[15] B. G. Pachpatte, *On a new Ostrowski type inequality in two independent variables*, Tamkang J. Math. **32**(1) (2001), 45–49.

[16] S. Rashid, M. A. Noor, K. I. Noor, and Y.-M. Chu, *Ostrowski type inequalities in the sense of generalized K-fractional integral operator for exponentially convex functions*, AIMS Math. **5**(3) (2020), 2629–2645.

[17] S. G. Samko, A. A. Kilbas, and O. I. Marichev, *Fractional Integrals and Derivatives*, Theory and Applications, Gordon and Breach, Yverdon, 1993.

[18] M. Z. Sarikaya, *On the Ostrowski type integral inequality*, Acta Math. Univ. Comenian, **79**(1) (2010), 129–134.

[19] M. Z. Sarikaya, *Ostrowski type inequalities involving the right Caputo fractional derivatives belong to L_p*, Facta Univ. Ser. Math. Inform. **27**(2) (2012), 191–197.
[20] M. Z. Sarikaya and H. Filiz, *Note on the Ostrowski type inequalities for fractional integrals*, Vietnam J, Math. **42**(2) (2014), 187–190.
[21] M. Z. Sarıkaya and H. Ogunmez, *On new inequalities via Riemann-Liouville fractional integration*, arXiv:1005.1167v1 [math.CA] 7 May 2010.
[22] N. Ujevic, *Sharp inequalities of Simpson type and Ostrowski type*, Comput. Math. Appl. **48** (2004), 145–151.
[23] H. Yildirim and Z. Kirtay, *Ostrowski inequality for generalized fractional integral and related inequalities*, Malaya J. Mat. **2**(3) (2014), 322–329.

Department of Mathematics, Faculty of Science and Arts, University of Kahramanmaraş Sütçü İmam,46000, Kahramanmaraş, Turkey
E-mail address: hyildir@ksu.edu.tr
E-mail address: sedakilincmath@gmail.com