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# Narayana numbers as sums of two base $b$ repdigits

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Abstract. In this study, we find all Narayana numbers which are expressible as sums of two base $b$ repdigits. The proof of the main result uses lower bounds for linear forms in logarithms of algebraic numbers and a version of the Baker–Davenport reduction method.

## 1. Introduction

For an integer $b \geq 2$, a positive integer $R$ is called a base $b$ repdigit if it has only one distinct digit in its base $b$ representation. In particular, such number has the form $a(b^m - 1)/(b-1)$ for $1 \leq a \leq b-1$. For $m = 1$, we get single repdigits and call them trivial repdigits in this paper. When $b = 10$, we omit the base and simply say $R$ is a repdigit.

Recently, Diophantine equations involving repdigits in linear recurrent sequences like Fibonacci, Lucas, Pell, Pell–Lucas, balancing, Lucas-balancing sequences, etc. have been considered by many authors. For instance, Luca [9] showed that 55 and 11 are the largest repdigits in the Fibonacci and Lucas sequences respectively. In 2015, Faye and Luca [7] proved that there are no Pell or Pell–Lucas numbers larger than 10 which are repdigits. Lucas, Pell and Pell–Lucas numbers as sums of two repdigits have been studied in [2, 3]. Rayaguru and Panda [11] searched the presence of repdigits in balancing or Lucas-balancing numbers. In [12], they found that 35 is the only balancing number which is concatenation of two repdigits. Later they found [13] all balancing and Lucas-balancing numbers expressible as sums of two repdigits. Bravo et al. [4] obtained all base $b$ repdigits which are sums of two Narayana numbers. They also showed that 88 is the only repdigit which

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is also a Narayana number. Here, we are interested in searching Narayana numbers which are sums of two base $b$ repdigits.

Narayana numbers originated from a herd of cows and calves problem which was proposed by the Indian mathematician Narayana Pandit [1]. The Narayana's cows sequence $\{N_n\}_{n\geq 0}$ is a third-order recurrence relation given by

$$N_{n+3} = N_{n+2} + N_n$$

for $n \geq 0$ with initial condition $(N_0, N_1, N_2) = (0, 1, 1)$. It is the sequence A000930 in the OEIS (On-line Encyclopedia of Integer Sequences). Each term in this sequence is a Narayana number. The first few Narayana numbers are

$$0, 1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, 41, \cdots .$$

The characteristic polynomial for the Narayana's cows sequence is given by $f(x) = x^3 - x^2 - 1$ which is irreducible in $\mathbb{Q}[x]$. The zeros of this polynomial are $\alpha$ ($\approx 1.46557$) and two conjugate complex zeros $\beta$ and $\gamma$ with $|\beta| = |\gamma| < 1$. The following are some properties of Narayana sequence (see Lemma 5 in [4]). The Binet's formula for the Narayana's cows sequence is given by

$$N_n = a\alpha^n + b\beta^n + c\gamma^n \text{ for all } n \geq 0,$$

where

$$a = \frac{\alpha}{(\alpha-\beta)(\alpha-\gamma)},\ b = \frac{\beta}{(\beta-\alpha)(\beta-\gamma)},\ c = \frac{\gamma}{(\gamma-\alpha)(\gamma-\beta)}.$$

This formula can also be written as $N_n = C_\alpha\alpha^{n+2} + C_\beta\beta^{n+2} + C_\gamma\gamma^{n+2}$ for all $n \geq 0$ where $C_x = \frac{1}{x^3+2}$ for $x \in \{\alpha, \beta, \gamma\}$. Numerically, the following estimates hold for $\alpha, C_\alpha$ and $C_\beta\beta^{n+2} + C_\gamma\gamma^{n+2}$:

$$1.45 < \alpha < 1.5;\ 5 < C_\alpha^{-1} < 5.15;\ |C_\beta\beta^{n+2} + C_\gamma\gamma^{n+2}| < 1/2 \text{ for all } \ n \geq 1.$$

Using induction it is easy to prove that

$$\alpha^{n-2} \leq N_n \leq \alpha^{n-1} \text{ for all } \ n \geq 1. \tag{1}$$

In this study, we solve the exponential Diophantine equation

$$N_n = d_1\left(\frac{b^{m_1}-1}{b-1}\right) + d_2\left(\frac{b^{m_2}-1}{b-1}\right) \tag{2}$$

for some integers $2 \leq m_1 \leq m_2$, $d_1, d_2 \in \{1, 2, \ldots, b-1\}$. We give an upper bound for the highest solution in every base $b$. As an illustration, we explicitly find the solutions to the equation (2) for the base $b = 10$. Our main result is the following.

**Theorem 1.** *The Diophantine equation*

$$N_n = d_1\left(\frac{b^{m_1}-1}{b-1}\right) + d_2\left(\frac{b^{m_2}-1}{b-1}\right)$$

*has finitely many solutions in integers* $(n, d_1, d_2, m_1, m_2, b)$ *where* $b$ *is the base with* $1 \le d_1, d_2 \le b-1$ *and* $2 \le m_1 \le m_2$. *Moreover* $n$ *is bounded by* $5.39 \cdot 10^{32} \log^5 b$. *In particular, the only Narayana numbers expressible as sums of two repdigits are* $N_{14} = 88 = 11+77 = 22+66 = 33+55 = 44+44$ *and* $N_{17} = 277 = 55+222$.

In order to prove Theorem 1, we need some elementary results which are mentioned in the next section.

## 2. Preliminaries

The following lemma gives a relation between $n$ and $m_2$ of (2).

**Lemma 1.** *All solutions of* (2) *satisfy* $(m_2-1)\log b < n\log\alpha < m_2\log b + 3$.

*Proof.* From (1), we have

$$\alpha^{n-2} \le N_n < 2 \cdot b^{m_2}.$$

Taking logarithm on both sides, we get

$$(n-2)\log\alpha < \log 2 + m_2 \log b,$$

which leads to

$$n\log\alpha < m_2\log b + 3.$$

Similarly, $b^{m_2-1} < N_n < \alpha^n$ gives

$$n\log\alpha > (m_2-1)\log b.$$

This completes the proof. □

Baker's theory plays an important role in reducing the bounds concerning linear forms in logarithms of algebraic numbers. Let $\eta$ be an algebraic number with minimal primitive polynomial

$$f(X) = a_0(X-\eta^{(1)})\dots(X-\eta^{(k)}) \in \mathbb{Z}[X],$$

where $a_0 > 0$, and $\eta^{(i)}$'s are conjugates of $\eta$. Then

$$h(\eta) = \frac{1}{k}\left(\log a_0 + \sum_{j=1}^{k}\max\{0, \log|\eta^{(j)}|\}\right)$$

is called the *logarithmic height* of $\eta$. In particular, if $\eta = a/b$ is a rational number with $\gcd(a,b) = 1$ and $b > 0$, then $h(\eta) = \log(\max\{|a|, b\})$. The following are some properties of the logarithmic height function:

- $h(\eta+\gamma) \le h(\eta) + h(\gamma) + \log 2$,

- $h(\eta\gamma^{\pm 1}) \leq h(\eta) + h(\gamma)$,
- $h(\eta^k) = |k|h(\eta)$.

With these notations, Matveev (see [10] or [5, Theorem 9.4]) proved the following result.

**Theorem 2.** *Let $\eta_1, \eta_2, \ldots, \eta_l$ be positive real algebraic integers in a real algebraic number field $\mathbb{L}$ of degree $d_{\mathbb{L}}$ and $b_1, b_2, \ldots, b_l$ be non-zero integers. If $\Gamma = \prod_{i=1}^{l} \eta_i^{b_i} - 1$ is not zero, then*

$$\log|\Gamma| > -1.4 \cdot 30^{l+3} l^{4.5} d_{\mathbb{L}}^2 (1 + \log d_{\mathbb{L}})(1 + \log D) A_1 A_2 \ldots A_l,$$

*where $D = max\{|b_1|, |b_2|, \ldots, |b_l|\}$ and $A_1, A_2, \ldots, A_l$ are positive real numbers such that*

$$A_j \geq max\{d_{\mathbb{L}} h(\eta_j), |\log \eta_j|, 0.16\} \text{ for } j = 1, \ldots, l.$$

We use the following reduction method of Baker–Davenport due to Dujella and Pethő [6, Lemma 5] for bound reduction.

**Lemma 2.** *Let $M$ be a positive integer and $p/q$ be a convergent of the continued fraction of the irrational number $\tau$ such that $q > 6M$. Let $A$, $B$, $\mu$ be some real numbers with $A > 0$ and $B > 1$. Let $\varepsilon := \|\mu q\| - M\|\tau q\|$, where $\|.\|$ denotes the distance from the nearest integer. If $\varepsilon > 0$, then there exists no solution to the inequality*

$$0 < |u\tau - v + \mu| < AB^{-w},$$

*in positive integers $u, v, w$ with*

$$u \leq M \text{ and } w \geq \frac{\log(Aq/\varepsilon)}{\log B}.$$

The following lemma will be used in our proof. It is seen in [8, Lemma 7].

**Lemma 3.** *Let $r \geq 1$ and $H > 0$ be such that $H > (4r^2)^r$ and $H > L/(\log L)^r$. Then*

$$L < 2^r H (\log H)^r.$$

## 3. Proof of Theorem 1

Our aim is to find upper bounds for the variables $n, m_1, m_2$ of (2). If $m_1 = m_2$, then we assume $d_1 \leq d_2$. Using Binet's formula of Narayana's cows sequence in (2), we get

$$C_\alpha \alpha^{n+2} + C_\beta \beta^{n+2} + C_\gamma \gamma^{n+2} = d_1 \left(\frac{b^{m_1} - 1}{b - 1}\right) + d_2 \left(\frac{b^{m_2} - 1}{b - 1}\right). \quad (3)$$

We examine (3) in two different steps.

Firstly, we write (3) in the following way

$$C_\alpha \alpha^{n+2} - \frac{d_2 b^{m_2}}{b-1} = \frac{d_1 b^{m_1}}{b-1} - \frac{(d_1+d_2)}{b-1} - \left(C_\beta \beta^{n+2} + C_\gamma \gamma^{n+2}\right).$$

Taking absolute values on both sides and dividing by $\frac{d_2 b^{m_2}}{b-1}$, we get

$$\left|\left(\frac{(b-1)C_\alpha}{d_2}\right)\alpha^{n+2} b^{-m_2} - 1\right| < \frac{3}{b^{m_2-m_1-1}}. \tag{4}$$

Put

$$\Gamma = \left(\frac{(b-1)C_\alpha}{d_2}\right)\alpha^{n+2} b^{-m_2} - 1. \tag{5}$$

We need to show that $\Gamma \neq 0$. Suppose $\Gamma = 0$, then

$$C_\alpha \alpha^{n+2} = \frac{d_2}{b-1} b^{m_2}. \tag{6}$$

To show the above equality is absurd, let $G$ be the Galois group of the splitting field of the characteristic polynomial $f(x)$ over $\mathbb{Q}$ and let $\sigma \in G$ be an automorphism such that $\sigma(\alpha) = \beta$. Applying $\sigma$ on both sides of (6) and taking their absolute values, we get

$$|C_\beta \beta^{n+2}| = \frac{d_2}{b-1} b^{m_2}.$$

But $|C_\beta \beta^{n+2}| < |C_\beta| = 0.407506\ldots < 1$, whereas $\frac{d_2}{b-1} b^{m_2} \geq 4$ for $m_2 \geq 2$ which is not possible. Therefore, $\Gamma \neq 0$. To apply Theorem 2 in (5), let

$$\eta_1 = \frac{(b-1)C_\alpha}{d_2},\ \eta_2 = \alpha,\ \eta_3 = b,\ b_1 = 1,\ b_2 = n+2,\ b_3 = -m_2,\ l = 3,$$

where $\eta_1,\ \eta_2,\ \eta_3\ \in \mathbb{Q}(\alpha)$ and $b_1,\ b_2,\ b_3 \in \mathbb{Z}$. The degree $d_{\mathbb{L}} = [\mathbb{Q}(\alpha) : \mathbb{Q}]$ is 3.

Since $b^{m_2-1} < N_n < \alpha^{n-1}$, we have inequality $m_2 < n$. Therefore, $D = \max\{1, n+2, m_2\} = n+2$. To estimate the parameters $A_1, A_2, A_3$, we calculate the logarithmic heights of $\eta_1, \eta_2, \eta_3$ as follows:

$$h(\eta_2) = h(\alpha) = \frac{\log\alpha}{3},\ h(\eta_3) = h(b) = \log b,$$
$$h(\eta_1) = h\left(\frac{(b-1)C_\alpha}{d_2}\right) \leq h(b-1) + h(C_\alpha) + h(d_2).$$

The minimal polynomial of $C_\alpha$ over $\mathbb{Z}$ is $31x^3 - 31x^2 + 10x - 1$ with all its zeros of modulus $< 1$. Hence,

$$h(\eta_1) < 2\log b + \frac{\log 31}{3} < 4\log b.$$

Thus, one can take

$$A_1 = 12\log b,\ A_2 = \log\alpha \text{ and } A_3 = 3\log b.$$

We apply Theorem 2 and find

$$\log|\Gamma| > -1.4\cdot 30^6 3^{4.5} 3^2 (1+\log 3)(1+\log(n+2))(12\log b)(\log\alpha)(3\log b).$$

Comparing the above inequality with (4) gives

$$\log 3 - (m_2-m_1-1)\log b > \log|\Gamma| > -3.7\cdot 10^{13}(1+\log(n+2))(\log^2 b),$$

which reduces to

$$\begin{aligned}(m_2-m_1-1)\log b &< \log 3 + 3.7\cdot 10^{13}(1+\log(n+2))(\log^2 b)\\ &< 3.8\cdot 10^{13}(1+\log(n+2))(\log^2 b).\end{aligned}$$

Then, we get

$$(m_2-m_1) < 3.9\cdot 10^{13}(1+\log(n+2))(\log b). \tag{7}$$

Secondly, we rewrite (3) as

$$C_\alpha\alpha^{n+2} - \frac{d_1 b^{m_1} + d_2 b^{m_2}}{b-1} = -\frac{d_1+d_2}{b-1} - \left(C_\beta\beta^{n+2} + C_\gamma\gamma^{n+2}\right).$$

Taking absolute values on both sides and dividing by $C_\alpha\alpha^{n+2}$, we obtain

$$\left|1-\alpha^{-(n+2)} b^{m_2}\left(\frac{d_1 b^{m_1-m_2}+d_2}{(b-1)C_\alpha}\right)\right| < \frac{2.5}{C_\alpha\alpha^{n+2}} < \frac{6}{\alpha^n}. \tag{8}$$

Put

$$\Gamma' = 1-\alpha^{-(n+2)} b^{m_2}\left(\frac{d_1 b^{m_1-m_2}+d_2}{(b-1)C_\alpha}\right).$$

Using similar arguments as before we can show that $\Gamma' \neq 0$. With the notations of Theorem 2, we take

$$\eta_1 = \alpha,\ \eta_2 = b,\ \eta_3 = \frac{d_1 b^{m_1-m_2}+d_2}{(b-1)C_\alpha},\ b_1 = -(n+2),\ b_2 = m_2,\ b_3 = 1,\ l = 3,$$

where $\eta_1,\ \eta_2,\ \eta_3\ \in \mathbb{Q}(\alpha)$ and $b_1,\ b_2,\ b_3 \in \mathbb{Z}$. The degree $d_{\mathbb{L}} = [\mathbb{Q}(\alpha):\mathbb{Q}]$ is 3.

Since $m_2 < n$, $D = n+2$. Computing the logarithmic heights of $\eta_1, \eta_2$ and $\eta_3$, we get

$$h(\eta_1) = \frac{\log\alpha}{3},\ h(\eta_2) = \log b$$

and

$$\begin{aligned}h(\eta_3) &\le h(d_1 b^{m_1-m_2}+d_2) + h((b-1)C_\alpha)\\ &\le h(d_1) + (m_2-m_1)\,h(b) + h(d_2) + h(b-1) + h(C_\alpha) + \log 2\\ &< 3\log b + \log 2 + \frac{\log 31}{3} + (m_2-m_1)\log b\\ &< 6\log b + (m_2-m_1)\log b.\end{aligned}$$

Hence from (7) we get

$$h(\eta_3) < 6\log b + 3.9\cdot 10^{13}(1+\log(n+2))\log^2 b.$$

So, we take

$$A_1 = \log\alpha,\ A_2 = 3\log b \text{ and } A_3 = 11.8\cdot 10^{13}(1+\log(n+2))\log^2 b.$$

Using all these values in Theorem 2, we have

$$\log|\Gamma'| > -1.4\cdot 30^6 3^{4.5} 3^2(1+\log 3)(1+\log(n+2))(\log\alpha)(3\log b) \cdot (11.8\cdot 10^{13}(1+\log(n+2))\log^2 b).$$

Comparing the above inequality with (8) implies that

$$n\log\alpha - \log 6 < 3.65\cdot 10^{26}(1+\log(n+2))^2\log^3 b.$$

Thus, we conclude that

$$n < 9.81\cdot 10^{26}(1+\log(n+2))^2\log^3 b < 1.56\cdot 10^{28}(\log n)^2\log^3 b.$$

With the notation of Lemma 3, we take $r = 2$, $L = n$ and $H = 1.56\cdot 10^{28}\log^3 b$. Applying Lemma 3, we have

$$\begin{aligned} n &< 2^2(1.56\cdot 10^{28}\log^3 b)(\log(1.56\cdot 10^{28}\log^3 b))^2 \\ &< (6.24\cdot 10^{28}\log^3 b)(65+3\log\log b)^2 \\ &< (6.24\cdot 10^{28}\log^3 b)(93\log b)^2 \\ &< 5.39\cdot 10^{32}\log^5 b. \end{aligned}$$

For a fixed base $b$, the equation (2) has only finitely many solutions. Once $b$ is fixed, we can determine all the solutions of (2) explicitly.

Now, as an illustration, we solve the equation (2) for $b = 10$. When $b = 10$, the bound on $n$ becomes

$$n < 3.4\cdot 10^{34}.$$

From Lemma 1, we find

$$m_1 \le m_2 < 5.64\cdot 10^{33}.$$

Our next aim is to reduce these bounds of (2). Put

$$\Lambda = (n+2)\log\alpha - m_2\log 10 + \log\left(\frac{9C_\alpha}{d_2}\right).$$

The inequality (4) can be written as

$$|e^\Lambda - 1| < \frac{3}{10^{m_2-m_1-1}}.$$

Observe that $\Lambda \neq 0$ as $e^{\Lambda}-1 = \Gamma \neq 0$. Assuming $m_2-m_1 \geq 2$, the right-hand side in the above inequality is at most $\frac{3}{10} < \frac{1}{2}$. The inequality $|e^z - 1| < y$ for real values of $z$ and $y$ implies $z < 2y$. Thus, we get

$$|\Lambda| < \frac{6}{10^{m_2-m_1-1}},$$

which implies that

$$\left|(n+2)\log\alpha - m_2\log 10 + \log\left(\frac{9C_\alpha}{d_2}\right)\right| < \frac{6}{10^{m_2-m_1-1}}.$$

Dividing both sides by $\log 10$ gives

$$\left|n\left(\frac{\log\alpha}{\log 10}\right) - m_2 + \left(\frac{\log(9\alpha^2 C_\alpha/d_2)}{\log 10}\right)\right| < \frac{2.7}{10^{m_2-m_1-1}}. \tag{9}$$

To apply Lemma 2 in (9), let

$$u = n,\ \tau = \left(\frac{\log\alpha}{\log 10}\right),\ v = m_2,\ \mu = \left(\frac{\log(9\alpha^2 C_\alpha/d_2)}{\log 10}\right),$$

$$A = 2.7,\ B = 10,\ w = m_2 - m_1 - 1.$$

Choose $M = 3.4\cdot 10^{34}$. We find $q_{61} = 837814603282183274510378124425469951$ exceeds $6M$ with $0.120711 < \varepsilon := \|\mu q_{61}\| - M\|\tau q_{61}\| < 0.454115$. Applying Lemma 2 for $1 \leq d_2 \leq 9$, we get

$$m_2 - m_1 - 1 \leq \frac{\log(2.7\cdot 837814603282183274510378124425469951/0.120711)}{\log 10}.$$

Thus, $m_2 - m_1 - 1 \leq 37$.

Now for $1 \leq d_1, d_2 \leq 9$ and $m_2 - m_1 \leq 38$, put

$$\Lambda' = -(n+2)\log\alpha + m_2\log 10 + \log\left(\frac{d_1 10^{m_1-m_2} + d_2}{9C_\alpha}\right).$$

From (3), we have

$$C_\alpha\alpha^{n+2}\left(1 - e^{\Lambda'}\right) = -\left(\frac{d_1+d_2}{9}\right) - \left(C_\beta\beta^{n+2} + C_\gamma\gamma^{n+2}\right).$$

Furthermore, we obtain

$$\frac{d_1+d_2}{9} + \left(C_\beta\beta^{n+2} + C_\gamma\gamma^{n+2}\right) > 0.$$

So $e^{\Lambda'} - 1 > 0$. Thus, $\Lambda' > 0$ and we have

$$0 < \Lambda' < e^{\Lambda'} - 1 = |\Gamma'| < \frac{6}{\alpha^n}.$$

This implies

$$\left|-(n+2)\log\alpha + m_2\log 10 + \log\left(\frac{d_1 10^{m_1-m_2} + d_2}{9C_\alpha}\right)\right| < \frac{6}{\alpha^n}.$$

Dividing both sides by $\log\alpha$ gives

$$\left|m_2\left(\frac{\log 10}{\log\alpha}\right)-n+\left(\frac{\log\left((d_1 10^{m_1-m_2}+d_2)/9C_\alpha\right)}{\log\alpha}-2\right)\right|<16\cdot\alpha^{-n}. \quad (10)$$

Now, let

$$u=m_2,\ \tau=\left(\frac{\log 10}{\log\alpha}\right),\ v=n,\ \mu=\left(\frac{\log\left((d_1 10^{m_1-m_2}+d_2)/9C_\alpha\right)}{\log\alpha}-2\right),$$

$$A=16,\ B=\alpha,\ w=n.$$

Choose $M=3.4\cdot 10^{34}$. Find $q_{61}=5030181332394063736620036033151353623$ exceeds $6M$ with $0.000137436<\varepsilon:=\|\mu q_{61}\|-M\|\tau q_{61}\|<0.499986$. Then we apply Lemma 2 to the inequality (10) for $1\le d_1,d_2\le 9$ and $m_2-m_1\le 38$ and get

$$n\le\frac{\log(16\cdot 5030181332394063736620036033151353623/0.000137436)}{\log\alpha}\le 251.$$

We compute all the solutions of the equation (2) using *Mathematica* for the above range and find the following solutions

$$\begin{aligned}N_{14}=88=11+77&=\frac{10^2-1}{9}+7\left(\frac{10^2-1}{9}\right),\\&=22+66=2\left(\frac{10^2-1}{9}\right)+6\left(\frac{10^2-1}{9}\right),\\&=33+55=3\left(\frac{10^2-1}{9}\right)+5\left(\frac{10^2-1}{9}\right),\\&=44+44=4\left(\frac{10^2-1}{9}\right)+4\left(\frac{10^2-1}{9}\right),\end{aligned}$$

and

$$N_{17}=277=55+222=5\left(\frac{10^2-1}{9}\right)+2\left(\frac{10^3-1}{9}\right).$$

Hence the theorem is proved.

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## References

[1] J. P. Allouche and T. Johnson, *Narayana's cows and delayed morphisms*, in articles of 3rd computer music conference JIM96, France, (1996).

[2] C. Adegbindin, F. Luca, and A. Togbé, *Pell and Pell–Lucas numbers as sum of two repdigits*, Bull. Malays. Math. Sci. Soc. **43** (2020), 1253–1271.

[3] C. Adegbindin, F. Luca, and A. Togbé, *Lucas numbers as sums of two repdigits*, Lith. Math. J. **59** (2019), 295–304.

[4] J. J. Bravo, P. Das, and S. Guzmán, *Repdigits in Narayana's cows sequence and their consequences*, J. Integer Seq. **23** (2020), Article 20.8.7, 15 pp.

[5] Y. Bugeaud, M. Mignotte, and S. Siksek, *Classical and modular approaches to exponential Diophantine equations I. Fibonacci and Lucas perfect powers*, Ann. Math. (2) **163** (2006), 969–1018.

[6] A. Dujella and A. Pethő, *A generalization of a theorem of Baker and Davenport*, Quart. J. Math. Oxford Ser. **49** (1998), 291–306.

[7] B. Faye and F. Luca, *Pell and Pell–Lucas numbers with only one distinct digit*, Ann. Math. Inform. **45** (2015), 55–60.

[8] S. Gúzman Sánchez and F. Luca, *Linear combinations of factorials and $s$-units in a binary recurrence sequence*, Ann. Math. Qué. **38** (2014), 169–188.

[9] F. Luca, *Fibonacci and Lucas numbers with only one distinct digit*, Port. Math. **57** (2000), 243–254.

[10] E. M. Matveev, *An explicit lower bound for a homogeneous rational linear form in the logarithms of algebraic numbers II*, Izv. Ross. Akad. Nauk Ser. Mat. **64** (2000), 125–180. Translation in Izv. Math. **64** (2000), 1217–1269.

[11] S. G. Rayguru and G. K. Panda, *Repdigits as product of consecutive Balancing or Lucas-Balancing numbers*, Fibonacci Quart. **56** (2018), 319–324.

[12] S. G. Rayguru and G. K. Panda, *Balancing numbers which are concatenation of two repdigits*, Bol. Soc. Mat. Mex. **26** (2020), 911–919.

[13] S. G. Rayguru and G. K. Panda, *Balancing and Lucas-balancing numbers expressible as sums of two repdigits*, Integers **21** (2021), Paper No. A7, 17 pp.

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