A Kantian Interpretation of Niels Bohr's Early Correspondence Principle: 1917–1924

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In the present paper I aim to discuss the philosophical foundations of the early correspondence principle (1917–1924), by comparing the conceptual structure underlying the first correspondence principle with the procedure of analogy that Immanuel Kant introduced in the *Critique of Judgment* from 1790. On such a comparison, I will seek to demonstrate the consistency of the conceptual ratio according to which the correspondence principle is to the classical "concepts" of space and time, as these *a priori* forms of intuition (space and time), in Kant, are related to the separate *faculty* of pure intuition. As a result, it will turn out that the conceptual structure of the correspondence principle suits to the Kantian doctrine of a separate *faculty* of pure intuition, which is divided from the *faculty* of understanding. The aim is to shed new light on the line of reasoning underlying Niels Bohr's *analogical* thinking in quantum physics.

Keywords: Bohr, Kant, atomic theory, correspondence principle, quantum physics

1. Introduction

The development of quantum physics took place in three stages. The first, roughly 1900–1913 saw the development of what is known as the "first quantum theory," which explained the full spectrum of thermal radiation. The second (1913–1925), known as the "old quantum theory" was a sort of hybrid theory, in that it retained much of the structure of classical mechanics and added postulates inspired by experimental results on light emitted and absorbed by atoms, postulates that did not cohere well with classical mechanics. The years 1925–1932 saw the formulation of what we now call quantum mechanics, which forms the basis of contemporary physics.

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Niels Bohr (1885–1962) was one of the architects of the old quantum theory, and he gave tremendous contributions to the elaboration of the new quantum mechanics as well. Moreover, his name is linked to the formulation of one of the most known and less understood principle of quantum theory: the correspondence principle.

Bohr formulated the correspondence principle in the context of the old quantum theory, but for some commentators and historians of physics its conceptual importance can be extended to quantum mechanics. What is undisputed is the fact that Niels Bohr's correspondence relation is today still a rather controversial scientific principle. Scientists, philosophers of science, and historians of science have indeed confronted each other on its philosophical foundations without ever reaching a real real agreement, perhaps the main reason being that the correspondence principle seems to assume different meanings with the passing of time.

At least up to the advent of the new quantum mechanics, in 1925, the correspondence principle was a *formal analogy* between atomic processes and classical harmonic components of motion, that is, an analogy was established between a certain high quantum number limit of the quantum-theoretical frequencies and the classical frequencies. To say it with Bohr's words, under certain conditions (for high quantum numbers, or with h, the Planck's constant, very small): "The possibility of an emission of a radiation of such a frequency [between two stationary states: i.e. any of several energy states an atom may occupy without emitting electromagnetic radiation] may also be interpreted from *analogy* with the ordinary electrodynamics" (Bohr 1913, 14), as an electron rotating round a nucleus in an elliptical orbit will emit a radiation whose frequency is the frequency of revolution of the electron. Therefore, if one had to drop any space-time pictures of events (as it happened after 1924 as a consequence of the crisis of the helium atom model), even the correspondence principle as a formal analogy had to be relinquished. And what remained was the general idea of the formal correspondence between classical harmonic components of motion (that is, broadly speaking, the frequency of the electron in the elliptical orbit) and quantum jumps (Darrigol 1997). It was at this stage of the development of quantum theory that atomic processes lost their corresponding orbital models, but the correspondence relation could still survive assuming a new meaning: it held in a purely symbolic manner, because transition amplitudes had replaced classical components of motion (Bohr 1925, 852). As a result, the established analogy between transition amplitudes and the mechanics of orbits was so *close* that the set of transitions amplitudes became the *true* atomic motion.

In the present paper, I want to discuss the philosophical foundations of the early correspondence principle (1917–1924), by comparing the conceptual structure underlying the first correspondence principle with the procedure of analogy used by Immanuel Kant.

The aim is to shed new light on the line of reasoning underlying Bohr's *analogical* thinking in quantum physics.

2. Bohr's first atomic theory

In this section, I will offer a glimpse of the formulation of the first atomic theory that Niels Bohr started to elaborate in 1912–1913. To begin with, it is worth remembering that, at the time, Bohr's aim was to explain the properties of chemical elements through the application of the quantum postulates (stationary states and frequency rule) to Rutherford's planetary model of atoms. In 1911, Ernest Rutherford proposed a planetary model of the atom with a concentrated, massive nucleus, which only could give account of the large-angle scattering observed in the experiments performed by Hans Geiger and Ernest Marsden at Manchester's laboratories.

Atomic models of the time were mainly based on ordinary electrodynamics, for this reason they sooner or later would have led to irreconcilable contradictions. But things started to change in the early 1910s, when most part of quantum physicists became aware of the inadequacy of the classical theories at the atomic scale. After the pioneering works of Planck (1900), who introduced the constant h, and Einstein (1905, 1906), who extended Planck's hypothesis by propounding the idea that even light was made of discrete quanta (and with his success in explaining the low-temperature degree in the specific heats of solids), others followed: Hendrik Lorentz advocated the hypothesis of an intimate connection between atomic structures and the quantum of action. Arthur Haas also carried on this program. However, the ideas were missing about how to explain the discrete emission spectra of atoms from their constitution with the help of quantum theory. Niels Bohr provided such ideas in the first of three fundamental papers published in 1913: "On the Constitution of Atoms and Molecules."

As is well known, Rutherford's model too was mechanically and electrodynamically unstable. To remedy the stability problem, Bohr introduced the concept of "stationary states" (in which the energy of the system is constant, as the electron orbits around the atomic nucleus without emitting radiation), which were subjected to the following assumptions:

(i) An atom can exist only in a discontinuous series of stationary states; (ii) the motion of electrons in a stationary state can be discussed by means of the ordinary mechanics (Bohr 1913, 7); (iii) the passing of electrons between stationary states cannot be described by means of ordinary mechanics and electrodynamics (Bohr 1913, 7); (iv) the frequency of the radiation emitted or absorbed during a transition between two stationary states is given by the difference in energy between these states divided by Planck's constant i.e. the frequency rule, $h\nu = E_1 - E_2$, where h is Planck's constant, ν is the frequency of the emitted radiation, E_1 and E_2 are respectively the energy of two stationary states.

As we can see, the assumption (i) can be also derived from Einstein's idea of quantization; moreover, Johannes Stark had already used the concept of stationary states in 1911.

By contrast, the assumption (iv) is a revolutionary step and implies the daring idea for which Bohr's atomic theory is known: the frequency of the motion of the electron differs from the frequency of the emitted radiation. In 1906, Einstein had already prospected this idea (the discrete selection of mechanical states) although he was not brave enough to carry it through.

Since the assumptions (ii) and (iii) were not sufficient to determine the energy of stationary states, Bohr had to introduce a "quantum rule": the energy of the n^{th} stationary state is $nh/2$ times the orbital frequency of the electron in this state. Assuming that the orbit of the electron is circular, this assumption is equivalent with the assumption that the angular momentum of the electron round the nucleus is equal to an entire multiple of $h/2\pi$.

In particular, by setting the frequency of the emitted radiation equal to half the final orbital frequency, Bohr renounced the classical relation between motion and emitted radiation, although he still required an analogous relation to subsist at the quantum level. This requirement is the conceptual embryo of what Bohr would later call the correspondence principle.

Between 1913 and 1916, a series of corroborated predictions followed, which, along with Einstein's and Sommerfeld's theoretical efforts, contributed to the consolidation of the two fundamental assumptions of stationary states and frequency rule, whose status of postulates would shift away in the course of time.

Bohr's atomic theory was able to account for some characteristic features of (i) the so-called Stark's effect, i.e. the effect of the electric field on spectral lines, (ii) the Zeeman's effect, i.e. the effect of the magnetic field on spectral lines, and (iii) the double spectral lines.

Above all, the theory explained the inelastic collision of an electron with a mercury atom observed during the experiments conducted by James Franck and Gustav Hertz in 1913. The Franck-Hertz experiments were an indirect confirmation of both the frequency rules and the existence of stationary states.

> Franck and Hertz assume that 4.9 volts corresponds to the energy necessary to remove an electron from the mercury atom, but it seems

> that their experiments may possibly be consistent with the assumption that this voltage corresponds only to the transition from the normal state to some other stationary state on the neutral atom. (Bohr 1915, 410–411)

In spite of the empirical evidence, Bohr doubted the generality of the frequency rule at least until 1915, to the extent that he hypothesized that a correction to this rule was needed for explaining the splitting of spectral lines in magnetic and electric fields. In this conceptual framework, Bohr went to mature the idea of the stability of stationary states as the only fundamental assumption, i.e. endowed with the necessary generality, of his atomic theory. To reinforce this idea, Bohr introduced this fundamental assumption:

$$\frac{\bar{T}}{\omega} = \frac{1}{2}hn \tag{2.1}$$

with $\bar{T}$ the average of the kinetic energy, and ω the frequency of the electron in its periodic orbit. This relation is a generalization of Planck's quantum hypothesis for the harmonic oscillator of one dimension (Rosenfeld and Hoyer 1981, 341); that is, the principle $E = nh\omega$. The relation $\frac{\bar{T}}{\omega}$ was suggested by Ehrenfest's theory of adiabatic invariance, according to which $\frac{\bar{T}}{\omega}$ was the fundamental quantity subject to a quantum condition.

The so-called principle of mechanical transformability of stationary states, better known as Ehrenfest's adiabatic theory, was regarded by Bohr as a fundamental assumption to be used "to fix" the stationary states of an atomic system among the continuous multitude of mechanically possible motions (Bohr 1918, 9).

As a matter of fact, the quantum conditions must be adiabatic invariant: the probability of the stationary states must remain unaltered during continuous transformations, which must be infinitely slow.

To summarize, Bohr needed a relation to define the variation of the stationary states in a system under external conditions. According to the relation (2.1), these variations cannot be calculated on the basis of the classical theories of mechanics and electrodynamics. But, if the variation of the external conditions is very slow (adiabatic), we may expect that the motion of the system in the stationary states can be defined by the application of ordinary mechanics and electrodynamics.

All other assumptions, including the already mentioned frequency rule, could hold only in the case of periodic systems. For this reason, Bohr, for the time being (until 1915), limited the application of ordinary mechanics to the motion in the stationary states of periodic systems like the harmonic oscillator and the hydrogen atom, without extending it to multi-periodic systems.

In 1916, Einstein published the article “Emission and absorption of radiation in quantum theory,” in which he showed that both Planck’s blackbody law and Bohr’s theory of spectra could be derived by assuming conditions for the statistical equilibrium on the relation between quantum transitions and the density of the associated emitted radiation.

Einstein’s achievement was a fundamental contribution to the consolidation of a more general frequency rule and the existence of stationary states.

A second effort in this direction came from Sommerfeld’s 1916-work. Sommerfeld and his collaborators applied some relativistic corrections to the electrons orbiting around the nucleus, which, according to Bohr’s theory of hydrogen atom, followed Kepler’s laws; that is: the electrons orbit the nucleus as a planet orbits the sun, in an ellipse with the sun at one focus, while the electron is influenced by an attraction inversely proportional to the square of the distance in accord with Newton’s law. In the case of electrons, however, the attraction at a given distance is not determined by mass, but by the electric charges of the electrons themselves. But at unchanged charges and mass variation, the electronic motion would be also modified. According to Einstein’s theory of special relativity, the mass of the electrons depends on the velocity, which varies in an elliptical orbit.

It turned out that the electronic motion assumed a value of more general nature with respect to the Kepler-like ellipses considered in the Bohr’s theory.

Einstein’s and Sommerfeld’s equations stood for “particular laws” that served the purpose to pinpoint the two postulates (stationary states and frequency rule), along with the empirical evidence offered by the Franck-Hertz experiments, on the way to the quantization of atomic systems. As Bohr pointed out, Einstein derived Planck’s law by introducing further assumptions regarding the probability of transition of a system between two stationary states, by means of Boltzmann’s principle on the relation between entropy and probability, and Wien’s displacement-law (cf. Bohr 1918, 7). Einstein’s attempt can therefore be regarded as a direct support to the generality of the frequency rule.

As is known, until 1915 Bohr had found a quantum rule only for periodic systems, since he had expected the frequency rule to breakdown for non-periodic systems. Sommerfeld was searching for a quantization method for multi-periodic systems, which included the relativistic correction for the Kepler problem, the Stark effect, and the Zeeman effect in relation to the model of the hydrogen atom. Yet the relation he found out would have soon shown a purely formal and arbitrary character (Kramers 1923, 148). Beyond a shadow of a doubt, Sommerfeld started from considerations similar to those Bohr had originally applied to the definition of the stationary states in the hydrogen atom. With his collaborators, Sommerfeld arrived at some formal quantum rules, which allowed fixing the stationary states in the hydrogen atom corresponding to the so-called double sets of quantum numbers. However, such method was still of purely formal nature, as Sommerfeld put forward a blind generalization of the quantum rules and of the frequency rules, as such his theory predicted more spectral lines than those expected by experimental results.

Sommerfeld's method, known as the principle of selection, did not succeed in providing empirical evidence in the case of the Stark and Zeeman effects in hydrogen atoms, but it held only if hydrogen atoms were undisturbed.

I point out that Bohr's main objective was to describe the interactions between radiation and matter. In order to pursue this aim, Bohr could not limit his theory to the undisturbed hydrogen atom, but he had to extend it to more complex systems. This fact implied the application of Bohr's atomic theory to multi-periodic systems. This was the roadmap to the formulation of the correspondence principle.

3. The conceptual structure of the correspondence principle

Bohr arrived at the correspondence principle by means of the following rational tools: Ehrenfest's adiabatic principle, Sommerfeld's selection rule, and Schwarzschild's and Epstein's analytical mechanics. It was the Epstein-Schwarzschild theory, in particular, which allowed extending the conditions of state to multi-periodic motions, i.e. to a set of systems more complex than those considered at first. It is worth pointing out that these tools allowed Bohr to protect and carry on his mechanical model of the atom, and, at the time, there was no other way (in this sense we can speak of "rational tools") "of describing in details the process of direct transition between two stationary states accompanied by an emission or absorption of radiation, and we cannot be sure beforehand that such a description will be possible at all by means of laws consistent with the application of the principle of conservation of energy" (Bohr 1923b).

The correspondence principle provided Bohr's theory with a wider generality with respect to previous attempts of quantization of atomic systems (Darrigol 1992, 125). Specifically, the correspondence principle allowed explaining the behavior of perturbed systems, which did not have to be multi-periodic. By the formulation of the correspondence principle, Bohr's theory permitted the derivation of selection rules, intensities, and polarizations of the perturbed systems, not only the spectra. The correspondence principle did not require the perturbed systems to be multi-periodic, provided that it applied to both periodic and multi-periodic systems. As a result, Bohr's method became a subtle selection rule of multi-periodic motions from a more fundamental non-multi-periodic system.

In 1917 Bohr formulated the concept of the "correspondence relation" in a draft of the paper "On the Quantum Theory of Line Spectra," which was published in 1918. There, Bohr was able to derive Sommerfeld's "selections rule" for the Stark and Zeeman effects and in the fine structure of the hydrogen atom. From his previous results in the case of strictly periodic motions, Bohr knew that each allowed quantum transition between stationary states corresponded to one harmonic component of the classical motion. By applying this relation to the Zeeman effect, he was able to derive Sommerfeld's selection rule.

Bohr argued that the correspondence principle was not another rule to add to the set of laws of the quantum theory, rather, in his view, it represented a fundamental law of much greater generality with respect to previous attempts of quantization of the theory.

Bohr wanted to re-establish continuity between the radiation associated to the transition between stationary states and the observed radiation according to classical electrodynamics. For this reason, we could think of this principle as an extension of previous attempts of quantization of the atom.

More specifically, the correspondence principle required that some of the relations satisfied by the classical harmonics of motion should be preserved (exactly or approximately) for the "corresponding" quantum-theoretical intensities (Darrigol 2014, 247). This is the inner meaning of the correspondence principle, which preserves the relation of the classical harmonics of motion for the corresponding quantum-theoretical intensities.

To sum up, until 1922 the correspondence principle played a guiding role in an open theory, as it showed a heuristic validity (Jammer 1966) in systems without the limitation of multi-periodicity. In particular, the correspondence principle permitted the first extension of the quantum theory to so-called perturbed systems, which did not have to be multi-periodic. Furthermore, when in 1920 Kramers published the results regarding the first application of the correspondence principle, these showed that this method succeeded in determining the effects of a small electric field on the fine structure of a hydrogen atom, contrary to Sommerfeld's selection rule, which in this case showed its impotence.

Yet, after two years, the American physicist John Van Vleck carried out a new calculation of the ionization potentials for the Bohr-Kemble model of the helium atom, whose results led Bohr to realize that the correspondence principle could not be used to calculate the energy of the stationary states in atoms with more than one electron.

It is worth remembering that through the introduction of the principle of mechanical transformability, Bohr wanted to show that the energy definition of stationary states depended on the possibility of continuously deforming the system in a way that would connect the stationary states (Darrigol 1992, 176). This property, which was introduced in order to define the energy for multi-periodic systems, and it was one of the fundamental assumptions of the correspondence principle, did not hold in the case of the helium atom. In fact, as Kramers showed, the motion in the normal state of the helium model was mechanically unstable. The failure of the Bohr-Kemble model was a breakthrough in the old quantum theory; as a consequence Bohr was brought to reject any space-time description of radiation process.

However, I want to point out that the problem of the instability of the helium atom was not Bohr's chief preoccupation at the time. As is well known, since mid-1922, both Bohr and Kramers had already become aware of the mechanical instability of the normal state of the helium atom. Notwithstanding the undisputable heuristic power of the correspondence principle—which in 1922 guided Coster and Hevesy to the discovery of the missing element 72 (hafnium)—Bohr did not overestimate the coherence of the theoretical structure of his atomic theory, which was based on the correspondence relation. Coherently, Bohr expressed a cautious optimism even in his Nobel lecture of 11 December 1922:

> By a theoretical explanation of natural phenomena we understand in general a classification of the observations of a certain domain with the help of *analogies* pertaining to other domains of observation, where one presumably has to do with simpler phenomena. The most that one can demand of a theory is that this classification can be pushed so far that one can contribute to the development of the field of observation by the prediction of new phenomena. When we consider the atomic theory, we are, however, in the peculiar position that there can be no question of an explanation in this last sense, since here we have to do with phenomena which from the very nature of the case are simpler than in any other field of observation, where the phenomena are always conditioned by the combined action of a large number of atoms. We are therefore obliged to be modest in our demands and content ourselves with concepts which are formal in the sense that they do not provide a visual picture [Anskuelighed] of the sort one is accustomed to in the explanations with which natural philosophy deals. (Bohr 1923c, 44, italics added)

That is to say that Bohr warned against false optimism, as far as he was aware of the formal character of the correspondence principle and the provisionality of atomic orbits, to the extent that he was ready to modify their configuration on the basis of new empirical findings.

Returning to the failure of the helium model, it is worth noting that the rejection of the so-called "second atomic theory" did not concern Bohr's previous use of space-time pictures of electronic motion.

> The quantum theory presents [...] a sharp departure from the ideas of classical electrodynamics in the introduction of discontinuities into the laws of nature. From the present point of view of physics, *however, every description of natural processes must be based on ideas which have been introduced and defined in the classical theory.* The question therefore arises, whether it is possible to present the principles of the quantum theory in such a way that their application appears free from contradiction. (Bohr 1923a, 1, italics added)

The space-time representation was the necessary basis for the application of the correspondence principle, and its conceptual structure was hence closely related to the use of classical concepts, specifically the classical concepts of space, time, and causality. Classical concepts would later become, for Bohr, the necessary conditions for an unambiguous communication and descriptions of quantum events.

Bohr aimed to define the relation between emitted radiation and electronic motion by using a "formal analogy" with the classical relation in the limit of high quantum numbers, although no causal and spatio-temporal representation of the quantum processes, connecting emitted radiation and electronic motion, could be given at the quantum level.

In spite of the a-causal nexus between emitted radiation and electronic motion, the empirical data related to quantum phenomena, according to Bohr, had to be subsumed under the category of causality. Bohr never departed from this conceptual stance.

> It should not be forgotten that the concept of causality underlies the very interpretation of each result of experiment, and that even in the co-ordination of experience one can never, in the nature of things, have to do with well-defined breaks in the causal chain. (Bohr 1937, 293)

Since the classical principle of causality should underlie the "very interpretation of each result of experiment," one would expect that a causal relation had to be established also between the frequency of the orbiting electron and the frequency of the emitted radiation.

In particular, the introduction by Bohr of the two assumptions determined the impossibility to provide a causal and spatio-temporal description of atomic processes, that is, the impossibility to define the nature of the emitted radiation. Bohr hence sought to establish a connection between the motion in the stationary states of an atomic system and the possibility of a transition between two of these states, which he found out in the correspondence principle. Nevertheless, the "correspondence relation" was a merely formal device: it succeeded, in fact, in describing the processes taking place during the formation and re-organization of the atom, without providing, though, the causal and spatio-temporal descriptions of quantum processes. Bohr recognized the "formal character" of the correspondence principle: since this principle could not explain the emitted radiation as an effect of the orbital frequency, it had to rely on the "formal analogy" between the structures of the quantum theory and classical electrodynamics to account for the nature of that radiation.

In a draft of the introduction to the 1918 paper "On the Quantum Theory of Line-Spectra," Bohr regarded the theory of line spectra, based on the relation $h\nu = E_1 - E_2$, as a "formal" tool, as such only in a formal way the quantum theory could be seen as a "natural generalization" of the classical theory of radiation. Furthermore, in 1920, Bohr used for the first time the expression "correspondence principle" in the paper "On the Series Spectra of the Elements," in which he conveyed the idea of a "formal analogy" between classical electrodynamics and the quantum theory of radiation.

> Although the process of radiation cannot be described on the basis of the ordinary theory of electrodynamics, according to which the nature of the radiation emitted by an atom is directly related to the harmonic components occurring in the motion of the system, there is found, nevertheless, to exist a far-reaching correspondence between the various types of possible transitions between the stationary states on the one hand and the various harmonic components of the motion on the other hand. This correspondence is of such a nature, that the present theory of spectra is in a certain sense to be regarded as a rational generalization of the ordinary theory of radiation. (Bohr 1920, 23–24)

In 1923, Bohr was still convinced of the formal character of the correspondence principle, as it is evident from the following quotation:

> If the correspondence principle cannot instruct us in a direct manner concerning the nature of the process of radiation and the cause of the stability of the stationary states, it does elucidate the application of the quantum theory in such a way that one can anticipate an inner consistency for this theory of a kind similar to the formal consistency of the classical theory. (Bohr 1923a, 25)

4. For a Kantian interpretation of the correspondence principle

It should be noted that some commentators (Shimony 1983; Honner 1987; Faye 1991; Faye 2014; Beller 1999; MacKinnon 2012) argued that in emphasizing the necessity of the classical concepts for the description of quantum phenomena, Bohr applied a transcendental method to his interpretation of quantum mechanics. For example, Shimony (1983) noted that Bohr claimed that an unequivocal description of an atomic phenomenon must include a description of all the relevant elements of the experimental apparatus as a consequence of the introduction of the quantum of action. This issue is an aspect of the indispensability of classical concepts as far as these concepts are a direct extension of our intuition: they are necessary to describe experiments and to communicate to others what we have done and learned. That is, an epistemological divide between classical concepts and quantum phenomena is triggered by the introduction of the Planck's h, which recalls the Kantian demarcation between "phenomenon" and "noumenon." What is important is not the world in it-self, but what can be described. Moreover, Bohr denied that classical concepts could be used to attribute properties to a physical world "in-itself" behind the phenomena, i.e. properties different from those being observed (Faye 2014). On the same track, Honner (1987) emphasized Bohr's insistence, following Heisenberg, that "quantum manipulations" be restricted to "observables."

Mara Beller (1999) rightly emphasized in Bohr's works the connections between key concepts, such as causality, visualizability, objectivity, and the distinction between subject and object, which firmly entrenched in Kantian philosophy.

According to MacKinnon, Bohr had been using a distinction between *descriptive* concepts and *formal* concepts, or concepts whose meaning depends on functioning in a system. This aspect of Bohr's insight into quantum physics fits neatly a transcendental interpretation of Bohr's thinking.

More specifically, other specialists (Chevalley 1994; Pringe 2007; 2009) recognized in the enunciation of the correspondence principle the mark of Kant's philosophy, as far as the analogical procedure underlying this principle would link to Kant's conception of symbolic presentation, which indeed uses analogy.

The origin of Kant's usage of analogy can be traced back to the *Anschauung-Symbol* distinction, which Kant dealt with in the *Critique of Judgment* from 1790.

In the *Critique of Pure Reason* from 1781, Kant had laid down the procedure of "Transcendental Schemata," according to which the understanding applies its categories to whatever is presented in *a priori* forms of intuition, i.e. space and time.

This procedural rule is established either in relation to pure intuition, as for mathematics, or in relation to empirical intuition, as for physics. When no sensation can be directly presented in intuition (as for the concept of God, which can only be thought by Reason), hypotheses, i.e. "presentations," have to be regarded as symbolical. That is to say: only an indirect presentation of the concept in intuition is allowed.

Kant argued that these hypotheses are either "schemata" (containing direct presentations of concepts in intuition) or "symbol" (containing indirect presentations of concepts in intuition).

Schemata perform this procedure demonstratively, whereas symbols do this by means of an analogy, in which the power of judgment plays a twofold role: on the one hand, it applies "the concept to the object of a sensible intuition," on the other hand, it applies "the mere rule of reflection on that intuition to an entirely different object, of which the first is only the symbol" (Kant 2000, 352).

Let us compare Kant's line of reasoning with regard to "analogy" and "symbol" with the conceptual structure of Bohr's correspondence principle.

The quantum theory is based on the quantum postulate as an empirical assumption. Nevertheless, this postulate undermines the possibility to apply the categories of understanding to a quantum object as an object of possible experience; that is, the quantum postulate does not permit a direct presentation of quantum concepts in intuition, as it violates a necessary condition for the interpretation of empirical results: the causal connection between electronic motion and radiation.

It turns out that quantum concepts can be represented only by analogy. The electronic motion in a stationary state, for instance, cannot be presented directly in intuition, as this motion lacks the empirical content associated to radiation spectra (Pringe 2007). It follows that the "presentation" of the motion of the electron in a stationary state assumes a symbolic character, i.e. it is merely formal. The symbolic presentation of quantum events uses the correspondence principle, which, indeed, requires that some of the relations satisfied by the classical harmonics of motion should be preserved for quantum-theoretical intensities.

On the one hand, the power of judgment applies a "concept" to the classical harmonics of motion, which stand for an object of the sensible intuition; on the other hand, the power of judgment applies "the mere rule" by which it reflects on "that sensible intuition" (classical harmonics of motion) to quantum-theoretical intensities, of which the classical harmonics of motion are the symbol. It is evident that the correspondence principle makes possible the re-presentation of quantum events (in analogous intuition) by a transfer of rule or structure (Chevalley 1994) between two quite different things: from classical harmonics of motion (which are objects of sensuous intuition) to quantum phenomena (to which "reflection" applies the structure of the object of sensuous intuition). The correspondence relation thus provides a quantum phenomenon with the "formal structure" of the "concept" of the corresponding classical harmonic component of motion, which is used as the symbol of that quantum phenomenon.

The correspondence principle is the analogical procedure by which an atomic process, for instance the transition of an electron between stationary states, can be predicted through the presence of the corresponding classical harmonic of motion.

> This correspondence between frequencies determined by the two methods must have a deeper significance and we are led to anticipate that it will also apply to the intensities. This is equivalent to the statement that, when the quantum numbers are large, the relative probability of a particular transition is connected in a simple manner with the amplitude of the corresponding harmonic component in the motion. This peculiar relation suggests *a general law for the occurrence of transitions between stationary states*. Thus we shall assume that even when the quantum numbers are small the *possibility of transition between two stationary states is connected with the presence of a certain harmonic component in the motion of the system*. (Bohr 1920, 27–28, italics added)

I want to make clear a point with regard to the relationships between Kant and Bohr. Namely, I do not want to argue that Bohr's correspondence principle stems from Kant's procedure of analogy. Neither do I want to lay emphasis on Bohr's debts towards Kant's philosophy,[1] except for the terminology. I am rather convinced that certain similarities between the logical procedures used by the two thinkers cannot be denied. Following a previous comparison between Kant's analogy and Bohr's correspondence principle, it seems to me that the correspondence principle is to the classical "concepts" of space and time, as these *a priori* forms of sensible intuition, in Kant, are related to the separate faculty of pure intuition. I argue once more that the correspondence relation, at least until 1924, was compatible with a Kantian-like scheme, and no longer could be an integral part of the 1925 Heisenberg's (and Max Born and Pascal Jordan) matrix mechanics, the reason being that a new type of correspondence would have been established in the new quantum mechanics, based on a different kind of relation.

5. Conclusion

The present discussion is a contribution to the long-standing philosophical debate concerning the conceptual foundations of the correspondence prin-

[1] As is well known, Bohr had little acquaintance with Kant's philosophy (Murdoch 1987), and what he knew of this probably stemmed from Harald Høffding's philosophy course, which Bohr attended during the first year of the university, and through discussions with his friends and colleagues at the "Ekliptica Club." However, it would be hard to argue that Bohr built his atomic theory on Kant's philosophy.

ciple. Specifically, I here sought to shed new light on the conceptual structure of the early version of the correspondence principle, whose similarity with Kant's procedure of analogy is so close that it was difficult to overlook it.

As is well known, Bohr was very interested in philosophical issues since his youth, and in many public occasions he recognized his cultural debt to Harald Høffding—a professor of philosophy at the University of Copenhagen, a close friend of Niels' father, and Niels' mentor during the first year at the university in Copenhagen—for having introduced him to the study of fundamental philosophical questions. However, Bohr was mainly a scientist, and he never blurred the boundaries between philosophy and scientific investigation. One fact cannot be denied: the striking conceptual similarity between the correspondence principle and Kant's procedure of analogy.

In particular, this similarity can help us to clarify the conceptual breakthrough characterizing the transition from the early correspondence principle to a new correspondence relation. For this reason, as I wanted to remind, the correspondence relation, at least until 1924, was compatible with a Kantian-like scheme, and no longer could be an integral part of the new matrix mechanics, the reason being that a new type of correspondence would have been established in 1925, based on a different kind of relation.

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