Semiotics and teaching

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Abstract. Proceeding from the subject and the aim of teaching, it is possible to distinguish the following levels or aspects: object language, metalanguage, syntax and semantics. The interrelations and the structure of these levels are studied here. Several examples of mathematical subjects (arithmetic, ALGOL-60) and other topics are presented from the semiotic aspect. The semiotic approach is expedient in cases when mathematical methods are to be applied in the optimizing of the teaching process. It is especially essential in programmed learning.

Keywords*:* edusemiotics; metalanguage; semiotics of learning; semiotics of teaching; teaching mathematics

The purpose of this article[1] is to provide an analysis of the learning process using the concepts of semiotics and mathematical logic. The author formed the views presented here when he was teaching mathematical disciplines (programming on computing machines, algorithm theory, mathematical logic, etc.), but they are to some extent applicable to the teaching of any disciplines and skills, to learning in general.

I. When starting to teach any discipline, it is first necessary to determine the goal of learning. The goals can be very different, even when teaching the same subject. For example, when teaching a foreign language, the goal can be to teach students to read mathematical texts with the help of a dictionary or to achieve complete mastery of the language.

[1] This article by Ivar Kull (1928–1989) was originally published as "*Семиотика и обучение*" (1965) in *Sign Systems Studies* 2: 11–21. The present translation from Russian is by Silvi Salupere; the English-language abstract and all footnotes are by the author. The text in Fig. 2 is slightly modified on the basis of the same figure that appeared in an Estonian-language article by the author (Kull, Ivar 1965. "Semiootika ja õppeprotsess". *Matemaatika ja kaasaeg* 8: 34–42).

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II. Precisely establishing the goal of learning makes it possible to determine the objects the construction of which should be taught. To illustrate this statement, we provide some examples below.

For example, when teaching a foreign language (at the level of practical mastery of the language), these objects are sentences and expressions used in the spoken language. The sentences and expressions are not, however, indivisible particles of the language. Such indivisible or elementary particles are sounds, the set of which is finite. From these, expressions and sentences in the language are constructed, using certain rules.

We provide a second example.[2] The subject of learning is the arithmetic of integers, the goal is to teach the calculation of the values of arithmetic expressions. The elementary particles here are the following symbols:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, −, ×, (,), =.

This list is called the alphabet of integer arithmetic. The symbols in this list (except, of course, for the comma and the full stop) are called the letters of this alphabet. Finite sequences of the letters of this alphabet are called words in this alphabet. For example, the words in this alphabet are

7094
08)(−×=
(9 − 4) × 3 + (− 12).

If we need to talk about unspecified words in this alphabet, we use Latin letters *P*, *Q*, *R*, etc. For example, the concept of 'joining two words' can be defined as follows: the joining of two words *P* and *Q* is called the word that is obtained by appending the word *Q* to the right of the word *P*. The joining of the words *P* and *Q* is denoted by *PQ*. For example, the joining of the words 70 and 94 gives the word 7094.

Not all words in the alphabet of integer arithmetic have the same importance in the construction of integer arithmetic, just as not all sound sequences in a natural language do. Special words, such as integers, arithmetic expressions, and calculations, play a special role in integer arithmetic.

For example, we will give an inductive definition of an integer:

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2 Regarding the constructions given in the example, see, for example, Markov 1954, ch. 1, and Shanin 1955, § 1–4.

(1) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, are unsigned integers;
(2) if P and Q are unsigned integers, then their joining PQ is an unsigned integer;
(3) if P is an unsigned integer, then $+P$ and $-P$ are signed integers;
(4) all unsigned integers and all signed integers are integers.

Examples of integers:

$$54, 0086, +0279, -31.$$

Next, we give the definition of an arithmetic expression:

(1) if P is an integer, then P is an arithmetic expression;
(2) if R and S are arithmetic expressions, then $(R) + (S)$, $(R) - (S)$, and $(R) \times (S)$ are arithmetic expressions;
(3) if R is an arithmetic expression, then $+(R)$ and $-(R)$ are arithmetic expressions.

Examples of arithmetic expressions:

$$9, 4, (9) - (4), ((9) - (4)) \times (3),$$
$$(((9) - (4)) \times (3)) + (-12). \tag{1}$$

To reduce the number of parentheses, it is advisable to adopt the following conventions:

(1) an unsigned integer can be written without parentheses;
(2) the $\times$ sign binds more closely than the + and – signs (i.e. if the parentheses do not determine otherwise, multiplication should be performed before addition and subtraction);
(3) expressions $+(+(R))$, $+(-(R))$, $-(+(R))$, and $(-(-R))$, where R is an arithmetic expression, can be replaced by the expressions $+(R)$, $-(R)$, $-(R)$, and $+(R)$, respectively.

Applying the given conventions, expression (1) can be rewritten as

$$(9 - 4) \times 3 - 12. \tag{2}$$

To calculate the value of an arithmetic expression, it is first necessary to determine the values of arithmetic operations (i.e. elementary arithmetic expressions):

$$(P) + (Q),$$
$$(P) - (Q),$$
$$(P) \times (Q),$$

where P and Q are integers. Suppose these values are already determined using the corresponding algorithms.

The calculation of the value of an arithmetic expression is called a word of the form

$$R_1 = R_2 = R_3 = \ldots = R_n,$$

where $R_1, R_2, R_3, \ldots, R_n$ is a sequence of arithmetic expressions, in which each R_i $(i \geq 2)$ is obtained from the previous one by replacing one elementary arithmetic expression with its value, and R_n is an integer. The number R_n is called the value of the original arithmetic expression R_1.

For example, we give the calculation of the value of arithmetic expression (2):

$$(9 - 4) \times 3 - 12 = 5 \times 3 - 12 = 15 - 12 = 3.$$

Thus, the calculation of the value of an arithmetic expression is reduced to constructing a certain word in the alphabet of integer arithmetic.

Analogously, corresponding alphabets can be selected for teaching any disciplines and skills. For example, for teaching how to operate a machine, all the complex control movements of the operator can be divided into elementary ones, which constitute the corresponding alphabet.[3]

III. As can be seen, teaching any subject can be considered as teaching the construction of certain words of a fixed alphabet. These words are called words of the so-called *object language*. The object language in the given examples was the foreign language considered, integer arithmetic, or operating machinery.

However, the presentation and construction of the object language is impossible using this very object language. For this, a broader system is necessary, which is called the *metalanguage* of this object language.

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3 Similar thoughts regarding sports are expressed in an article by Voronin *et al.* 1964.

For example, when teaching a foreign language in a Russian school, the metalanguage is usually the Russian language. When teaching the Russian language in a Russian school, the metalanguage and the object language seem to coincide. Yet in reality, when presenting the morphology and syntax of the Russian object language, we assume that students understand the corresponding explanations in Russian that acts as a metalanguage in the explanations.

In teaching a foreign language by the so-called direct method (i.e. without using the native language), such means as showing objects, pictures, physical actions, dramatizations and, for explanations, already learned elements of the language being studied are used. The listed means of information transmission form the corresponding metalanguage.

The example of integer arithmetic can convince us that the definitions of concepts and constructions in the object language are impossible without metalanguage symbols (letters P, Q, R, R_1, R_2, etc.).

It should be noted that it is impossible to do without a metalanguage even when training animals.

Below, we will provide some examples of the mixing of the metalanguage with the object language:

1. Showing the name of a geographical object on a map instead of the object itself.
2. Using, in the metalanguage of the ALGOL-60 language (input language for automatic programming), an equality

$$\pm 0\mathrm{C}.\mathrm{C}0_{10}\pm 0\mathrm{C} = \pm \mathrm{C}.\mathrm{C}_{10}\pm \mathrm{C}\,,$$

where C denotes an unsigned integer. Actually, the symbol C is a letter of the object language, and it is not advisable to use it as a metalanguage symbol. In this case, it would be necessary to formulate this, for example, as follows:

$$\pm 0\mathfrak{A}.\mathfrak{B}0_{10}\pm 0\mathfrak{C} = \pm \mathfrak{A}.\mathfrak{B}_{10}\pm \mathfrak{C}\,,$$

where $\mathfrak{A}$, $\mathfrak{B}$, and $\mathfrak{C}$ are metalanguage symbols denoting unsigned integers.

3. In Korea, there is a custom requiring the groom to demonstrate his wit to the young villagers on his wedding night.[4]

Once a thirteen-year-old boy was married off to a twenty-year-old girl. The groom was very afraid of the upcoming discussion, but the bride promised to prompt him. She stood by the wall, and he by the window.

— Can you hear? – asked the wife.

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4 The following example is taken from Garin-Mihajlovskij 1958: 490–491.

— Can you hear? – the husband loudly repeated.
— We hear, we hear, – they cheerfully answered him from the yard.
— I'm asking you, – the wife whispered.
The husband, however, again loudly repeated this metalanguage phrase. Soon, the people in the yard realized that he was repeating his wife's words, and shouted to him:
— Well, your wife has a fool for a husband.

IV. As can be seen, when teaching the object language, a corresponding metalanguage is necessary. The question, however, is how to find the optimal ratio between these languages (optimality is established here based on the effectiveness of teaching). There are many possible ratios. We provide some typical cases (Fig. 1).

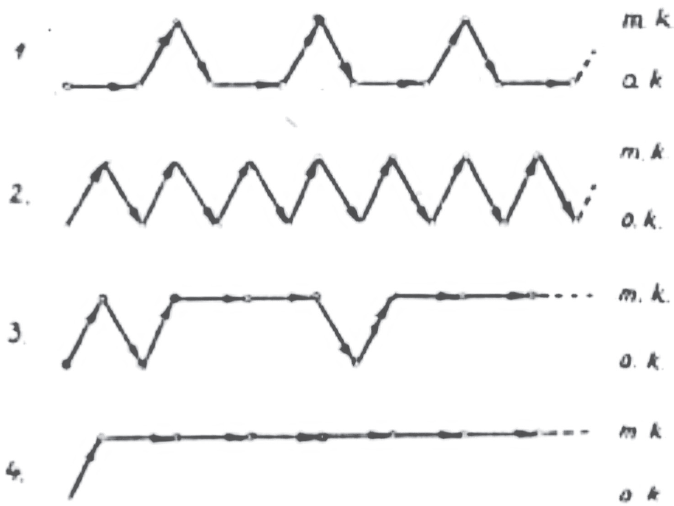

Figure 1. Four cases of using metalanguage and object language in teaching (m. k. – metalanguage level; o. k. – object language level).

Case 1. After presenting the alphabet of the object language, some leading examples in the object language are given. Then a precise definition of the concept or construction (in the metalanguage) is given, which ends with the construction of the corresponding constructions in the object language (exercises, reinforcement of the material), etc.

In *Case 2*, the stages of leading examples are absent. From constructions in the metalanguage follow corresponding examples in the object language, they move to new constructions in the metalanguage. The example of constructing integer arithmetic fits within the framework of the second case.

Case 3 differs from the previous one in that examples in the object language are not given always, but only when necessary.

In *Case 4*, except for the alphabet of the object language, the whole presentation is conducted exclusively in the metalanguage.

These cases have been provided in order of increasing speed of presenting new material. Yet Case 4 can hardly be considered the best, since students may have difficulties in implementing the corresponding constructions in the object language, which contradicts the goal of learning. It is advisable to consider a combination of the first three cases, also taking into account the complexity of the material and the preparedness of the students.

We will make some additional remarks:

(1) Using already learned elements of the language studied (for example when teaching foreign languages or mathematical logic) in the functions of the metalanguage will undoubtedly give an additional positive effect in learning.

(2) It is usually said that the difficulties in learning mathematics are due to the abstractness of this discipline. However, this does not reveal the actual state of affairs. Difficulties may arise because the teaching is conducted exclusively or almost exclusively at the level of the metalanguage (without proper reinforcement in the object language). For example, such expressions as $f(x)$, $P(x) = 0$, $F(x, y, y', y'', ..., y^{(n)}) = 0$, etc., belong to the metalanguage. Secondly, difficulties may arise because the object language has a complex structure. For example, the object language has a large number of words of different types. However, this is not abstractness yet. Reproach for abstractness in the presentation of the object language without semantic interpretation is generally meaningless. In fact, the letters of the object language alphabet can be considered quite concrete things, which are no more abstract than nuts and washers in the work of a locksmith.

V. Regarding the construction of the object language without semantic interpretation, we note the following.

All expressions in the object language can be considered as words in a certain fixed finite alphabet. Words have a finite number of letters, but this number is not limited from above. Such objects as words in a certain alphabet are called constructive objects.

In the constructions of the object language, only finite sets (of any cardinality) are used, i.e. what is accepted is the abstraction of the potential, but not actual infinity.

The rules for forming constructions in the object language should be given algorithmically (if possible), but, in any case, as formal rules. Otherwise, it is impossible to proceed, since the letters of the object-language alphabet contain no information – except for the information that, for example, letter 1 is namely 1, and not 2, 3, 4, etc., and information about the internal functions of these letters in the construction of the object language.

Considering all these points, the author finds it advisable to apply not classical, but constructive logic in the construction of the object language. The essence of constructive logic is that it allows such proofs, conclusions, constructions, etc., which, proceeding from constructive objects, again result in constructive objects.

The study of constructive logic began in the 1920s–1930s. This approach is associated with the names of Brouwer, Heyting, Kolmogorov, Glivenko, Gödel, Gentzen, and others. The scope of the article does not allow for a review of constructive logic.[5] We will only note some of its differences from classical logic.

In classical logic, the statement 'A or not-A' (in the form of the formula $A \vee \neg A$) is considered identically true, which is not the case in constructive logic. In classical logic, the statement A and its double negation $\neg\neg A$ are logically equivalent. Based on constructive logic, A is a stronger statement than $\neg\neg A$. In constructive logic, from the statement $\exists x\ F(x)$ (there exists an x such that $F(x)$ holds) follows $\neg\forall\ x\ \neg F(x)$ (it is not true that $\mathrm{F}(x)$ does not hold for all x), but not the other way round. Classically, these two statements are equivalent. The difference between constructive and classical logic is due to the different interpretation of individual basic concepts. For example, in constructive logic, the existential quantifier $\exists$ is understood in such a way that the object with the required properties must be specified algorithmically. In the case of $A \vee \neg A$, it is necessary to specify exactly which of the statements A or ¬A holds. None of this is required in classical logic.

VI. Using only formal rules for constructing the object language, as outlined in Part V above, we construct the object language purely *syntactically*. Yet the words of the object language, generally speaking, do not represent the immediate level of our knowledge of reality and require *semantic interpretation*.[6] Such an interpretation is necessary for the application of this object language.

In native language teaching, the main attention is usually paid to the syntactic[7] construction of the language, as the semantic side of the language is more or less

5 On this matter, see Goodstein 1961; Kleene 1957.

6 Essentially, semantic interpretation is a special level of metalanguage (in the general sense), which can be called the metalanguage of the semantics of a given object language. To distinguish the metalanguage discussed in Sections III and IV, it can be called the metalanguage of the syntax of a given object language.

7 The concept of syntax in semiotics also encompasses the concept of morphology in grammar.

known to the students. If necessary, means of the same language are used for the semantic interpretation of individual words and expressions.

In foreign language teaching, semantic interpretation (with the help of the native language or already semantically learned elements of the language being studied[8]) is given in parallel with the syntactic construction of this language. Of course, for semantic interpretation, one can use the demonstration of objects, pictures, and other methods.

It is interesting to note that in natural languages, in some cases, there are such rules for constructing the language that are based on semantics (so-called semantic construction rules). Such rules include, for example, some rules for determining articles in the German language, the rule according to which words like 'scissors', 'tongs', 'trousers', and some others have only the plural form. Of course, semantic construction rules can be formulated in a non-semantic way. But since semantics is still necessary, it will make most sense to give these rules in a semantic form.

In the process of teaching mathematical disciplines, at least two different levels of semantic interpretation can be distinguished. One of them is geometric interpretation, which is very widely used in mathematics. The use of this fairly universal and understandable interpretation undoubtedly facilitates the learning process. However, it should be noted that there are some tendencies to exaggerate the role of this interpretation (in non-geometric disciplines). These are manifest in the fact that the construction of non-geometric disciplines sometimes essentially relies on geometric interpretation (for example, in the treatment of curvilinear and surface integrals in many textbooks of mathematical analysis). This means that the object language (the corresponding section of mathematical analysis) is already constructed with the help of semantic interpretation, which introduces elements of uncertainty and vagueness into the construction of the object language. Moreover, such an approach distorts the idea of the correct construction of mathematical disciplines. Such techniques can be used only when students fully understand the logical-semiotic essence and limits of these techniques.

Another level of semantic interpretation of mathematical disciplines is the interpretation of the words of the constructed object language with the help of non-mathematical (mechanical, physical, chemical, biological, economic, etc.) concepts. It must be said that such interpretations are unfortunately given insufficient attention in mathematics. However, such a situation does not promote understanding of the practical applications of mathematics and causes some students to lose interest in mathematics.

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8 See, for example, the dictionaries: West, Endicott 1935; Flood, West 1962. The explanations in these dictionaries are given using a minimal vocabulary.

In recent years, many artificial languages have been created by mathematicians: FORTRAN, IT, ALGOL-60, etc., for automatic programming and other purposes. In presenting these languages, such aspects as metalanguage, object language, syntax, and semantics are usually highlighted or can be easily distinguished. Sometimes, unfortunately, semantics is not given due attention, as, for example, in the official report on the ALGOL-60 language. Naturally, such works cannot serve as the first manuals for studying these languages, as they are difficult to understand and can induce a feeling of hopelessness in a person trying to get acquainted with them.[9]

When teaching practical skills (operating machinery, typing, gymnastics, sports, etc.), the level of the object language (i.e. the corresponding physical actions) is a level that does not require special semantic interpretation. The situation is different with art. Despite the fact that, for example, choreography has similarities with gymnastics, semantic interpretation is very significant in the former, while it is simply absent in the latter.

In general, it can be said that the semantics of the object language has a complex structure, as has already been demonstrated for mathematical disciplines.

VII. In recent years, much attention has been paid to the so-called dialectical logic. There is no generally accepted definition of dialectical logic yet. Of course, many aspects and facets of dialectical logic have been more or less clarified. For example, dialectical logic is associated with such a property of human thinking as the ability to formalize something. Many elements of dialectical logic can be found, for example, in the metalogic of formalized logical calculi.

Elements of dialectical logic are found at all semiotic levels of the teaching process, but mainly in the link leading from the analysis of reality to the construction of metalanguage. The least dialectical elements are usually found in the syntactic construction of the object language.

It is also appropriate to apply factors of a dialectical-psychological nature, such as emotions, humour, etc., in the teaching process, taking into account the level of development of the learners.[10] The main area of their application is, apparently, the metalanguage and semantics of the object language being studied.

The relationship between the different levels of the teaching process can be schematically represented as shown in Fig. 2.

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[9] Ageev 1964: 3.

[10] For example, first-formers should not be taught rules of conduct in the style of the comedian Jüri Järvet (Järvet 1962).

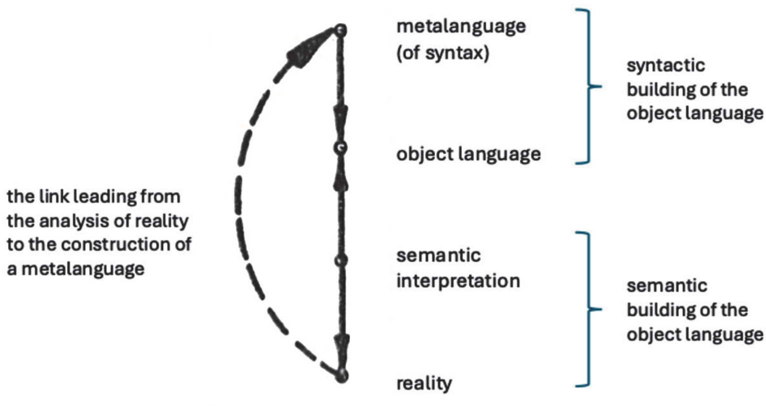

Figure 2. The relationship between the different levels of the teaching process.

The teaching objective mentioned in Part I is achieved by mastering the syntax and semantics of the object language. At the same time, it is advisable to develop the students' understanding of the basis of metalanguage construction as much as possible.

The diagram in Fig. 2 can be supplemented with two more components: student–teacher (or the latter's replacement), connected to each other by feed-forward and feedback channels.

VIII. The logical-semiotic point of view is essential in the analysis of the learning process in attempts to improve the learning process using mathematical methods (mathematical statistics, information theory, algorithm theory, programming, etc.). This starting point is particularly important in programmed learning. The principles of programmed learning are the following:
(a) a strict, detailed course structure based on the logical structure of the material;
(b) dosing of the learning material, i.e. presentation of the material in portions;
(c) mandatory assessment of the material after each portion;
(d) direct notification of students about the results of assessment.[11]
As can be seen, the principles of programmed learning are fully consistent with the logical-semiotic model of learning. Moreover, programmed learning requires precisely such a logical-semiotic basis. So far, it does not yet have such a basis. The

[11] Agur 1964.

author is of the opinion that the main area of application of programmed learning is the syntax of the object language.

IX. Familiarity with the basic concepts of semiotics and mathematical logic makes it possible to understand the structure of the learning process better. In the author's opinion, mastering these concepts would be very useful for teachers (especially university lecturers), and it is necessary to familiarize students – future teachers – with these issues.

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Семиотика и обучение

Исходя из предмета и цели преподавания, можно выделить следующие уровни или аспекты: объектный язык, метаязык, синтаксис и семантика. Изучаются взаимосвязи и структура этих уровней. В семиотическом аспекте представлено несколько примеров математических дисциплин (арифметика, АЛГОЛ-60) и других тем. Семиотический подход целесообразен в тех случаях, когда математические методы применяются для оптимизации процесса обучения. Это особенно важно в программированном обучении.

Semiootika ja õpetamine

Lähtudes õppeainest ja õpetamise eesmärgist, on võimalik eristada järgmisi tasandeid või aspekte: objektkeel, metakeel, süntaks ja semantika. Uuritakse nende tasandite omavahelisi seoseid ja struktuuri. Semiootilisest aspektist esitatakse mitmeid näiteid matemaatikast (aritmeetika, ALGOL-60) ja muudest teemadest. Semiootiline lähenemine on otstarbekas juhtudel, kui matemaatilisi meetodeid tuleb rakendada õppeprotsessi optimeerimisel. See on eriti oluline programmeeritud õppimisel.