The Fibonacci sequence and the nature of mathematical discovery: A semiotic perspective

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Abstract. This study looks at the relation between mathematical discovery and semiosis, focusing on the famous Fibonacci sequence. The serendipitous discovery of this sequence as the answer to a puzzle designed by Italian mathematician Leonardo Fibonacci to illustrate the efficiency of the decimal number system is one of those episodes in human history which show how serendipity, semiosis, and discovery are intertwined. As such, the sequence has significant implications for the study of creative semiosis, since it suggests that symbols are hardly arbitrary products of human reason, but rather unconscious probes of reality.

Introduction

One of the most famous mathematical discoveries of all time is the one that pertains to a sequence of integers connected by the following simple rule — for every three consecutive integers, the sum of the first two integers produces the third in the sequence, {1, 1, 2, 3, 5, 8, 13, …}. So, for example, 3 = 1 + 2, 5 = 2 + 3, 8 = 3 + 5, and so on. The sequence is known as the "Fibonacci Sequence" (henceforward FS). So much has been written on this sequence that it would be presumptuous to claim that anything new can be said about it that has not already been said. Indeed, mathematicians have been studying the FS ever since its discovery in 1202 by Italian mathematician Leonardo Fibonacci (1170–1240). However, lacking from the relevant literature is a semiotic consideration of the implications this sequence has for understanding the nature of discovery. The purpose of the present paper is to do exactly that — to reflect upon the FS and its relation to mathematical discovery from a semiotic perspective.[1]

Resorting to semiotically-based ideas (whether overtly or indirectly) in order to investigate mathematical features and facts is not new — indeed, over the last few decades it has become quite common to do so (e.g. Rotman 1988; Reed 1994; MacNamara 1996; Radford, Grenier 1996; Antenos-Conforti *et al*. 1997; English 1997; Lakoff, Nuñez 2000; Anderson *et al*. 2000). The mindset that guides this line of inquiry in general is the connection between symbols, mathematics, and discovery. It is, in my view, an important perspective because it leads to an insightful reformulation of the classic questions of mathematical philosophy that originated with the ancient Pythagoreans: What is mathematics? Why does it allow us to discover natural laws? As Arthur Koestler (1959: 34) so eloquently put it: "Nobody before the Pythagoreans had thought that mathematical relations held the secret of the universe. Twenty-five centuries later, Europe is still blessed and cursed with their heritage." And as the great Neapolitan philosopher Giambattista Vico (1688–1744) argued throughout his life, such relations do not come about by an exercise of strict logical thinking, but rather through a creative form of understanding that he called the *fantasia* — a unique blend of imagination and reasoning (Bertland 2004).

The fact that the FS is the direct product of a clever puzzle constructed by Fibonacci to show how Hindu-Arabic numerals can be used efficiently, bears great relevance to the question at hand. Puzzles are as old as civilization. There has never been a period of time, nor has there ever been a culture, without some kind of puzzle tradition. Very few other kinds of artifacts have had the broad appeal that puzzles have. Throughout history, riddles, mazes, magic squares, geometrical puzzles, and the like have been used for pedagogical, recreational, and various other kinds of social functions. The "puzzle instinct," as it can be called (Danesi 2002), continues to manifest itself in the widespread popularity today of modern puzzle artifacts, from crosswords to the Rubik's Cube. It may even go back as far as 10,000 years BCE, as evidenced by several bones found in the Ishango Tribe that have marks on them representing numbers and which were

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probably used to carry out numerical games (Heinzelin 1962). But are puzzles just playful texts or objects, intended merely to train the mind or to entertain? Or are they products of something more fundamental in the human species? Do they reveal, in fact, something about a truly enigmatic interplay between *fantasia* and discovery in mathematics, given that many classic mathematical puzzles have led to subsequent discoveries in mathematics opening up new fields of inquiry?

One puzzle that stands out in this regard is Fibonacci's famous Rabbit Puzzle, which the medieval mathematician created primarily to illustrate the practicality of using the decimal number system to his fellow Italians (on this point see, for example, Ouaknin 2004: 133–140). As it turns out, no other puzzle has had as many implications for the study of mathematical pattern; and no other puzzle has had as many "reifications" in the study of nature. There is no evidence to suggest that Fibonacci himself was aware of the implications and applications that the solution to his puzzle would turn out to have. It was the French mathematician François Edouard Anatole Lucas (1842–1891) who noticed some of these in the nineteenth century. Since Lucas's observations, the amount of mathematical properties that the FS has been found to conceal and the number of reifications that it has been found to have in nature and human life have been absolutely astounding. The question that the FS begs is an obvious one: How could such a simple puzzle, designed originally to show the efficiency of decimal numerals over Roman ones, contain so many "secrets of the universe," so to speak?

As Umberto Eco (1998) has cogently argued in regard to discovery in general, the crystallization of the FS from a simple puzzle is one of those episodes in human history which show how serendipity and discovery are intertwined. It is an episode with enormous implications for the study of creative semiosis, since it suggests that symbols are hardly arbitrary products of human reason, but rather unconscious probes of reality. Was this, in fact, the "secret" that got the Pythagoreans into trouble in the ancient world, leading to their systematic killing by those who may have feared what they knew? As is well known, Pythagoras (c. 582–500 BCE) and his followers taught that number was the essence of all things. They associated numbers with virtues, colors, and many other ideas. To study the relation between number and reality Pythagoras founded a school called the "Brotherhood" among the aristocrats of the city of Crotona. As history records, the people of that city became suspicious of the Brotherhood — a suspicion that led eventually to an uprising and an extermination of its members.

Discovery in Mathematics

In his monumental history of semiotics, John Deely (2001) argues essentially that we can only understand the history of knowledge by mapping it against the development of sign theory. His discussion of the relation between signs and knowledge, especially as to how we attain it and how it is symbolized, is based on the premise that signs give shape to formless ideas, not in an arbitrary fashion, but in response to inferential processes that are tied to our experience of reality.

Knowledge systems vary throughout the world. But such variation is, upon closer scrutiny, superficial. Below the surface of these systems are sign creation processes that reflect universals in how reality is perceived. The problem is that we never get the "whole picture" at once. This is why special theories of the physical universe are possible and highly useful, but general ones are not. In other words, our knowledge systems can only give us partial glimpses of reality. What is important to note is that the elements that constitute these systems are hardly the products of firm reasoning processes; rather they seem to come to consciousness as if by magic. Discovery, in other words, cannot be forced by logical analysis. It simply *happens*. But it is not totally random, as the Fibonacci Rabbit Puzzle episode shows. It is probably tied to unconscious modes of interconnecting experiences and their meanings. This is perhaps the reason why a sign (a word, text, formula, theory, puzzle, etc.) invented in one realm of representation leads, subsequently, to discovery in other realms. Signs are thus both encoders and guides of reality. St. Augustine appropriately characterized this aspect of human semiosis as a blending of our experience of natural signs *(signa naturalia)* with conventionalized knowledge (*signa data)*. Another way to put it, using the ideas of the Tartu School, is to say that there is an interplay between our existence in the biosphere and our existence in the semiosphere (Lotman 1990). This interplay is what leads, arguably, to discoveries.

The word *serendipity*, incidentally, was coined by Horace Walpole in 1754, from the title of the Persian fairy tale *The Three Princes of Serendip*, whose heroes make many fortunate discoveries accidentally (Merton, Barber 2003). The tale goes somewhat as follows. Three princes from Ceylon were journeying in a strange land when they came upon a man looking for his lost camel. The princes had never seen the animal, but they asked the owner a series of seemingly pertinent questions: Was it missing a tooth? Was it blind in one eye? Was it lame? Was it laden with butter on one side and honey on the other? Was it being ridden by a pregnant woman? Incredibly, the answer to all their questions was yes. The owner instantly accused the princes of having stolen the animal since, clearly, they could not have had such precise knowledge otherwise. But the princes merely pointed out that they had observed the road, noticing that the grass on either side was uneven and this was most likely the result of the camel eating the grass. They had also noticed parts of the grass that were chewed unevenly, suggesting a gap in the animal's mouth. The uneven patterns of footprints indicated signs of awkward mounting and dismounting, which could be related to uneven weights on the camel. Given the society of the era, this suggested the possibility that the camel was ridden by a pregnant woman, creating a lack of equilibrium and thus an uneven pattern of footprints. Finally, in noticing differing accumulations of ants and flies they concluded that the camel was laden with butter and honey — the natural attractors of these insects. Their questions were, as it turns out, inferences based on astute observations, or to use Peircean terminology, "abductions" of a logico-inferential nature.

Ceylon's ancient name was Serendip, and it was Walpole who, after having read the tale, decided to introduce the word *serendipity* into the English language. The princes made their discovery of the facts of the matter as a result of what Walpole called "accidental sagacity." Serendipity characterizes the history of discovery in mathematics and science — Wilhelm Conrad Roentgen (1845–1923) accidentally discovered X-rays by seeing their effects on photographic plates; Alexander Fleming (1881–1955) serendipitously discovered penicillin by noticing the effects of a mold on bacterial cultures; and the list could go on and on (e.g. Roberts 1989). Incidentally, Roentgen called his discovery "X-rays" because he simply didn't know what to call the rays, so he resorted to the traditional use of "X" as an "unknown" in mathematics. The historical record suggests that discovery is hardly the product of a systematic search for truth, but rather a serendipitous consequence of using our *fantasia*. Perhaps the most famous of all serendipitous episodes in the history of science is Archimedes' discovery of a law of hydrostatics (known as Archimedes' Principle) as he was purportedly taking a bath. After visualizing the law in his mind through a flash of insight, he is said to have run out into the streets of Syracuse naked, crying "Eureka," meaning "I have found it." Since then, such flashes of insight have been called "Eureka moments."

What is perhaps even more astounding is the fact that serendipity plays a role in reification — the manifestation of a form in knowledge domains other than the original one in which it was forged. A perfect example of this are the reifications of π (pi) = 3.14 (Beckmann 1971; Blatner 1997; Eymard *et al*. 2004; Posamentier 2004). Pi is the ratio that results when the circumference of a circle is divided by its diameter. Although discovered in the ancient world, the Greek letter π was first used in 1706 by English mathematician William Jones (1675–1749) and adopted by Swiss mathematician Leonhard Euler (1707–1783) in 1737. Serendipitously, π appears in a number of mathematical calculations and formulas, such as the one used to describe the motion of a pendulum or the vibration of a string. It also turns up in equations describing the DNA double helix, rainbows, ripples spreading from where a raindrop falls into water, all kinds of waves, navigation systems, and the list could go on and on. Does this mean that the circle form that produced π is implicit in these new domains? What is the connecting link between the circle form that produced the notion of π and other forms such as rainbows?

In a fascinating 1998 movie, titled *π: Faith in Chaos*, by American director Darren Aronofsky, a brilliant mathematician, Maximilian Cohen, teeters on the brink of insanity as he searches for an elusive numerical code hidden in π. For the previous ten years, Cohen was on the verge of his most important discovery, attempting to decode the numerical pattern beneath the ultimate system of ordered chaos — the stock market. As he verges on a solution, real chaos is swallowing the world in which he lives. Pursued by an aggressive Wall Street firm set on financial domination and a Kabbalah sect intent on unlocking the secrets hidden in their ancient holy texts, Cohen races to crack the code, hoping to defy the madness that looms before him. Instead, he uncovers a secret for which everyone is willing to kill him.

As the movie's subtext implies, the stream of digits of π seems to challenge us to try to find a pattern within them. The greatest challenge to date, however, has been the race to simply compute π farther than before. The further it has been computed, the more old theories about patterns within are dispelled and new ones created. So far, π has been computed to over 51 billion digits. What is our attraction to this number? Is it perhaps the fact that a circle is probably the most perfect and simple form known to human beings? And why does π appear in statistics, biology, and in many other domains of knowledge? It simply keeps cropping up, reminding us that it is there, and defying us to understand why. Very much like the universe itself, the more technologically advanced we become and as our picture of π grows ever more sophisticated, the more its mysteries grow. There is a beauty to π that keeps our interest in it. One can argue, as does Beckman (1971), that π is one of those products of human effort that is a mirror of human history — it starts out in one domain of activity (geometry) and ends up in others and is probably everywhere (if we look for it).

Although the idea that signs are both reactions to experience and subsequent locators of new experiences is an extremely problematic one for many philosophers and mathematicians, it offers crucial insights in any attempt to approach (if not answer) one of the oldest questions in philosophy and mathematics: Is mathematics invented or discovered? Those supporting the view that mathematics as an invention or creation of the human mind include Augustus de Morgan, Janos Bolyai, David Hilbert, Albert Einstein, and George Pólya (Dewdney 1999). Those supporting the view that mathematics is the means by which we consciously discover truths are Archimedes, Isaac Newton, Leonhard Euler, and G. H. Hardy (Dewdney 1999). Semiotically, however, it can be argued that both perspectives are accurate. As the Pythagoreans believed, numbers do indeed seem to hold the key to the universe at the same time that they emanate from human perspectives of that same universe. The Pythagoreans lasted a long time, from about 500 BC until well into the Islamic era. Common wisdom holds that theirs was a pre-scientific system of belief, a close cousin of astrology and numerology, rendered obsolete by the rise of rationalist science in the late Renaissance that provided more effective explanations of natural events. But science has now come virtually full circle, restoring mathematics to a throne not unlike that imagined by the ancient Pythagoreans. Whether we recognize it or not, the information age in which we live confronts us once again with the ancient mystery of why the universe is so mathematical: Does the cosmos make mathematics, or does mathematics make the cosmos?

Differences in numerical notation (Roman, decimal, etc.) are, of course, culture-based and invented; but the similarities captured by all such systems goes beyond culture. Numbers are thus both invented and discovered, giving them a unique status in the history of human ingenuity (Menninger 1969). The human mind creates numbers in the same sense that it creates colors. Yet the colors we perceive correspond to something real outside the mind. In this sense, we are discovering numbers all the time. Paradoxical as it may sound, only the possibility of being wrong will save mathematics from becoming a purely cultural exercise.

Mounting evidence in the neurosciences suggests that the rudiments of arithmetic are anchored in our genes, that infants are born with a capacity for recognizing and distinguishing among small numerical referents, etc. If such research is indeed correct, then the discovery of mathematical patterns is something we are programmed to do from birth. Although the structures of the cosmos certainly predate the human mind, they are not understood or even existent outside of human minds. The human brain, equipped by evolution, seems to be inclined to translate these structures into mathematics.

The Fibonacci sequence

When all is said and done, the question of where invention ends and discovery begins seems to defy a satisfactory answer. The case of Fibonacci's Rabbit Puzzle is a truly remarkable one in this regard, because it is, without question, a simple invention, and yet it contains within its solution so many discoveries that it truly boggles the mind to come up with a rational explanation as to why this is so.

The puzzle is found Fibonacci's *Liber Abaci*, published in 1202. Fibonacci designed his book as a practical introduction to the Hindu-Arabic number system, which he had learned to use during his extensive travels in the Middle East. His method of exposition was based on the creation of puzzles that illustrated how easily the Hindu-Arabic system could be used to solve what would otherwise constitute intractable problems with the Roman numeral system. In the *Liber Abaci* he also introduced the word *cephirum* for "zero" as a *figura nihili* ("a sign of nothing") in Latin. For historical accuracy it should be mentioned that the zero concept started out as *sunya* in sixth-eighth century Sanskrit, was then adapted as *sift* in ninth-century Arabic, introduced as *cephirum* through Fibonacci in thirteenth-century Latin (with variants *cifa, zefirum,* and *zephirum*), developing finally to *zero* in fourteenth-century Italian — a word adopted by English in the fifteenth century.

The puzzle is found in the third section of the *Liber Abaci:*

> A certain man put a pair of rabbits, male and female, in a very large cage. How many pairs of rabbits can be produced in that cage in a year if every month each pair produces a new pair which, from the second month of its existence on, also is productive?

There is 1 pair of rabbits in the cage at the start. At the end of the first month, there is still only 1 pair, for the puzzle states that a pair is productive only "from the second month of its existence on." It is during the second month that the original pair will produce its first offspring pair. Thus, at the end of the second month, a total of 2 pairs, the original one and its first offspring pair, are in the cage. Now, during the third month, only the original pair generates another new pair. The first offspring pair must wait a month before it becomes productive. So, at the end of the third month, there are 3 pairs in total in the cage — the initial pair, and the two offspring pairs that the original pair has thus far produced. If we keep tabs on the situation month by month, we can show the sequence of pairs that the cage successively contains as follows: 1, 1, 2, 3. The first digit represents the number of pairs in the cage at the start; the second, the number after one month; the third, the number after two months; and the fourth, the number after three months.

During the fourth month, the original pair produces yet another pair. At that point in time the first offspring pair produces its own offspring pair. The second pair produced by the original rabbits has not started producing yet. Therefore, during that month, a total of 2 newborn pairs of rabbits are added to the cage. Altogether, at the end of the month there are the previous 3 pairs plus the 2 newborn ones, making a total of 5 pairs in the cage. This number can now be added to our sequence: 1, 1, 2, 3, 5. During the fifth month, the original pair produces yet another newborn pair; the first offspring pair (now fully productive) produces another pair of its own as well; and now the second offspring pair produces its own first pair. The other rabbit pairs in the cage have not started producing offspring yet. So, at the end of the fifth month, 3 newborn pairs have been added to the 5 pairs that were previously in the cage, making the total number of pairs in it: 5 + 3 = 8. We can now add this number to our sequence: 1, 1, 2, 3, 5, 8. Continuing to reason in this way, it can be shown that after twelve months, there are 233 pairs in the cage. Now, the intriguing thing about this puzzle is the sequence of pairs itself, on a month-by-month basis:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233

The salient characteristic of this sequence, as mentioned above, is that each number in it is the sum of the previous two: e.g. 2 (the third number) = 1 + 1 (the sum of the previous two); 3 (the fourth number) = 1 + 2 (the sum of the previous two); etc. This pattern can of course be extended ad infinitum, by applying the simple rule of continually adding the two previous numbers to generate the next:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, …

Little did Fibonacci know how significant his sequence would become. Over the years, the properties of the Fibonacci numbers have been extensively studied, resulting in a considerable literature. The basic pattern hidden in the FS was studied first by the French-born mathematician Albert Girard (1595?–1632?) in 1632. It is expressed with the formula: $F_n = F_{n-2} + F_{n-1}$ where F_n stands for any number in the sequence and F_{n-1} the number before it and F_{n-2} the second number before it. At about the same time, the astronomer Johannes Kepler (1571–1630) noticed that the FS converges to the *golden ratio*, whose value is .618… (Darling 2004: 116) — a finding confirmed in 1753 by the Scottish mathematician Robert Simson (1687–1768). As is well known, the ratio results from two divisions of a line such that the smaller is to the larger as the larger is to the sum of the two, a ratio of roughly three to five. If we take the stretch of numbers in the FS starting with 5 and ending with 34, and take successive ratios they approach the golden ratio:

3/5	= .6
5/8	= .625
8/13	= .615
13/21	= .619
21/34	= .617

The golden ratio has been found to produce aesthetic effects and has itself been found to have an astounding number of reifications in nature (Livio 2002). This adds even more a sense of Pythagorean mystery to the FS: Why would there be a connection between a sequence of numbers produced by a puzzle about copulating rabbits and one of the most enigmatic ratios in the history of human civilization? The plot thickens, so to speak. In the nineteenth century the term *Fibonacci Sequence* was coined by the French mathematician Edouard Lucas, as mentioned, and mathematicians from many domains of inquiry began to discover myriads of numerical patterns hidden within in it (e.g. Ogilvy, Anderson 1966: 133–144; Stewart 2004: 87–93). Not only, but stretches of the sequence started cropping up in nature — in the spirals of sunflower heads, in pine cones, in the regular descent (genealogy) of the male bee, in the logarithmic (equiangular) spiral in snail shells, in the arrangement of leaf buds on a stem, in animal horns, in the botanical phenomenon known as phyllotaxis whereby the arrangement of the whorls on a pinecone or pineapple, in the petals on a sunflower, in the branches of some stems, and so on and so forth. In most flowers, for example, the number of petals is one of: 3, 5, 8, 13, 21, 34, 55, or 89 (lilies have 3 petals, buttercups 5, delphiniums 8, marigolds 13, asters 21, daisies 34 or 55 or 89). In sunflowers, the little florets that become seeds in the head of the sunflower are arranged in two sets of spirals: one winding in a clockwise direction, the other counterclockwise. The number in the clockwise is often 21, 34 and counterclockwise 34, 55, sometimes 55 and 89, and sometimes 89 and 144 in the spirals of sunflower heads, in pine cones (examples cited in Stewart 1995 and Devlin 2004).

The list of such reifications is truly startling — so much so that a journal, called *The Fibonacci Quarterly*, was established in 1963 to publish findings related to the FS. Why would the solution to a simple puzzle produce numbers that are interconnected with patterns in nature and human life? There is, to the best of my knowledge, no definitive answer to this question. Maybe the puzzle instinct itself is at the root of such serendipities. As mathematician Ian Stewart puts it (2001: v), "simple puzzles could open up the hidden depths of the universe." As a "serendipitous sign" the FS seems to have led to an incredible discovery — namely that a simple recursive pattern constitutes the fabric of a large slice of nature. Devlin (2005: 105) sees the FS as essentially a descriptive statement — a model — of a growth process: "The Fibonacci sequence is one of a number of very simple mathematical models of growth processes that happens to fit a large variety of real-life growth processes." While this turns out to be true, what still remains perplexing is that Fibonacci hardly devised the FS to describe nature. He did not come up with it from studying plants. Rather, the FS is the outcome of a puzzle about rabbits.

Incidentally, Lucas came up with his own sequence of numbers, now called the *Lucas numbers*, which he started with 1 and 3:

1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, …

As in the case of the Fibonacci Sequence, any number in the Lucas Sequence is obtained by summing the previous two. As it turns out, the Lucas numbers also have surprising properties and reifications (Ball 2003). The question now becomes, does any recursive sequence produce serendipitous reifications? If so, what is reality? Were the Pythagoreans correct after all in saying that, fundamentally, there is continuity between the human mind and nature and that the language of this continuity is that of number?

As Devlin suggests, the predictive power of signs lies, arguably, in the fact that they are models of things (Sebeok, Danesi 2000). Model-making constitutes a truly astonishing evolutionary attainment, without which it would be virtually impossible for humans to carry out their daily life routines. I would like to suggest that numerical sequences, such as the Fibonacci one, are models of intrinsic pattern — whether the pattern is felt unconsciously or expressed overtly. In previous work, I have labeled such models *metaforms* (e.g. Danesi 2003; 2004), since they tend to result from creative associations (inferences, abductions, etc.) that are expressed as metaphors in language and as related symbols in nonverbal domains. Metaforms are common in scientific theory formulation. By making new connections and relating concepts, scientists seek to give structure to the world of matter. Science involves things we cannot see, hear, touch, etc. — atoms, waves, gravitational forces, magnetic fields, etc. So, scientists use their imagination and their capacity to metaphorize in order to get a look, so to speak, at this hidden matter. Waves are said to *undulate* through empty space like water waves rippling through a still pond; atoms are conceived as little balls leaping from one quantum state to another; electrons are portrayed as traveling in circles around an atomic nucleus; and so on. This form of reasoning is extremely powerful. It is a product of innate feeling structures, as Langer (1948) called them, that result from our interactions with the world.

The following question can now be asked: Is the FS a metaform? If it is, then it suggests that metaforms are slices of truth, constituting powerful evidence that discovery lies in the ability of the human mind to visualize the universe as interacting with itself. The FS is a classic, albeit mind-boggling, exemplar of the *verum-factum* principle in philosophy. Although there are precedents for it, no one was able to discuss it as insightfully as Vico did. This principle can be explained as the ability of the human imagination to discover patterns in the world because the human mind already has such patterns built into it. As Bergin and Fisch (1984: xlv) have perceptively pointed out, in being makers of things, Vico believed that human beings were themselves made to do just that: "Men have themselves made this world of nations, but it was not without drafting, it was even without seeing the plan that they did just what the plan called for." As Peirce similarly put it, the mind has "a natural bent in accordance with nature" (CP 6.478). This blending of mind and nature becomes perception, which Peirce called the "outward clash" of the physical world on the senses (see also Fann 1970; Eco, Sebeok 1983; Merrell, Quieroz 2005).

In effect, there are two parts to the human mind, expressed in most traditions of the world in various ways. The Greeks used the terms *mythos* and *logos*, with the former being the intuitive sense for pattern and the latter the ability to reflect upon it and give it a form. Form and content (the real world) are thus inextricable — products of two interacting parts of the brain. Signs give expression to this inextricability and, thus, invariably shed light on snippets of reality. The problem has always been devising an overall picture of that reality. Signs are metaforms leading to discovery not because they were designed as “knowledge-productive” but because they are imaginative artifacts.

This interplay between mythos and logos would explain why the early histories of mathematics, magic, and puzzle-making overlap considerably. The ancient magicians, mathematicians, and puzzlists (mainly makers of riddles and anagrams, Danesi 2002) were concerned with basically the same thing — unraveling hidden patterns. Indeed, no distinction was made between *numeration* and *numerology*. Numerologists translated an individual’s name and birth date into numbers which, in turn, were believed to reveal the individual’s basic character and destiny. Numerology started with the Pythagoreans, who taught that all things were numbers, and that all relationships could be expressed numerically. In Hebrew the same symbols are used for digits as for letters, and the ancient art of *gematria*, or “divination,” claimed that the letters of any word or name found in sacred scripture could be interpreted as digits and rearranged to form a number that contained secret messages encoded in it. The earliest recorded use of gematria was by the Babylonian king Sargon II in the eighth century BC, who built the wall of the city of Khorsbad exactly 16,283 cubits long because this was the numerical value of his name.

A thick volume could be written about the many meanings ascribed to specific numbers across the world and across history. Take, for example, the number 7. It is found, for instance, in the Old Testament where, as part of God’s instructions to Moses for priests making a blood offering we find the following statement: “And the priest shall dip his finger in the blood, and sprinkle of the blood seven times before the Lord, before the veil of the sanctuary” (Leviticus 4:6). It is also noteworthy that God took six days to make the world and then rested on the seventh. The number 13, too, has a long history associated with mysticism. So widespread is the “fear of the number 13” that it has even been assigned a name: *triskaidekaphobia*. In Christianity, 13 is linked with the Last Supper of Jesus and his twelve disciples and the fact that the thirteenth person, Judas, betrayed Jesus. Other similarly “unlucky numbers” exist in different parts of the world. And across cultures, people tend to think of certain things such as dates, street addresses, or certain numbers as having great significance. Human beings seem to possess the basic notion that the world is itself a magical pattern of small numbers arranged in patterns.

It was only after the Renaissance that numerology was relegated to the status of a pseudoscience. Paradoxically, the Renaissance at first encouraged interest in the ancient magical arts and in their relation to philosophical inquiry. Intellectuals such as Italian philosopher Giovanni Pico della Mirandola (1463–1494) rediscovered the occult roots of classical philosophy, and protoscientists such as Swiss physician Philippus Aureolus Paracelsus (1493–1541) affirmed these practices, partly in defiance of medieval religiosity. Both the Roman Catholic Church and the new Protestantism, however, turned sharply against magic and the occult arts in the fifteenth and sixteenth centuries. Mathematics was subsequently completely liberated from the occult mysticism in which it was shrouded in the ancient world.

But the connection between mysticism and mathematics has hardly been lost. Solving puzzles, proving a difficult theorem, or observing a mysterious manifestation of Fibonacci numbers in nature continues to cast a "magical spell" over us. In fact, to this day, the boundaries between mathematics and magic are rarely clear-cut. Every mathematical idea is caught up in a system of references to other ideas, patterns, and designs that humans are inclined to dream up. And this imparts an aura of Pythagorean mysticism to that very system.

The production of metaforms suggests that we are "programmed" to discover things serendipitously, just as Vico claimed. In observing the facts of existence, we constantly stumble across hidden patterns. The FS brings this out perfectly. It emphasizes rather dramatically that the line between myth and logic is a very fine one indeed. In the original tale, from which the concept of serendipity is derived, the three princes made their deductions by noticing anomalies that suggested explanations. These spurred their insights. Maybe Fibonacci saw something in a rabbit pen that tickled his fancy and spurred his insight, leading to his puzzle, and to the hidden reifications that it contains.

Whatever the truth, Fibonacci's Rabbit Puzzle continues to reverberate with implications in all kinds of knowledge domains. This paper has only skimmed the surface of these implications. A similar argument could be made for as whole host of mathematical metaforms, such as e, $e^{i\pi} + 1 = 0$, among many others (e.g. Maor 1994), which have turned out to have a wide variety of serendipitous applications. The number e was discovered by Leonhard Euler in 1727 as the limit of the expression $(1 + 1/n)^n$ as n becomes large without bound. Its limiting value is approximately 2.7182818285. Unlike π, *e* has no simple geometric interpretation. Yet it forms the base of natural logarithms; it appears in the fundamental function for equations describing growth and many other processes of change; it surfaces serendipitously as well in the formulas for many curves; it crops up frequently in the theory of probability and in formulas for calculating compound interest; and the list could go on ad infinitum. Now, why Euler devised that formula in the first place is not clear. He certainly could not have known the kinds of ideas and applications it would have led to, since these came after its formulation. The number *e* is a perfect example of a metaform.

Euler is also responsible for the extraordinary equation, $e^{i\pi} + 1 = 0$, also written as $e^{ix} = -1$, in which *i* is the square root of –1. In addition to its many practical applications — it has wide application, for instance, in understanding the motion of any type of wave, including light — this formula is unique in that it combines five fundamental numbers in mathematical discovery — 0, 1, π, *i*, and *e*. Now, it is clear that what distinguishes metaforms such as the FS and *e* from so-called "universal laws" in science is that they are not devised to reveal a deep principle about how the world as ordered; rather they issue forth from flights of fancy.

Concluding remarks

From the Pythagorean practice of giving sacrifice to the gods for mathematical discoveries to the seventeenth century practice on the part of the Japanese of giving *sangaku* (the Japanese word for "mathematical tablet") to the spirits for discovering mathematical proofs, there seems to be a universal feeling across the world that discoveries reveal the world to us in bits and pieces. This is why the ancients thought that a causal connection existed between earthly matters and the stars. Those who could use numbers to calculate forthcoming events, such as the next planting season, garnered great power unto themselves, becoming wizards, mathematicians, and astronomers. The concept of metaforms provides a framework for understanding why discoveries are made. As products of our innate capacity to model the world, they are products of the most creative modeling system that nature has thus far produced — the human mind (Cassirer 1944; Bonner 1980; Adam 2004).

But even the notion of metaform really does not penetrate the substance of the enigma at hand. Nor does it really answer the two questions enunciated above. Semiotics is a descriptive science, after all, not an explanatory one. So we are left with the same kinds of questions with which I started off this paper: Why does mathematics work as a model to explain the physical world? Why is the Pythagorean Theorem, for instance, real, explaining a whole range of phenomena? This is a true mystery. As Jacob Bronowski has aptly put it:

> To this day, the theorem of Pythagoras remains the most important single theorem in the whole of mathematics. That seems a bold and extraordinary thing to say, yet it is not extravagant; because what Pythagoras established is a fundamental characterization of the space in which we move, and it is the first time that it is translated into numbers. And the exact fit of the numbers describes the exact laws that bind the universe. If space had a different symmetry the theorem would not be true. (Bronowski 1973: 168)

And as Clawson (1999: 284) has suggested, mathematics might even explain the laws of unknown universes: "Certain mathematical truths are the same beyond this particular universe and work for all potential universes."

But again: Why should this be so? Why does there seem to be continuity between mind matter and physical matter? Is it possible to discover the larger pattern from which the fabric of metaforms of reality have been cut to produce a "broader picture" of the universe? It is, after all, this desire to see the broader picture that the reifications of the FS stimulate in us. But it is an elusive picture, and we seem destined never to get a total look at it, just tantalizing serendipitous glimpses of it here and there. All that can be said is the Pythagorean view that numbers and symbols were mirrors of nature is not just rhetorical flourish. As Ghyka (1997), Schneider (1994), Adam (2004), and many others have abundantly illustrated mathematical principles are mysterious because they manifest themselves serendipitously in flowers, shells, crystals, plants, and the human body, as well as in the symbolic language of folk sayings, fairy tales, myths, religions, art forms, and architecture. But why this is so remains one of the greatest puzzles of all times.

References

Adam, John A. 2004. *Mathematics in Nature: Modeling Patterns in the Natural World*. Princeton: Princeton University Press.
Anderson, Myrdene; Sáenz-Ludlow, Adalira; Cifarelli, Victor 2000. Musement in mathematical manipulation. In: Gimate-Welsh, Adrián (ed.), *Ensayos Semióticos*. Mexico: Porrúa, 663–676.
Antenos-Conforti, Enza; Barbeau, Ed; Danesi, Marcel 1998. *Problem-Solving in Mathematics: A Semiotic Perspective with Educational Implications*. (Toronto: Monograph Series of the Toronto Semiotic Circle, 20.) Welland: Soleil.
Ball, Keith 2003. *Strange Curves, Counting Rabbits, and Other Mathematical Explorations*. Princeton: Princeton University Press.
Beckmann, Petr 1981. *A History of π*. New York: St. Martin's.
Bergin, Thomas Goddard; Fisch, Max Harold 1984. *The New Science of Giambattista Vico*. 2nd ed. Ithaca: Cornell University Press.
Bertland, Alexander U. 2004. Vico's reasoning concerning the origin of number. *New Vico Studies* 22: 49–62.
Blatner, David 1997. *The Joy of Pi*. Harmondsworth: Penguin.
Bonner, John Tyler 1980. *The Evolution of Culture in Animals*. Princeton: Princeton University Press.
Bronowski, Jacob 1973. *The Ascent of Man*. Boston: Little, Brown, and Co.
Cassirer, Ernst A. 1944. *An Essay on Man*. New Haven: Yale University Press.
Clawson, Calvin C. 1999. *Mathematical Sorcery: Revealing the Secrets of Numbers*. Cambridge: Perseus.
Danesi, Marcel 2002. *The Puzzle Instinct: The Meaning of Puzzles in Human Life*. Bloomington: Indiana University Press.
— 2003. Metaphorical 'networks' and verbal communication: A semiotic perspective of human discourse. *Sign System Studies* 31(2): 341–364.
— 2004. *Poetic Logic: The Role of Metaphor in Thought, Language, and Culture*. Madison: Atwood Publishing.
Darling, David 2004. *The Universal Book of Mathematics*. Hoboken: John Wiley.
Deely, John 2001. *Four Ages of Understanding: The First Postmodern Survey of Philosophy from Ancient Times to the Turn of the Twenty-First Century*. Toronto: University of Toronto Press.
Devlin, Keith 2004. Cracking the Da Vinci Code. *Discover* 25(6): 64–69.
— 2005. *The Math Instinct*. New York: Thunder's Mouth Press.
Dewdney, Alexander Keewatin 1999. *A Mathematical Mystery Tour: Discovering the Truth and Beauty of the Cosmos*. New York: John Wiley & Sons.
Eco, Umberto 1998. *Serendipities: Language and Lunacy*. (Weaver, William, trans.) New York: Columbia University Press.
Eco, Umberto; Sebeok, Thomas A. (eds.) 1983. *The Sign of Three: Dupin, Holmes, Peirce*. Bloomington: Indiana University Press.
English, Lyn D. (ed.) 1997. *Mathematical Reasoning: Analogies, Metaphors, and Images*. Mahwah: Lawrence Erlbaum Associates.

Eymard, Pierre; Lafon, Jean-Pierre; Wilson, Stephen S. 2004. *The Number Pi.* New York: American Mathematical Society.
Fann, Kuang Tih 1970. *Peirce's Theory of Abduction*. The Hague: Martinus Nijhoff.
Ghyka, Matila 1977. *The Geometry of Art and Life*. New York: Dover.
Heinzelin, Jean de 1962. Ishango. *Scientific American* 206(6): 105–116.
Koestler, Arthur 1959. *The Sleepwalkers*. London: Penguin.
Lakoff, George; Núñez, Raphael E. 2000. *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being*. New York: Basic Books.
Langer, Susanne 1948. *Philosophy in a New Key*. Cambridge: Harvard University Press.
Livio, Mario 2002. *The Golden Ratio*. New York: Broadway.
Lotman, Yuri 1990. *The Universe of the Mind: A Semiotic Theory of Culture.* Bloomington: Indiana University Press.
Lucas, François Édouard Anatole 1882-1894. *Récreations mathématiques,* 4 vols. Paris: Gauthier-Villars.
MacNamara, Olwen 1996. Mathematics and the sign. *Proceedings of PME* 20: 369–378.
Maor, Eli 1994. *e: The Story of a Number*. Princeton: Princeton University Press.
Menninger, Karl 1969. *Number Words and Number Symbols: A Cultural History of Number*. Cambridge: MIT Press.
Merrell, Floyd; Quieroz, João (eds.) 2005. *Abduction: Between Subjectivity and Objectivity*. Special Issue of *Semiotica* 153(1/4). Berlin: Mouton de Gruyter.
Merton, Robert K.; Barber, Elinor 2003. *The Travels and Adventures of Serendipity: A Study in Sociological Semantics and the Sociology of Science*. Princeton: Princeton University Press.
Norman, Donald A. 1993. *Things that Make Us Smart: Defending Human Attributes in the Age of the Machine*. Reading: Addison-Wesley.
Ogilvy, Charles Stanley; Anderson, John T. 1966. *Excursions in Numbers Theory*. New York: Dover.
Ouaknin, Marc-Alain 2004. *The Mystery of Numbers*. Paris: Assouline.
Peirce, Charles Sanders 1931–1958. *Collected Papers*. Cambridge: Harvard University Press.
Posamentier, Alfred S. 2004. *Pi: A Biography of the World's Most Mysterious Number*. New York: Prometheus.
Radford, Luis; Grenier, Monique 1996. On dialectical relationships between signs and algebraic ideas. *Proceedings of PME* 20: 179–186.
Reed, David 1994. *Figures of Thought: Mathematics and Mathematical Texts.* London: Routledge.
Roberts, Royston M. 1989. *Serendipity: Accidental Discoveries in Science*. New York: John Wiley.
Rotman, Brian 1988. Towards a semiotics of mathematics. *Semiotica* 72: 1–35.
Schneider, Michael S. 1994. *Constructing the Universe: The Mathematical Archetypes of Nature, Art, and Science*. New York: Harper Collins.
Sebeok, Thomas A.; Danesi, Marcel 2000. *The Forms of Meaning: Modeling Systems Theory and Semiotics*. Berlin: Mouton de Gruyter.

Stewart, Ian 1995. *Nature's Numbers*. New York: Basic Books.
— 2001. Foreword. In: Moscovich, Ivan, *1000 PlayThinks*. New York: Workman Publishing, v.
— 2004. *Math Hysteria: Fun and Games with Mathematics*. Oxford: Oxford University Press.

Последовательность чисел Фибоначчи и сущность математического открытия

В статье рассматривается связь между математическим открытием и знаковым процессом на примере последовательности чисел Фибоначчи. Случайное открытие этой последовательности как ответ на задачу, сформулированную знаменитым итальянским математиком Леонардо Фибоначчи для иллюстрации эффективности десятеричной системы, является одним из тех случаев в истории человечества, где явственно сплетаются случай, семиозис и открытие. Последовательность Фибоначии позволяет изучить созидающий семиозис и дает понять, что символы не являются арбитрарными продуктами человеческого сознания, а подсознательными “зондами” реальности.

Fibonacci rida ja matemaatilise avastuse loomus: Semiootiline vaade

Artikkel vaatleb suhet matemaatilise avastuse ja märgiprotsessi vahel, kuulsa Fibonacci rea näitel. Selle rea juhuslik avastamine kui vastus itaalia matemaatiku Leonardo Fibonacci poolt sõnastatud ülesandele illustreerida kümnendsüsteemi efektiivsust, on üks neid juhtumeid inimajaloos, mis näitab, kuidas juhus, semioos ja avastus on põimunud. Sellisena on Fibonacci rida oluliste tulemitega loova semioosi uurimiseks, kuivõrd ta viitab, et sümbolid pole inimmõistuse arbitraarsed produktid, vaid alateadvuslikud reaalsuse sondid.