PHOTOMETRIC MEASURES ON THE MOON AND THE EARTH-SHINE

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I. On the Illumination-Law of the Moon's Surface.

1. Introduction.

In 1914 a series of lunar photographs was obtained by the writer with the aid of a 160 mm visual refractor (focal length 274 cm) of the private observatory of Mr. Trindin in Moscow; on the same plate near the focal image of the Moon extrafocal photographs of *α* Lyrae or *α* Aurigae were taken; these photographs with varying exposure formed a scale of photographic density with which the density of different parts of the Moon's surface could be compared. In 1922 these photographs were measured with the aid of a microphotometer at the Tartu Observatory; the measures were undertaken chiefly as a training in microphotometer readings; nevertheless certain results of interest can be deduced from them. On account of the non-uniformity of the extrafocal images of the comparison star, due to the chromatic aberration of the visual objective, much weight cannot be attributed to the absolute brightness deduced from the measures; variations of atmospheric transparency added to the uncertainty, so that the probable error of one plate 1) resulted as ±0.18 st. magnitudes. On the contrary, the difference of brightness of the single points on the Moon's surface could be deduced with greater precision, the differential measures being affected only by the variations in the sensitiveness of the plate at different points: the probable error of the differential brightness of a point measured on a single plate came out as ±0.07 st. mg. Thus the derivation of the distribution of brightness over the Moon's surface at different phase-angles may be regarded as the chief purpose of the present investigation; as the final

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1) For one photograph this error may be regarded as systematical, affecting equally all measured points.

result the observational data are represented by an empirical law, giving the surface brightness as the function of the angles of incidence and reflection.

It is well known that all so-called "theoretical laws of illumination" fail to represent the actual variation with phase-angle of the brightness of the planets and the Moon; since the physical conditions on the surface of different celestial bodies may present an infinite variety, the optical properties of these bodies must show the same variety, so that a single "law" cannot hold in all cases, and without knowing the true structure of the surface the brightness at given conditions cannot be theoretically computed. On the contrary, by inverting the problem — i. e. by determining empirically the illumination-law and by trying to find a model of the reflecting surface that fits best the observational data — considerable advance in the knowledge of planetary surfaces can be expected.

With these ideas in view the measurements discussed in this paper were undertaken.

2. Measurement and Reduction of the Plates.

Table 1 contains the data relating to the photographs and their reduction.

The separate columns of this table contain: Sec z — the mean secant of the zenith distance, ψ — the phase-angle, ϱ — the apparent radius of the Moon; B_g and L_g — the selenocentric selenographic latitude and longitude of the observer, $B_\odot$ and $L_\odot$ — the corresponding selenocentric coordinates of the sun; Δf — distance from focus of the extrafocal images of the star; n — number of exposures of the star; exposures max. and min. — the longest resp. shortest exposures of the star, the other exposures forming a regular progression between these extreme values. The quantities ψ, ϱ, B_g, L_g and z for the Moon are values corrected for parallax. The kind of plates used were Lumière extra-rapides, size $4\frac{1}{2} \times 6$; developer Metol-Hydrochinon. For the Moon diaphragms of 39.5, 25.3 and 20.2 mm were used, as indicated in the table; the purpose was to obtain greater accuracy by lengthening the time of exposure and by approaching the surface brightness of the focal image of the Moon to the surface brightness of the extra-

Table 1.

a) Photographs of the Moon.

Negative	Date, G.M.T. 1914	Exposure	Diaphragm	Sec z	ψ	ϱ	B_g	L_g	$B_\odot$	$L_\odot$
	h m	s	mm							
9. I[1]	April 4, 5 32	3.0	39.5	1.18	78.°7	15′57.″0	−4.°5	−7.°7	+0.°8	70.°9
„ II	„ , 5 36	3.0	„	1.18	„	„	„	„	„	„
10.	„ , 6 47	4.0	„	1.27	78. 3	15 56. 6	−4. 4	−7. 9	„	70. 3
14.	April 5, 9 42	3.0	„	1.81	64. 1	16 10. 6	−2. 9	−7. 5	„	56. 6
26.	April 30, 5 46	7.0	„	1.61	120. 7	15 27. 7	−5. 3	−7. 1	+1. 3	113. 6
27.	„ , 5 49	8.0	„	1.62	„	„	„	„	„	„
28. I	„ , 5 51	4.0	„	1.63	„	„	„	„	„	„
„ . II	„ , 5 53	12.0	„	1.64	„	„	„	„	„	„
32.	May 3, 6 09	9.0	25.3	1.40	83. 6	16 06. 0	−2. 0	−6. 8	„	76. 8
35.	May 6, 7 32	7.0	„	1.95	43. 1	16 40. 2	+3. 1	−3. 5	+1. 4	39. 6
36.	May 7, 9 47	9.0	20.2	2.97	28. 1	16 44. 7	+4. 7	−1. 8	„	26. 2
37.	„ , 9 48	10.0	„	2.97	„	„	„	„	„	„

b) Comparison Star.

Negative	Star	Date 1914	Δf	n	Exposures Max.	Exposures Min.	Sec z
			mm		s	s	
9	α Aurigae	April 4	33.1	13	82	7	1.28
10	„	April 4	33.0	10	52	7	1.46
14	„	April 5	33.1	10	60	8	2.38
26	α Lyrae	April 30	31.1	9	20	2	1.10
„	„	„	41.1	13	40	2	1.10
„	„	„	71.1	9	60	6	1.09
27	„	April 30	41.1	15	45	2	1.18
„	„	„	81.1	10	116	8	1.15
„	„	„	31.1	11	21	2	1.11
28[2]	„	May 5	41.1	13	37	2	1.21
32[2]	„	May 5	41.1	13	44	3	1.18
„	„	„	81.1	11	120	9	1.16
„	„	„	31.1	13	24	2	1.13
35	„	May 6	81.1	11	120	8	1.24
„	„	„	41.1	8	40	6	1.18
36	„	May 7	51.1	14	60	3	1.18
37	„	May 7	51.1	15	60	3	1.24

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[1] This photograph was obtained 24.8 mm out of focus. Other photographs were focal.

[2] The comparison star for Neg. 28 and 32 was photographed 5 resp. 2 days after the lunar photograph.

focal stellar images; a considerable improvement of the quality of the images of the Moon was also obtained, the chromatic aberration of the visual objective being of no significance for the small diaphragms used.

Table 2 contains the selenographic coordinates of the measured points. A few points, included originally in the program, were not measured, chiefly on account of the irregularities in brightness[1]) which did not allow to obtain accurate readings on the microphotometer; the numeration originally adopted was not changed, and so the gaps in the numeration of the table were produced. The points were chosen so that the chief formations, the dark "maria" and the bright "continentes", were represented at various conditions of illumination and observation. The points on the continentes were chosen where no great irregularities of the surface occurred; thus they may be regarded as representing, on the average, the photometric conditions of a horizontal surface. All points may be divided into two groups: points of the regular program, from 1 to 64, measured on every photograph where the point was illuminated by the sun; and supplementary points, marked by the letter s, from 1_s to 45_s, chosen near the terminator where the variation of the brightness is extremily rapid; the supplementary points were measured only on one day, when the terminator passed near them.

On the microphotometer the points were found from their rectangular coordinates computed in advance; the scales of the microphotometer could be read with accuracy to 0.05 mm, or about 1:500 of the Moon's diameter; the actual accuracy of pointing as controlled by lunar craters with known coordinates was somewhat less. The computation of the coordinates was made as follows: the rectangular coordinates of each point were found from their selenographic coordinates (B, L of table 2) and the optical libration (B_g, L_g, table 1), and were converted into millimeters with the aid of the known focal distance and the apparent radius (ϱ, table 1); two conspicuous markings (artificial or defects of the plate), distant enough, were chosen on the plate; the orientation of the Moon's disk with respect to these markings was determined by measuring on a Repsold machine points

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1) Produced by the irregular topography of the surface.

Table 2.
Selenographic Coordinates of Measured Points.

№	Formation	B	L
1	Cont., N Polar Region	+73.°7	+ 0.°0
2	„	+68. 8	+ 0. 0
3	„	+65. 0	0. 0
5	Cont., NW Limb	+66. 5	+54. 2
6	„	+62. 0	+40. 8
7	Mare Frigoris	+56. 5	+27. 2
8	Cont. NW Limb	+39. 0	+70. 5
9	„	+37. 1	+59. 6
10	„	+34. 6	+49. 5
11	„	+32. 0	+41. 0
12	„	+29. 2	+33. 8
13	Cont. Equator. Limb	0. 0	+74. 5
14	Mare Crisium	+17. 1	+58. 1
15	Cont. near M. Crisium	+ 7. 1	+57. 0
16	Cont. near Proclus	+14. 5	+43. 2
16 a	Proclus	+16. 2	+46. 4
17	Mare Foecunditatis	— 2. 0	+51. 9
19	Cont. near centre	+ 2. 2	+11. 7
20	Mare Tranquillitatis	+ 5. 5	+29. 0
21	„	+ 2. 3	+26. 6
22	Mare Serenitatis	+23. 7	+22. 3
23	„	+31. 7	+21. 2
23 a	Mare Seren., on Ray of Tycho	+31. 7	+23. 2
24	Mare Seren.	+26. 2	+13. 5
25	Contin. SW Limb	—34. 5	+69. 0
26	„	—32. 5	+61. 8
27	„	—29. 5	+50. 7
28	„	—27. 0	+42. 9
29	„	—23. 8	+36. 9
31	Mare Nectaris	—13. 9	+32. 6
32	„	—13. 1	+36. 5
33	„	—18. 5	+36. 6
34	Cont. between M. Nect. and M. Foec.	— 5. 3	+34. 8
35	Contin.	—34. 2	+36. 0
36	Contin.	— 8. 0	+11. 2
37	Contin.	—27. 2	+17. 1
38	Contin.	— 4. 4	+19. 8
39	Contin.	—27. 4	+ 5. 3
40	Contin. SW Limb	—61. 6	+51. 0
40 a	„	1)	1)
41	„	—58. 1	+41. 0
42	„	—52. 0	+27. 2
43	„	—45. 9	+20. 8
44	„	—41. 0	+15. 2

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1) On Neg. 26—28 $B = -68.^{\circ}1$ $L = +66.^{\circ}0$; Neg. 9, 10 $B = -67.5$ $L = +71.0$; Neg. 14 $B = -66.2$ $L = +66.0$; Neg. 32 $B = -66.^{\circ}9$ $L = +64.^{\circ}0$.

NB. On Negatives 36 and 37 the coordinates of the actually measured points differed by several tenths of a degree from the coordinates given in the table. In the reduction of the measures this difference was taken into account.

Table 2. Continued.

№	Formation	*B*	*L*
49	Ptolemaeus	− 9.°0	− 2.°1
50	Cont. near Ptol.	− 7.2	− 6.0
52	Cont. near Tycho	−37.8	− 4.1
53	Mare Humorum	−23.7	−38.0
54	Oceanus Procellarum	− 4.5	−36.1
55	Mare near centre	+ 7.6	− 8.1
55 a	Dark Spot between Rays of Copern.	+ 6.0	− 8.1
57	Rays of Copernicus	+12.4	−17.6
58	Mare Imbrium	+21.3	− 7.7
59	„	+36.5	−14.0
60	Oceanus Procellarum	+22.3	−33.5
61	Sinus Iridum (mare)	+43.8	−32.9
62	Near Kepler (bright)	+ 6.5	−36.5
63	Contin. near Sin. Iridum	+44.0	−43.0
64	Plato	+50.8	−10.2
1 s	Mare Crisium	+18.4	+69.1
2 s	„	+18.0	+65.0
3 s	„	+17.7	+59.6
4 s	„	+17.3	+55.4
5 s	Mare Serenitatis	+20.3	+28.7
6 s	„	+19.3	+27.8
7 s	„	+18.0	+26.5
8 s	Mare Tranquillitatis	+12.3	+39.6
9 s	„	+ 9.0	+35.0
10 s	„	+ 5.5	+31.5
11 s	Oceanus Procellarum	− 1.0	−30.3
12 s	„	− 1.0	−35.0
13 s	„	− 1.0	−38.6
14 s	„	− 1.0	−41.8
15 s	„	+33.0	−23.5
16 s	„	+32.5	−27.5
17 s	„	+32.5	−30.0
18 s	„	+32.5	−34.2
19 s	„	+32.5	−37.1
20 s	„	+32.5	−41.4
21 s	„	− 0.3	−43.0
22 s	„	− 0.4	−48.4
23 s	„	+29.3	−37.6
24 s	„	+32.9	−43.5
25 s	„	+35.1	−50.9
26 s	Contin. to E from Tycho	−44.4	−12.4
27 s	„	−44.1	−22.8
28 s	„	−44.1	−28.0
29 s	„	−44.1	−33.5
30 s	„	−44.1	−44.8
31 s	„	−44.1	−50.8
32 s	Contin. to N from Plato and Sin. Iridum]	+52.3	−21.3
33 s	„	+52.3	−22.2
34 s	„	+51.2	−29.0
35 s	„	+50.0	−34.8
36 s	„	+44.9	−36.6
37 s	„	+40.3	−39.1
38 s	„	+39.5	−41.2

Table 2. Continued.

№	Formation	B	L
39 s	Contin. to E from Tycho	−39. 5	−35. 8
40 s	"	−44. 5	−44. 6
41 s	"	−50. 2	−58. 3
42 s	"	−49. 7	−57. 8
43 s	Contin. near Mare Humorum	−15. 5	−41. 3
44 s	"	−17. 0	−46. 6
45 s	"	−18. 5	−53. 8

on the lunar surface with known coordinates (Tycho, Plato, Kepler, Proclus and other craters were used); on the microphotometer the position of the two markings was measured and the orientation of the Moon's axis thus determined; the orientation being known, the rectangular coordinates were transformed into coordinates of the microphotometer; the computation was controlled by measuring on the microphotometer the coordinates of the lunar formations that were measured previously on the Repsold machine[1]).

The area measured on the microphotometer covered a circle of 0.11 mm diameter or about 0.5 selenocentric degrees at the centre of the disk; two readings of the photometric wedge were made on each point by moving the wedge in opposite directions; the accuracy of the readings being greater than the differences produced by the unequal sensitiveness of the plate at different points, a greater number of readings was considered as useless. In the few cases, chiefly on the continentes, when the area to be measured was not uniform, the reading was made so that one-half of the area appeared brighter, and one-half — fainter than the wedge; since points with very great contrasts in the brightness were rejected, in the case of moderate contrasts the way of measurement used is quite legitimate and the result may be regarded as representing the average surface brightness of the area.

The microphotometer readings on the extrafocal stellar images were made always on a point, called afterwards "a", corres-

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[1]) These formations could not be directly used to determine the orientation on the microphotometer because of the difficulty of finding these objects with the aid of the small field (about $\frac{1}{2}$ mm) covered by the microphotometer.

ponding to a definite point on the objective, 22 mm from its centre. The very centre of the image could not be used for this purpose on account of the considerable chromatic aberration. The extrafocal images revealed a non-uniformity due to the following causes: 1) the chromatic aberration, producing a gradual decrease of the intensity from the centre towards the edges of the image, the intensity- gradient decreasing with the increasing distance from the focus; and 2) certain irregularities in the transparency of the objective, producing a network of fine details which remained invariable at different distances from the focus. The effect of the chromatic aberration was neglected in the reduction; this is the weak point of the work, and an error in the zero-point and in the scale of magnitudes could hence arise. As to the details produced by the unequal transparency of the objective, they favoured the identification of the point "a"; the coordinates of the latter were measured on the Repsold machine separately for each image and on the microphotometer the points were found in the same manner as the points of the lunar surface.

Table 3 contains the microphotometer readings made on the point "a" of the extrafocal stellar images; the logarithms of the single exposures are reduced to a certain mean zenith distance by the application of small corrections for atmospherical absorption; these corrections did not exceed a few units of the third decimal.

The data of table 3 were used to fix the scale of stellar magnitudes. For purposes of interpolation Schwarzschild's formula was adopted: $i t^p = i_1 t_1^p$ . . . . (1), where i and i_1 are the intensities giving equal photographic effect with the exposures t and t_1. It may be questioned whether the more general formula of E. Kron[1]) must be used here; certain departures from Schwarzschild's law were found and examined from the standpoint of Kron's formula; the most probable value of the "optimal intensity" resulted as ∞, for which Kron's formula is equivalent to the formula of Schwarzschild; this circumstance, as well as the low value found for the exponent p, makes quite legitimate the use of Schwarzschild's formula in the present occasion.

The exponent p was determined from the data of table 3

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1) Publ. d. Astrophys. Observatoriums zu Potsdam, № 67.

Table 3.

Microphotometer Readings on the Extrafocal Stellar Images.

Measurements of a point 0.27 radii from the centre.

D = photographic density or reading of the photometric wedge.
Lge = logarithm of exposure in seconds, reduced to mean zenith distance.

D	Loge
Neg. 9 α Aurigae $\Delta f = 33.1$	
4.86	0.845
5.11	1.000
5.38	1.114
5.58	1.230
5.97	1.342
6.16	1.431
6.56	1.556
6.84	1.681
7.20	1.806
7.50	1.914
5.26	1.079
5.62	1.204
5.96	1.342
Neg. 10 α Aurigae $\Delta f = 33.0$	
4.87	1.000
4.45	0.845
5.26	1.114
5.58	1.255
5.88	1.380
6.36	1.568
6.69	1.716
6.14	1.477
5.78	1.342
Neg. 14 α Aurigae $\Delta f = 33.1$	
6.28	1.778
5.98	1.681
5.64	1.580
5.36	1.477
5.13	1.380
4.84	1·279
4.62	1.176
4.46	1.079
4.20	0.954
4.24	0.903

D	Loge
Neg. 26 α Lyrae $\Delta f = 31.1$	
4.89	0.699
5.16	0.778
5.54	0.903
5.86	1.041
6.24	1.176
6.55	1.301
$\Delta f = 41.1$	
3.00	0.301
3.32	0.477
3.66	0.602
3.88	0.699
4.19	0.778
4.44	0.845
4.72	0.903
4.82	1.000
5.14	1.114
5.51	1.230
5.82	1.362
6.06	1.491
6.32	1.602
$\Delta f = 71.1$	
5.25	1.778
4.80	1.653
4.63	1.531
4.32	1.415
4.10	1.301
3.72	1.176
3.35	1.041
2.96	0.903

D	Loge
Neg. 27 α Lyrae $\Delta f = 41.1$	
3.08	0.291
3.48	0.467
3.83	0.592
3.74	0.592
4.32	0.768
4.48	0.835
4.57	0.893
4.79	0.990
4.93	1.069
5.31	1.166
5.48	1.291
5.84	1.405
6.26	1.521
6.45	1.643
$\Delta f = 81.1$	
3.00	1.041
3.20	1.146
3.76	1.301
4.16	1.431
4.36	1.531
4.65	1.653
5.14	1.778
5.51	1.908
5.84	2.064
$\Delta f = 31.1$	
3.58	0.309
4.38	0.485
4.74	0.610
4.42	0.485
4.72	0.610
5.13	0.707
5.50	0.853
5.84	0.962
6.14	1.087
6.44	1.212

D	Loge
Neg. 28 α Lyrae $\Delta f = 41.1$	
3.86	0.699
3.01	0.301
3.51	0.477
4.20	0.699
4.50	0.778
4.92	0.903
5.20	1.000
5.41	1.079
5.82	1.204
6.48	1.447
6.90	1.568
Neg. 35 α Lyrae $\Delta f = 81.1$	
6.62	2.071
6.28	1.979
5.81	1.849
5.21	1.724
4.78	1.594
4.44	1.469
4.15	1.354
3.73	1.222
3.28	1.106
3.12	0.992
$\Delta f = 41.1$	
7·59	1.610
7.20	1.485
6.70	1.350
6.28	1.212
5.74	1.087
5.52	1.008
5.14	0.911
4.72	0.786

D	Loge
Neg. 32 α Lyrae $\Delta f = 41.1$	
7.74	1.638
7.36	1.512
6.84	1.393
6.57	1.296
5.99	1.171
5.75	1.074
5.06	0.898
5.94	1.199
4.65	0.773
4.14	0.694
3.94	0.597
3.47	0.472
$\Delta f = 81.1$	
6.34	2.079
6.06	1.964
5.58	1.839
5.26	1.716
4.46	1.462
4.04	1.342
3.25	1.079
2.93	0.954
$\Delta f = 31.1$	
7.70	1.388
7.44	1.263
7.02	1.154
6.76	1.049
6.24	0.911
5.91	0.786
5.41	0.610

D	Loge
Neg. 36 α Lyrae $\Delta f = 51.1$	
4.00	0.477
4.06	0.613
4.29	0.699
4.76	0.778
6.07	1.041
6.54	1.146
6.95	1.230
6.84	1.230
7.17	1.322
7.59	1.415
7.93	1.519
8.16	1.643
Neg. 37 α Lyrae	
3.98	0.602
4.05	0.602
3.92	0.477
4.23	0.699
4.44	0.778
5.15	0.903
5.44	0.954
5.82	1.041
6.47	1.146
6.73	1.230
7.14	1.322
7.45	1.415
7.77	1.531
7.97	1.653
8.30	1.778

Table 4.

Determination of Schwarzschild's Exponent p.

Δf_1 Δf_2	31.1 41.1			31.1 71.1	31.1 81.1		41.1 71.1	41.1 81.1			Average	
2 (Log Δf_2— —Log Δf_1)	0.242			0.718	0.832		0.476	0.590			p	n
Negative	26	27	32	26	27	32	26	27	32	35		
D (Density)	Log e_2 — Log e_1											
7.5	—	—	0.256	—	—	—	—	—	—	—	0. 95	1
7.1	—	—	0.282	—	—	—	—	—	—	—	0. 86	1
6.7	—	—	0.305	—	—	—	—	—	—	0.744	0. 69	2
6.3	0.396	0.407	0.316	—	—	1.132	—	—	0.813	0.756	0.717	6
5.9	0.348	0.415	0.350	—	1.070	1.147	—	0.650	0.796	0.760	0.751	8
5.5	0.342	0.421	0.373	—	1.078	1.157	—	0.658	0.783	0.777	0.744	8
5.1	0.325	0.396	—	0.970	1.075	—	0.643	0.679	0.767	0.782	0.759	8
4.7	—	0.362	—	—	1.053	—	0.638	0.690	0.748	0.773	0.779	6
4.3	—	0.315	—	—	1.037	—	0.592	0.720	0.718	—	0.807	5
3.9	—	0.257	—	—	0.985	—	0.550	0.724	0.686	—	0.853	5
3.5	—	—	—	—	—	—	0.544	0.742	0.660	—	0.851	3
3.1	—	—	—	—	—	—	0.585	0.783	—	—	0.779	2
Average p	0.701			0.740	0.769		0.804	0.801			0.775	
n	17			1	9		6	22			55	

on the assumption that the intensity of the extrafocal images varied as the inverse square of the distance from the focus, Δf1). If e_1 and e_2 are the exposures producing equal density $D_1 = D_2$ on the distances Δf_1 and Δf_2 from the focus, then

$$p = \frac{2\,(\mathrm{Log}\,\Delta f_1 - \mathrm{Log}\,\Delta f_2)}{\mathrm{Log}\,e_1 - \mathrm{Log}\,e_2} \quad \ldots \ (2).$$

When a series of different determinations of p is available, the mean value is given by

$$\bar{p} = 2\,\frac{\Sigma\,(\mathrm{Log}\,\Delta f_1 - \mathrm{Log}\,\Delta f_2)}{\Sigma\,(\mathrm{Log}\,e_1 - \mathrm{Log}\,e_2)} \quad \ldots \ (2^1).$$

The *Log e* of table 3 were plotted with the D as abscissae and smooth curves drawn through the points; for the negatives 26, 27, 32 and 35, on which exposures at different distances from the focus were made, the logarithmic difference of the expos-

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1) The effective focus of the photographic radiation may be regarded as known with the accuracy of about ± 0.2 mm.

ures producing equal density was determined graphically from these curves; table 4 contains the result. The values of p, computed according to formula (2^1), are grouped in this table according to the density and the distances from the focus; the density has evidently no systematic influence on p; however, a systematic decrease of p with decreasing Δf (or increasing intensity of illumination) seems to be indicated in the table; the change takes place in a sense opposite to the direction demanded by Kron's formula, so that the use of this formula can give no improvement; besides, the systematic variation cannot be much relied upon, since the greatest deviation occurs where the difference of Δf is the smallest, and where a systematic error in the adopted brightness at $\Delta f = 31.1$ of about 0.06 st. mg. may account for the deviation. The source of such an error may be, e. g., a systematic error of pointing, the point measured on the small extrafocal images being systematically somewhat too far from the centre as compared with the position of the point "a" on the large images; the error described would tend to act in a direction opposite to the effect of chromatic aberration; the chief reason why the chromatic aberration was not taken into account was that, whereas the latter should increase the intensity at the point "a" at small distances from the focus (the increase relatively to the inverse-square law was estimated for $\Delta f = 31.1$ as 0.24 st. magnitudes), the observations indicated the existence of an unknown systematic influence which not only counterbalanced the supposed effect, but even changed the sign of the deviation. Under these circumstances the simple assumption of the inverse-square law seemed to be the best which could be made. Finally the average value

$p = \mathbf{0.775}$ was adopted.

It is interesting to compare the result here obtained with results derived from eye-estimates; in 1914 such estimates were made by the writer in connection with an investigation on the brightness of Mars[1]; from a greater number of plates than the number used here the following values were found[2]:

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[1] Zum Lichtwechsel d. Planeten Mars. Astronomische Nachrichten 5162, pp. 17—24.

[2] Loc. cit. p. 18, Tabelle 1.

Star: *α* Lyrae.

Negat.	23	23	24	24	26	26	27	27	32	32	31	31	35	38
Δf_1	41.1	31.1	46.1	31.1	31.1	41.1	41.1	31.1	41.1	31.1	33.1	46.1	41.1	63.1
Δf_2	61.1	41.1	81.1	46.1	41.1	71.1	81.1	81.1	81.1	81.1	46.1	81.1	81.1	88.1
p	0.810	0.737	0.784	0.780	0.759	0.774	0.805	0.748	0.751	0.727	0.680	0.713	0.756	0.770
weight	52	10	30	4	11	32	46	82	54	42	14	32	34	8

Grouping the data according to the values of Δf, we obtain:

	Average p
$\Delta f =$ 31.1—41·1 31.1—46.1 33.1—46.1	0.727 ± 0.017
31.1—81.1 31.1—71.1	0.741 ± 0.010
41.1—71.1 41·1—61.1 41.1—81.1 46.1—81.1	0.773 ± 0.006
63.1—88.1	0.770 ± 0.040
Average of all	0.760 ± 0.005

The eye-estimates show a better internal agreement than the microphotometric measures; this may be explained by the circumstance that irregularities in the extrafocal images have little influence on such estimates, the images being compared as a whole; a slight systematic change of p is indicated, in the same direction as found from the microphotometer measures, although in a considerably less degree. The agreement of the final values of p derived by the two different methods is very satisfactory, so that they may be regarded as supporting one another. This is of importance in our present case, since the effect of chromatic aberration on the eye-estimates is, probably, negligible.

Since the photographs of the Moon and of the point "*a*" on the stellar images corresponded to different parts of the objective, a correction for differences in the transmission seemed necessary à priori; to determine the correction, the distribution of the brightness in the inner part of the extrafocal images was determined on different plates and images, chiefly for $\Delta f = 81.1$ and 71.1. The photographic density was converted into intensity logarithms by using the time-scale of the negative (table 3) and by multiplying the *log e* by the factor $p = 0.775$. The following results were obtained:

Table 5.

Ratio of the Average Transmission of the Diaphragms to the Transmission at the Point "*a*".

Diam. of Diaphragm, mm.	20.3	25.3 Ratio	39.5
	1.028	1.034	0.960
	1.006	1.002	1.008
	0.959	0.952	1.040
	1.074	1.052	0.993
	1.034	1.006	0.976
	0.978	0.975	
Mean	1.013 ± 0.011	1.004 ± 0.010	0.995 ± 0.009

The difference in the mean transmission comes out very small, indeed; for convenience' sake the correction for transmission was applied, though it could as well be neglected.

The surface brightness of the Moon was expressed in stellar magnitudes, *the zero point of which* was taken *equal to the surface brightness of the extrafocal photographs of a Lyrae at the distance 31.1 mm from the focus;* the surface brightness was reduced to the diameter of the diaphragm = 39.5 mm. The surface brightness so obtained was called the "*standard magnitude*" or the "standard surface brightness". For each plate a table was constructed for converting the microphotometer readings into standard magnitudes. For this purpose the following corrections were applied.

1) Correction for diaphragm and transmission of the objective; taking the intensity of illumination proportional to d^2, d being the diameter of the diaphragm, and using the values of the transmission given in table 5, the following quantities to be added to the observed surface brightness were adopted:

Diaphragm mm	20.3	25.3	39.5
Correction of Surf. Br. (Stellar Magn.)	—1.430	—0.965	—0.005;

with the aid of these corrections the surface brightness can be reduced to the aperture 39.5 mm and transmission of the point "*a*".

2) Correction for atmospheric absorption; the

mean coefficient of absorption found from another series of photographs made in spring, 1914, in Moscow[1] was adopted as **0.255 of the logarithm of exposure per unit of air mass:** it is obvious that, where Schwarzschild's formula holds, the correction can be applied to the logarithm of exposure as a linear function of Sec z.

3) The logarithm of the exposure-ratio of the Moon and the comparison star was multiplied by $2.5p = 1.938$ and so converted into stellar magnitudes.

4) The relative intensities of the images of the comparison star were adopted according to the inverse-square law as follows:

α Lyrae, $\Delta f =$	31.1	41.1	51.1	71.1	81.1	α Aurigae, $\Delta f =$	33.0	33.1
Surface Brightness, Stellar Magn.	0.000	+0.605	+1.075	+1.795	+2.080		+0.930	+0.935

The zero-point of the surface brightness corresponds to α Lyrae photographed 31.1 mm behind the focus; the photographic difference of magnitude of α Aurigae — α Lyrae was adopted, according to *Harvard Annals*, 71, equal to +0.80 st. mg.

With the aid of these data and of smooth curves drawn through the points represented by table 3, table 6 was constructed. This table gives the *standard surface brightness* of the Moon in stellar magnitudes as the function of the photographic density. When several series of extrafocal images at different distances from the focus were available, independent values of the standard magnitude were determined from each series and the mean taken. The computation was executed in the following way: for each value of the density the $loge$ was found from curves representing table 3; if e_0 is the exposure for the lunar photograph, $2.5\ p\ (loge_0 - loge)$ gives the difference in surface brightness of the Moon and the stellar images; by adding the corrections 1), 2) and 4) described above the standard surface brightness of the Moon is obtained.

The difference $loge_0 - loge$ indicates the "range of extrapolation": an error in the factor p or deviations from Schwarzschild's law will the more influence the result the greater the difference is. The "range of extrapolation" can be easily deter-

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[1] Astr. Nachrichten loc. cit. p. 19.

mined for every value of the photographic density by taking *loge* from table 3 and e_0 — the exposure used for the Moon — from table 1.

Table 7 contains the result of measurement of the lunar photographs. In this table each point is indicated by the number assigned to it in table 2; *s* and *I* mean the standard surface

Table 6.

For converting Photographic Density (*D*) of the Moon into Standard Surface Brightness (*s*) (Stellar Magnitudes).

D	*s*	*D*	*s*	*D*	*s*	*D*	*s*	*D*	*s*	*D*	*s*
Neg. 9 I and II		Neg. 14		Neg. 27		Neg. 28 I Continued		Neg. 32 Continued		Neg. 36 and 37	
4.9	+ 0.300	4.2	+ 0.365	3.0	+ 1.565	5.2	— 0.390	7.0	— 1.440	4.2	— 0.650
5.1	+ 0.065	4.3	+ 0.220	3.2	+ 1.410	5.4	— 0.520	7.2	— 1.545	4.4	— 0.785
5.3	— 0.150	4.5	— 0.020	3.4	+ 1.255	5.6	— 0.660	7.4	— 1.660	4.6	— 0.855
5.5	— 0.320	4.7	— 0.230	3.6	+ 1.040	5.8	— 0.790	7.6	— 1.770	4.8	— 0.930
5.7	— 0.470	4.9	— 0.390	3.8	+ 0.925	6.0	— 0.925	7.8	— 1.890	5.0	— 0.990
5.9	— 0.620	5.1	— 0.535	4.0	+ 0.800	6.2	— 1.060			5.2	— 1.060
6.1	— 0.765	5.3	— 0.695	4.2	+ 0.680	6.4	— 1.195	Neg. 35.		5.4	— 1.135
6.3	— 0.900	5.5	— 0.830	4·4	+ 0.560	6.6	— 1.330	3.1	+ 0.400	5.6	— 1.215
6.5	— 1.050	5.7	— 0.965	4.6	+ 0.410	6.8	— 1.460	3.3	+ 0.265	5.8	— 1.290
6.7	— 1.190	5.9	— 1.090	4.8	+ 0.270	6.9	— 1.525	3.5	+ 0.150	6.0	— 1.370
6.9	— 1.340	6.1	— 1.210	5.0	+ 0.130			3.7	+ 0.010	6.2	— 1.440
7.1	— 1.470	6.3	— 1.315	5.2	— 0.010	Neg. 32		3.9	— 0.115	6.4	— 1.520
7.3	— 1.615			5.4	— 0.155	3.0	+ 0.920	4.1	— 0.245	6.6	— 1.610
7.5	— 1.750	Neg. 26		5.6	— 0.295	3.2	+ 0.780	4.3	— 0.380	6.8	— 1.700
Neg. 10		3.0	+ 1.380	5.8	— 0.430	3.4	+ 0.560	4.5	— 0.540	7.0	— 1.795
		3.2	+ 1.210	6.0	— 0.580	3.6	+ 0.435	4.7	— 0.660	7.2	— 1.910
4.5	+ 0.505	3.4	+ 1.045	6.2	— 0.770	3.8	+ 0.320	4.9	— 0.780	7.4	— 2.025
4.7	+ 0.385	3.6	+ 0.900	6.4	— 0.920	4.0	+ 0.200	5.1	— 0.900	7.6	— 2.160
4.9	+ 0.265	3.8	+ 0.755	6.6	— 1.160	4.2	+ 0.080	5.3	— 0.995	7.8	— 2.305
5.1	+ 0.115	4.0	+ 0.625			4.4	— 0.040	5.5	— 1.080	8.0	— 2.470
5.3	— 0.035	4.2	+ 0.500	Neg. 28 I 1)		4.6	— 0.150	5.7	— 1.180		
5.5	— 0.185	4.4	+ 0.365	3.0	0.970	4.8	— 0.260	5.9	— 1.275		
5.7	— 0.335	4.6	+ 0.220	3.2	+ 0.825	5.0	— 0.370	6.1	— 1.385		
5.9	— 0.495	4.8	+ 0.070	3.4	+ 0.670	5.2	— 0.480	6.3	— 1.495		
6.1	— 0.655	5.0	— 0.085	3.6	+ 0.545	5.4	— 0.540	6.5	— 1.605		
6.3	— 0.805	5.2	— 0.230	3.8	+ 0.420	5.6	— 0.665	6.7	— 1.720		
6.5	— 0.970	5.4	— 0.340	4.0	+ 0.300	5.8	— 0.785	6.9	— 1.845		
6.7	— 1.115	5.6	— 0.480	4.2	+ 0.185	6.0	— 0.905	7.1	— 1.950		
		5.8	— 0.625	4.4	+ 0.080	6.2	— 1.025	7.3	— 2.065		
		6.0	— 0.790	4.6	— 0.040	6.4	— 1.140	7.5	— 2.180		
		6.2	— 0.970	4.8	— 0.160	6.6	— 1.215	7.7	— 2.305		
		6.4	— 1.055	5.0	— 0.270	6.8	— 1.335				
		6.6	— 1.200								

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1) For Neg. 28 II *s* can be obtained by adding + 0.925 mg. to the *s* of Neg. 28 I.

Table 7.

Measured Brightness of different Points on the Moon's Surface.

s = Standard Surface Brightness in stellar magnitudes; $I = 2.512^{-s}$; i = angle of incidence of the solar rays; ε = angle of reflection; ψ = phase-angle.

	April 30. $\psi = 120.^{0}7$.								Neg. 32. May 3. $\psi = 83.^{0}6$				
Point	s				Neg. 26, 27, 28 I, 28 II Mean				Point				
	Neg. 26	Neg. 27	Neg. 28 I	Neg. 28 II	s	I	Cosi	ε		s	I	Cosi	ε
5	−0.15	−0.20	−0.41	−0.32	−0.27	1.28	0.215	$82.^{0}5$	5	−1.45	3.81	0.389	$79.^{0}5$
6	−0.12	−0.15	−0.24	−0.22	−0.18	1.18	0.150	76.4	6	−1.37	3.53	0.400	73.0
8	−1.06	−0.74	—	−0.70	−0.90	2.29	0.577	83.0	7	−0.83	2.15	0.375	64.0
9	−0.91	−0.61	—	−0.65	−0.79	2.07	0.476	73.9	8	−1.43	3.74	0.788	79.9
10	−0.54	−0.26	—	−0.38	−0.46	1.53	0.368	66.2	9	−1.46	3.84	0.775	71.5
11	−0.26	−0.16	—	−0.05	−0.22	1.22	0.259	58.5	10	−1.28	3.25	0.742	63.0
12	+0.30	+0·36	—	+0.45	+0.31	0.752	0.160	51.9	11	−1.22	3.08	0.698	55.9
13	−1.09	−0.69	−1.37	—	−1.04	2.61	0.774	80.6	12	−1.08	2.70	0.651	49.7
14	−0.56	−0.22	−0.75	−0.52	−0.51	1.61	0.544	67.8	13	−1.24	3.14	1.000	79.6
15	−0.80	−0.48	−1.09	—	−0.78	2.05	0.549	64.0	14	−0.85	2.19	0.910	66.5
16	−0.34	−0.15	−0.42	−0.17	−0.27	1.28	0.325	53.5	15	−1.15	2.89	0.934	63.1
17	−0.30	−0.10	−0.48	—	−0.28	1.30	0.471	58.9	16	−1.10	2.75	0.811	52.0
20	+1.32	+1.52	—	+1.43	+1.36	0.285	0.086	37.3	16a	−1.55	4.18	0.833	55.0
21	—	—	—	+1.66	+1.63	0.223	0.049	34.5	17	−0.75	2.00	0.905	58.1
25	−1.04	−0.68	−1.31	−1.08	−1.03	2.58	0.577	74.6	19	−1.04	2.61	0.419	19.0
26	−1.05	−0.77	−1.34	−1.18	−1.08	2.70	0.513	69.4	20	−0.49	1.58	0.665	36.0
27	−0.77	−0.59	−1.13	−0.87	−0.84	2.17	0.390	60.0	21	−0.53	1.64	0.640	33.3
28	−0.16	−0.03	−0.59	−0.22	−0.25	1.26	0.288	52.0	22	−0.55	1.67	0.540	38.1
29	−0.04	+0.14	−0.04	+0.01	+0.02	0.982	0.202	46.0	23	−0.50	1.59	0.494	42.7
31	+0.63	+0.70	+0.47	+0.62	+0.61	0.570	0.144	39.7	23a	−0.72	1.95	0.519	43.6
32	+0.20	+0.32	+0.05	+0.16	+0.18	0.848	0.214	43.7	24	−0.38	1.42	0.411	34.1
33	+0.30	+0.42	+0.18	+0.24	+0.29	0.752	0.206	44.5	25	−1.36	3.50	0.801	75.4
34	+0.24	+0.22	+0.05	+0.22	+0.18	0.848	0.192	41.3	26	−1.36	3.50	0.799	70.0
35	+0.06	+0.27	+0.05	−0.11	+0.07	0.938	0.168	49.2	27	−1.37	3.53	0.771	61.3
40a	−0.60	−0.30	−0.62	−0.48	−0.50	1.59	0.238	83.0	28	−1.13	2.83	0.727	53.5
41	+0.30	−0.16	—	—	−0.01	1.01	0.143	64.1	29	−1.02	2.56	0.691	47.4
1s	−0.66	—	—	—	−0.66	1.84	0.679	75.0	31	−0.85	2.19	0.688	40.5
2s	−0.58	—	—	—	−0.58	1.71	0.630	71.2	32	−0.85	2.19	0.735	44.1
3s	−0.54	—	—	—	−0.54	1.65	0.564	66.5	33	−0.71	1.93	0.717	45.2
4s	−0.53	—	—	—	−0.53	1.63	0.509	63.1	34	−1.02	2.56	0.737	41.5
5s	+1.21	+1.37	—	—	+1.21	0.328	0.082	42.5	35	−1.00	2.51	0.611	50.9
6s	+1.26	+1.42	—	—	+1.26	0.314	0.071	40.8	36	−0.88	2·25	0.406	18.4
7s	+1.40	+1.47	—	—	+1.36	0.286	0.053	39.6	37	−0.76	2.02	0.438	33.7
8s	+0.34	+0.47	+0.19	+0.43	+0.36	0.719	0.270	48.5	38	−1.13	2.83	0.539	26.7
9s	+0.72	+0.79	+0.48	+0.68	+0.67	0.540	0.193	43.3	39	−0.55	1.67	0.271	27.8
10s	+1.09	+1.14	+0.90	+1.09	+1.06	0.377	0.137	39.5	40	−1.06	2.65	0.409	72.7
									40a	−1.22	3.08	0.363	80.3
									41	−0.92	2.33	0.408	66.5
									42	−0.78	2.05	0.382	57.1
									43	−0.87	2.23	0.373	50.0
									49	−0.04	1.04	0.185	8.4
									50	+0.52	0.620	0.119	5.0

Table 7. Continued.

April 4. $\psi = 78^{0}.7$.

Point	s			Neg. 9 I. 9 II. 10 Mean			
	Neg. 9 I 1)	Neg. 9 II	Neg. 10	s	I	Cosi	ε
5	—	—1.02	—1.20	—1.18	2.97	0.397	80.°9
6	—	—1.00	—1.12	—1.14	2.86	0.419	75. 1
7	—	—0.51	—0.62	-0.64	1.81	0.411	66. 2
8	—	—1.24	- 1.13	—1.26	3.19	0.786	82. 8
9	—1.54	—1.36	—1.18	—1.36	3.50	0.789	73. 9
10	—1.37	—1.09	—1.13	—1.20	3.02	0.773	65. 5
11	—1.30	—1.06	—1.02	—1.13	2.83	0.742	58. 3
12	—1.13	—0.86	—0.73	—0.91	2.31	0.707	50. 6
13	—	—1.31	—0.92	—1.19	2.99	0.999	80. 4
14	—0.99	—0.82	—0.57	—0.80	2.09	0.934	67. 9
15	—1.37	—1.21	—0.89	—1.16	2.91	0.962	64. 8
16	—	—1.13	—0.94	—1.11	2.78	0.861	53. 4
16a	—	—1.56	—1.38	—1.55	4.18	0.879	56. 4
17	—0.89	—0.80	—0.43	—0.71	1.93	0.944	59. 6
19	—1.02	—0.85	—0.94	—0.94	2.38	0.514	20. 2
20	—0.57	—0.47	—0.22	—0.42	1.48	0.742	38. 0
21	—0.61	—0.51	—0.33	—0.48	1.56	0.717	35. 2
22	—0.51	—0.33	—0.32	—0.39	1.44	0.610	40. 5
23	—0.56	—0.40	—0.27	—0.41	1.46	0.558	45. 2
23a	—	—0.45	—0.45	—0.53	1.64	0.583	46. 1
24	—0.41	—0.21	—0.28	—0.30	1.32	0.492	37. 1
25	—	—1.52	—1.10	—1.39	3.60	0.813	75. 3
26	—1.73	—1.53	—1.22	—1.49	3.95	0.824	70. 0
27	—1.50	—1.50	—1.12	—1.37	3.53	0.813	60. 4
28	—1.31	—1.26	—0.89	—1.15	2.89	0.782	52. 7

April 4. Continued.

Point	s			Neg. 9 I. 9 II. 10 Mean			
	Neg. 9 I 1)	Neg. 9 II	Neg. 10	s	I	Cosi	ε
29	—1.26	— 1.13	— 0.83	—1.07	2.68	0.753	47.°3
31	—	— 0.82	— 0.67	—0.82	2.13	0.760	40. 5
32	—	— 0.85	— 0.52	—0.76	2.02	0.800	44. 0
33	—	— 0.83	— 0.53	—0.76	2.02	0.780	45. 2
34	—	— 1.06	— 0.72	—0.97	2.44	0.806	42. 5
35	—1.22	— 1.13	— 0.80	—1.05	2.63	0.669	50. 0
36	—0.88	— 0.75	— 0.81	—0.81	2.11	0.497	18. 6
37	—0.90	— 0.80	— 0.57	—0.76	2.02	0.521	32. 4
38	—1.24	— 1.05	— 0.97	—1.09	2.73	0.627	27. 5
39	—0.73	— 0.59	— 0.56	—0.63	1.79	0.364	25. 9
40	—1.24	— 1.17	— 0.81	—1.07	2.68	0.437	68. 8
40a	—	— 1.33	— 0.92	—1.20	3.02	0.371	86. 0
41	—1.10	— 1.15	— 0.85	—1.03	2.58	0.446	64. 7
42	—1.03	— 0.93	— 0.68	—0.88	2.25	0.435	55. 1
43	—0.93	— 0.95	— 0.57	—0.82	2.13	0.437	47. 7
44	—0.88	— 0.89	— 0.62	—0.80	2.09	0.419	41. 3
49	—0.33	— 0.11	— 0.21	—0.22	1.22	0.289	6. 8
50	—	0.00	— 0.03	—0.06	1.06	0.225	2. 9
52	—	— 0.22	— 0.31	—0.34	1.37	0.198	32. 9
55	—	—	+ 0.24	+0.08	0.930	0.196	11. 8
55a	—	—	+ 0.42	+0.26	0.788	0.196	10. 2
58	—	—	+ 0.24	+0.08	0.930	0.198	25. 5
59	—	—	+ 0.58	+0.42	0.680	0.080	41. 0
64	—	—	+ 0.39	+0.23	0.810	0.114	54. 0

Neg. 35. May 6. $\psi = 43^{0}.1$.

Point	s	I	Cosi	ε
1	— 1.63	4.50	0.239	72.°8
2	— 1.65	4.58	0.301	65. 7
3	— 1.44	3.77	0.347	61. 9
5	— 1.71	4.84	0.401	74. 5
6	— 1.71	4.84	0.490	67. 2
7	— 1.28	3.25	0.554	58. 0

Neg. 35. May 6. Continued.

Point	s	I	Cosi	ε
8	— 1.91	5.81	0.680	75.°1
9	— 1.94	5.97	0.763	66. 5
10	— 1.92	5.86	0.822	58. 0
11	— 1.80	5.25	0.860	50. 6
12	— 1.61	4.41	0.883	43. 8
13	— 1.98	6.20	0.818	78. 0

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1) Neg. 9 I, being taken about 25 mm out of focus, could not furnish reliable measures on points where great contrasts of brightness existed in the neighbourhood; such points were therefore omitted and only those were measured which were surrounded by a comparatively uniform surface. The diameter of the bundle of rays for this photograph was about 0.35 mm (corresponding to the diaphragm 39.5 mm).

Table 7. Continued.

Neg. 35. May 6. $\psi = 43^0.1$.					Neg. 35. May 6. Continued.				
Point	s	I	Cosi	ε	Point	s	I	Cosi	ε
14	— 1.50	3.98	0.910	61°.7	44	— 1.80	5.25	0.671	47°.2
15	— 1.92	5.86	0.948	59 .9	49	— 1.38	3.57	0.732	12 .0
16	— 1.76	5.07	0.971	47 .7	50	— 1.40	3.63	0.692	10 .4
16a	— 2.24	7.88	0.958	50 .7	52	— 1.88	5.65	0.557	40 .8
17	— 1.44	3.77	0.975	55 .5	53	— 0.06	1.06	0.188	43 .0
19	— 1.76	5.07	0.882	15 .3	54	— 0.01	1.01	0.243	33 .4
20	— 1.17	2·94	0.975	32 .5	55a	— 0.85	2.19	0.668	4 .8
21	— 1.28	3.25	0.971	30 .3	57	— 1.19	2.99	0.532	16 .9
22	— 1.19	2.99	0.881	32 .6	58	— 1.03	2.58	0.640	18 .6
23	— 1.24	3.14	0.816	37 .0	59	— 0.87	2.23	0.488	34 .4
23a	— 1.34	3.44	0.820	37 .9	60	— 0.22	1.22	0.278	35 .0
24	— 1.21	3.05	0.813	28 .4	61	— 0.23	1.24	0.232	48 .3
25	— 2.13	7.12	0.702	77 .8	62	— 0.69	1.89	0.239	33 .1
26	— 2.27	8.10	0.766	71 .0	63	— 0.24	1.25	0.130	51 .8
27	— 2.14	7.19	0.842	61 .5	64	— 0.85	2.19	0.427	49 .4
28	— 1.91	5.81	0.880	53 .6	11s	— 0.46	1.53	0.341	26 .5
29	— 1.74	4.97	0.905	47 .8	12s	— 0.20	1.20	0.263	30 .9
31	— 1.58	4.29	0.955	39 .6	13s	+ 0.02	0.983	0.202	34 .6
32	— 1.52	4.06	0.964	43 .0	14s	+ 0.30	0.759	0.148	37 .7
33	— 1.46	3.84	0.939	45 .2	15s	— 0.64	1.81	0.390	35 .9
34	— 1.65	4.58	0.989	39 .1	16s	— 0.48	1.56	0.337	37 .7
35	— 1.80	5.25	0.812	52 .8	17s	— 0.49	1.58	0.306	39 .5
36	— 1.68	4.71	0.864	18 .4	18s	— 0.18	1.18	0.244	42 .3
37	— 1.64	4.54	0.809	36 .1	19s	+ 0.08	0.930	0.203	44 .4
32s	— 1.04	2.61	0.317	51 .2	20s	+ 0.32	0.745	0.143	47 .6
33s	— 1.06	2.65	0.307	51 .6	26s	— 1.94	5.97	0.422	47 .2
34s	— 0.84	2.17	0.246	53 .0	27s	— 1.46	3.84	0.314	48 .8
35s	— 0.56	1.68	0.189	54 .6	28s	— 1.32	3.37	0.258	50 .3
38	— 1.80	5.25	0.934	24 .7	29s	— 1.13	2.83	0.191	52 .0
39	— 1.75	5.01	0.721	31 .6	30s	— 0.69	1.89	0.051	56 .9
40	— 1.94	5.97	0.446	76 .5	31s	+ 0.06	0.88	0.000	59 .8
41	— 1.88	5.65	0.509	69 .1	36s	— 0.44	1.51	0.184	52 .8
42	— 1.86	5.55	0.582	60 .5	37s	— 0.18	1.18	0.163	50 .5
43	— 1.74	4.97	0.642	52 .9	38s	+ 0.06	0.88	0.137	51 .9

Neg. 14. April 5. $\psi = 64^0.1$.					Neg. 14. April 5. Continued.					Neg. 14. April 5. Continued.				
Point	s	I	Cosi	ε	Point	s	I	Cosi	ε	Point	s	I	Cosi	ε
5	— 1.36	3.50	0.411	80°.3	14	— 0.99	2.49	0.959	67°.3	23	— 0.74	1.98	0.699	43°.8
6	— 1.37	3.53	0.462	74 .4	15	— 1.28	3.25	0.993	64 .4	24	— 0.65	1.83	0.659	35 .3
7	— 0.97	2.44	0.489	65 .2	16	— 1.21	3.05	0.942	52 .9	25	— 1.30	3.31	0.794	76 .0
8	— 1.42	3.70	0.761	80 .5	16a	— 1.53	4.10	0.946	55 .9	26	— 1.31	3.34	0.831	70 .8
9	— 1.47	3.88	0.806	73 .4	17	— 0.83	2.15	0.996	59 .3	27	— 1.27	3.22	0.860	61 .0
10	— 1.28	3.25	0.824	64 .3	19	— 1.04	2.61	0.707	19 .8	28	— 1.09	2.73	0.857	53 .4
11	— 1.28	3.25	0.821	56 .8	20	— 0.66	1.84	0.881	37 .1	29	— 1.01	2.53	0.855	47 .7
12	— 1.16	2.91	0.812	50 .5	21	— 0.62	1.77	0.865	34 .3	31	— 0.85	2.19	0.881	41 .0
13	— 1.26	3.19	0.939	80 .3	22	— 0.63	1.79	0.760	39 .6	32	— 0.85	2.19	0.908	44 .5

Table 7. Continued.

Neg. 14. April 5. Continued.					Neg. 14. April 5. Continued.					Neg. 14. April 5. Continued.				
Point	s	I	Cosi	ε	Point	s	I	Cosi	ε	Point	s	I	Cosi	ε
33	— 0.78	2.05	0.885	45°.6	40a	— 0.96	2.42	0.386	86°.0	55	— 0.21	1.21	0.423	10°.3
34	— 1.03	2.58	0.923	41 .5	41	— 0.88	2.25	0.496	66 .3	55a	— 0.03	1.03	0.423	8 .7
35	— 0.90	2.29	0.764	51 .0	42	— 0.75	2·00	0.525	56 .3	57	— 0.26	1.27	0.268	18 .0
36	— 0.94	2.38	0.693	18 .8	43	— 0.74	1.98	0.555	49 .4	58	— 0.23	1.24	0.409	24 .0
37	— 0.99	2.49	0.680	33 .3	44	— 0.78	2.05	0.558	42 .7	59	— 0.15	1.15	0.272	39 .5
38	— 1.14	2.86	0.794	27 .2	49	— 0.51	1.61	0.511	7 .8	64	— 0.26	1.27	0.260	54 .5
39	— 0.74	1.98	0.548	27 .5	50	— 0.45	1.52	0.453	4 .1					
40	— 0.86	2.21	0.462	71 .6	52	— 0.66	1.84	0.376	34 .6					

May 7. $\psi = 28^{0}.1$.							May 7. Continued.						
Point	s1)		Neg. 36, 37 Mean		Cosi	ε	Point	s1)		Neg. 36, 37 Mean		Cosi	ε
	Neg. 36	Neg. 37	s	I				Neg. 36	Neg. 37	s	I		
13	— 2.27	— 2.23	—2.25	7.97	0.662	75°.8	55a	— 1.25	—	—1.25	3.17	0.818	5°.4
14	— 1.80	— 1.66	—1.73	4.93	0.805	61 .1	57	— 1.72	— 1.71	—1.72	4.88	0.705	17 .6
15	— 2.50	— 2.44	—2.47	9.74	0.841	59 .3	58	— 1.52	— 1.43	—1.48	3.91	0.778	17 .9
16	— 2.27	— 2.13	—2.20	7.59	0.922	46 .1	59	— 1.54	— 1.42	—1.48	3.91	0.618	34 .0
17	— 1.74	— 1.69	—1.72	4.87	0.891	54 .8	60	— 1.12	— 1.12	—1.12	2.80	0.466	35 .8
19	— 2.26	— 2.18	—2.22	7.74	0.966	13 .7	61	— 1.20	— 1.10	—1.15	2.89	0.372	49 .6
20	— 1.65	— 1.58	—1.62	4.45	0.994	30 .8	62	— 1.54	— 1.58	—1.56	4.21	0.451	35 .3
21	— 1.59	— 1.60	—1.60	4.37	1.000	28 .5	63	— 1.42	— 1.33	—1.38	3.57	0.252	54 .6
22	— 1.61	— 1.49	—1.55	4.18	0.921	30 .3	64	— 1.52	— 1.36	—1.44	3.77	0.527	47 .5
23	— 1.78	— 1.64	—1.71	4.84	0.857	35 .5	21s	—	— 0.98	—0.98	2.47	0.356	41 .5
24	— 1.68	— 1.57	—1.63	4.50	0.880	30 .0	22s	—	— 0.86	—0.86	2.21	0.262	46 .4
31	— 1.80	— 1.74	—1.77	5.11	0.954	39 .3	23s	—	— 1.10	—1.10	2.75	0.398	41 .5
33	— 1.75	— 1.73	—1.74	4.97	0.919	45 .3	24s	—	— 0.96	—0.96	2.42	0.302	47 .7
34	— 2.17	— 2.16	—2.17	7.39	0.979	38 .2	25s	—	— 0.82	—0.82	2.13	0.167	54 .7
36	— 2.15	— 2.24	—2.20	7.59	0.950	17 .7	39s	—	— 1.77	—1.77	5.12	0.347	54 .2
37	— 2.04	— 2.32	—2.18	7.46	0.865	37 .7	40s	—	— 1.50	—1.50	3.98	0.217	62 .2
39	— 2.02	— 2.26	—2.14	7.19	0.818	33 .4	41s	—	— 1.10	—1.10	2.75	0.040	72 .3
49	— 1.78	— 1.82	—1.80	5.25	0.862	13 .6	42s	—	— 0.99	—0.99	2.50	0.048	72 .3
50	— 1.82	— 1.84	—1.83	5.40	0.833	12 .6	43s	—	— 1.45	—1.45	3.81	0.360	44 .0
53	— 0.75	— 1.14	—0.94	2.38	0.375	46 .1	44s	—	— 1.18	—1.18	2.97	0.274	48 .8
54	— 1.03	— 1.09	—1.06	2.65	0.451	35 .9	45s	—	— 0.98	—0.98	2.47	0.159	55 .6
55	— 1.42	— 1.42	—1.42	3.70	0.818	7 .0							

brightness expressed in stellar magnitudes and ordinary intensity-units respectively; *Cosi* determines the intensity of illumination of a horizontal surface at the point measured; ε is the angle of reflection or the zenith distance of the observer as seen from the point on the Moon. For the days when several pho-

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1) On Neg. 36 and 37 a number of points on the continentes could not be measured, being overexposed.

tographs were obtained, the mean of the single measures as well as the mean of the *Cosi* and ε was taken. In deriving the mean value of s systematic differences of the individual plates were taken into account, as will be explained later on. Some points were occasionally omitted in the measures; the special reason of omission on Neg. 9 I, 36 and 37 is explained in the footnotes of the table; in the remaining instances the points were omitted either on account of defects of the plate or because an error was made in pointing, so that a wrong point was measured[1]).

Photographs made on the same day show systematic differences which must be attributed to variations of atmospheric absorption (acting on the whole photograph) and to a systematic variation of the sensitiveness of the plate (depending on the coordinates of the measured point). From the common points the following systematic differences of the photographs were found:

Table 8. Systematic Differences of Photographs.

	April 30.					April 4.				May 7.
Negative	27—26	28_{I}—26	28_{II}—26	26—26	Mean	9_{I}—10	9_{II}—10	10—10	Mean	36—37
Syst. Difference, St. Mg.	+0.16	—0.19	+0.03	0.00	0.00	—0.31	—0.17	0.00	—0.16	—0.02
Number of Points	24	18	21	—	—	28	45	—	—	34

From these differences the following systematic corrections of the single photographs were adopted:

Table 9. Systematic Corrections of the Photographs.

Negative	26	27	28_{I}	28_{II}	9_{I}	9_{II}	10	36, 37 *adopted*
Correction, St. Mg.	0.00	—0.16	+0.19	—0.03	+0.15	+0.01	—0.16	0.00

By applying these corrections, the measured brightness was reduced to the average system of all negatives obtained on the corresponding day, and in this manner the *mean* values of table 7 were found.

The deviations of the individual differences from the mean of all points can give us an information on the precision of the

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1) The coordinates on the microphotometer were noted during the measurement, so that the position of the measured point could be controlled afterwards.

differential measures of surface brightness; the pairs of negatives 27 — 26, 28_{I} — 26, 28_{II} — 26, 9_{I} — 10, 9_{II} — 10, 36 — 37 gave the following number of deviations (rounded off to 0.05 mg.) from the mean differences:

Deviation	±0.00	±0.05	±0.10	±0.15	±0.20	±0.25	±0.30	±0.35	±0.40	±0.62	Total
Number	23	42	33	30	23	12	3	2	1	1	170

From these data the mean square deviation of the difference in s for one point measured on two plates comes out as ±0.15 st. mg.; this corresponds to the following value of the probable error of *differential* measures of s on one negative:

$$\text{p. e.} = \pm 0{,}674 \cdot \frac{0{,}15}{\sqrt{2}} = \pm 0.072 \text{ st. magn.}$$

This error includes 1) variations in the sensitiveness of the plate from one point to another, 2) errors in the microphotometer readings and 3) errors in the scale of the comparison star; the first source of error must be considered as the chief one.

3. Derivation of the Preliminary Illumination-Law.

The only way to determine the illumination-law of the Moon's surface consists in the assumption that similar formations of the surface have on the average the same reflecting power and that the same illumination-law holds for them; the deviations in the reflecting power must be regarded as accidental errors.

Therefore the measured points were joined into the following groups, according to their supposed reflecting power:

Ist Group.	Ordinary *continentes:* points 5, 6, 8, 9, 10, 11, 12, 13, 15, 16, 19, 25, 26, 27, 28, 29, 34, 35, 36, 37, 38, 39, 40, 40_a, 41, 42, 43, 44, 43_s, 44_s, 45_s, 32_s, 33_s, 34_s, 35_s, 36_s, 37_s, 38_s.
IInd „	Exceptionally bright *continentes*, like the region around Tycho: points 16_a, 52, 26_s, 27_s, 28_s, 29_s, 30_s, 31_s, 39_s, 40_s, 41_s, 42_s.
IIIrd „	*Continentes* below normal brightness: points 49, 50.
IVth „	The "rays" around Copernicus and Kepler: points 57, 62.
Vth „	Ordinary *maria*: points 14, 17, 20, 21, 22, 23, 24, 31, 32, 33, 53, 54, 55, 58, 59, 60, 61, 64, 1_s, 2_s, 3_s,

Vth Group.		4_s, 5_s, 6_s, 7_s, 8_s, 9_s, 10_s, 11_s, 12_s, 13_s, 14_s, 15_s, 16_s, 17_s, 18_s, 19_s, 20_s, 21_s, 22_s, 23_s, 24_s, 25_s.
VIth	„	*Maria* above normal brightness: mare Frigoris, point 7.
VIIth	„	Mare Serenitatis, on bright “ray“ of Tycho; point 23 a.
VIIIth	„	Dark spot, point 55 a.

It was assumed, as a first approximation, that all points of one group have identical optical properties. Only groups I and V could be used in the derivation of the law of illumination, the other groups being represented by too small a number of points. Points 1, 2, 3 and 63 were not included originally in the discussion, though they evidently belong to group I.

The points were classified according to the arguments ε (angle of reflection) and i (angle of incidence) and average values were formed; table 10 contains the result. Whereas in the case of observations of the same point the mean of the stellar magnitudes (logarithmic mean) must be taken, only the logarithmic

Table 10.

Average Surface Brightness arranged according to the Angles of Illumination and Reflection.

Continentes, Group I.

April 30 (4 Negatives).

Average Cos i	0.180	0.152	0.150	0.202	0.274	0.227	0.361	0.476	0.531	0.577	0.774
„ ε	45°.2	58.0	76.4	46.0	55.3	82.8	59.9	73.9	66.7	78.3	80.6
„ I	0.893	0.881	1.18	0.982	1.24	1.44	1.66	2.07	2.38	2.44	2.61
Points	34; 35.	12; 41.	6.	29.	11; 28.	5; 40 a.	10; 16; 27.	9.	15; 26.	8; 25.	13.

May 3 (1 Negative).

Average Cos i	0.271	0.412	0.400	0.395	0.390	0.539	0.714	0.646	0.829	0.841
„ ε	27°.8	18.7	41.8	61.8	76.4	26.7	44.4	54.8	61.7	76.6
„ I	1.67	2.44	2.12	2.19	3.27	2.83	2.56	2.87	3.17	3.55
Points	39.	19; 36.	37; 43.	41; 42.	5; 6; 40; 40 a.	38.	29; 34.	10; 11; 12; 28; 35.	15; 16; 26; 27.	8; 9; 13; 25.

April 4 (3 Negatives).

Average Cos i	0.430	0.429	0.439	0.396	0.570	0.595	0.724	0.780	0.836	0.847
„ ε	22°.2	45.0	62.9	80.7	23.6	41.2	54.4	44.9	61.1	78.0
„ I	1.95	2.11	2.50	2.95	2.56	2.32	2.57	2.56	3.18	3.32
Points.	36; 39.	43; 44.	40; 41; 42.	5; 6; 40 a.	19; 38.	35; 37.	11; 12.	29; 34.	10; 15; 16; 26; 27; 28.	8; 9; 13; 25.

Table 10. Continued.

Continentes, Group I.

April 5 (1 Negative).

Average Cos i	0.510	0.430	0.548	0.556	0.731	0.680	0.764	0.778	0.855	0.835	0.818	0.923	0.968	0.939
„ ε	$61^0.3$	78.0	27.5	46.6	21.9	33.3	51.0	78.2	47.7	57.2	72.1	41.5	58.6	80.3
„ I	2.12	2.92	1.98	2.02	2.62	2.49	2.29	3.50	2.53	3.07	3.61	2.58	3.15	3.19
Points	41; 42.	5; 6; 40; 40_a.	39.	43; 44.	19; 36; 38.	37.	35.	8; 25.	29.	10; 11; 12; 27; 28.	9; 26.	34.	15; 16.	13.

May 6 (1 Negative).

Average Cos i	0.168	0.290	0.527	0.424	0.696	0.642	0.691	0.873	0.846
„ ε	$52^0.5$	51.9	65.6	75.5	39.4	52.9	76.5	16.8	40.0
„ I	1.31	2.48	5.35	5.40	5.13	4.97	6.46	4.90	4.48
Points	35_s; 36_s; 37_s; 38_s;	32_s; 33_s; 34_s.	6; 41; 42.	5; 40.	39; 44.	43.	8; 25.	19; 36.	12; 37.

May 6 (1 Negative). Continued.

Average Cos i	0.832	0.792	0.934	0.955	0.948
„ ε	57.2	74.5	24.7	44.9	59.9
„ I	5.89	7.15	5.25	4.87	5.86
Points	9; 10; 11; 27; 28; 35.	13; 26.	38.	16; 29; 34.	15.

May 7 (2 Negatives).

Average Cos i	0.317	0.159	0.662	0.844	0.841	0.958	0.950
„ ε	$46^0.4$	55.6	75.8	35.6	59.3	15.7	42.2
„ I	3.39	2.47	7.97	7.32	9.74	7.66	7.49
Points	43_s; 44_s.	45_s.	13.	37; 39.	15.	19; 36.	16; 34.

Maria, Group V.

April 30 (4 Negatives).

Average Cos i	0.068	0.069	0.140	0.221	0.471	0.539	0.654
„ ε	$35^0.9$	41.0	39.6	45.0	58.9	65.8	73.1
„ I	0.252	0.309	0.474	0.715	1.30	1.63	1.78
Points	20; 21.	5_s; 6_s; 7_s.	31; 10_s.	32; 33; 8_s; 9_s;	17.	14; 3_s; 4_s.	1_s; 2_s.

May 3 (1 Negative).

Average Cos i	0.452	0.652	0.614	0.726	0.908
„ ε	$38^0.4$	34.6	39.7	44.6	62.3
„ I	1.50	1.61	1.93	2.06	2.10
Points	23; 24.	20; 21.	22; 31.	32; 33.	14; 17.

April 4 (3 Negatives).

Average Cos i	0.197	0.080	0.114	0 553	0.730	0.780	0.939
„ ε	$18^0.6$	41.0	54.0	40.9	36.6	43.2	63.8
„ I	0.930	0.680	0.810	1.40	1.52	2.06	2.01
Points	55; 58.	59.	64.	22; 23; 24.	20; 21.	31; 32; 33.	14; 17.

Table 10. Continued.

Maria, Group V.

April 5 (1 Negative).

Average Cosi	0.272	0.260	0.416	0.679	0.873	0.842	0.908	0.978
„ ε	39°.5	54.5	17.2	39.6	35.7	42.1	44.5	63.3
„ I	1.15	1.27	1·22	1.90	1.80	2.01	2.19	2.32
Points	59.	64.	55; 58.	23; 24.	20; 21.	22; 31; 33.	32.	14; 17.

May 6 (1 Negative).

Average Cosi	0.198	0.202	0.341	0.270	0.321	0.439	0.427	0.640
„ ε	35°.2	45.1	26.5	33.0	38.6	35.2	49.4	18.6
„ I	0.917	1.03	1.53	1.21	1.57	2.02	2.19	2.58
Points	13_s; 14_s; 54.	18_s; 19_s; 20_s; 53; 61.	11_s	12_s; 60.	16_s; 17_s.	15_s; 59.	64.	58.

May 6 (1 Negative). Continued.

Average Cosi	0.813	0.848	0.973	0.953	0.942
„ ε	28.4	34.8	31.4	42.6	58.6
„ I	3.05	3.06	3.10	4.06	3.88
Points	24.	22; 23.	20; 21.	31; 32; 33.	14; 17.

May 7 (2 Negatives).

Average Cosi	0.282	0.167	0.377	0.374	0.458	0.618	0.527	0.798	0.868	0.848	0.972	0.936
„ ε	47°.0	54.7	41.5	47.8	35.8	34.0	47.5	12.4	32.8	58.0	29.8	42.3
„ I	2.32	2.13	2.61	2.64	2.72	3.91	3.77	3.80	4.67	4.90	4.33	5.04
Points	22_s; 24_s.	25_s.	53; 61.	21_s; 23_s.	54; 60.	59.	64.	55; 58.	23; 24.	14; 17.	20; 21; 22.	31; 33.

errors of observation being responsible for the discrepancies, in the case of average values for *different* points the variety in brightness must be regarded as real, and it is therefore natural to find the mean of the intensities themselves instead of their logarithms. This is the reason why in table 7 the observed surface brightness (s) expressed in stellar magnitudes was converted into ordinary intensity (I). Table 10 gives the average values of *Cosi*, ε and I. In such a form the table is yet not ready for discussion; since the surface brightness depends upon three arguments, ψ, ε and i, it is desirable to obtain series of data where two of the arguments remain invariable, so that the influence of the third may be separated. The phase-angle remains practically constant for photographs taken on one day; we shall therefore arrange the measures of one day into groups with one of the arguments, ε or i constant; in the following ε was actually chosen. The values of I and *Cosi* of table 10 were changed so as to make them correspond to round values

Table 11.

Normal Values of the Surface Brightness.

ψ = Phase-Angle; i = Angle of Incidence; $j = -2{,}5 \log (\mathrm{Cos}\,i)$;
ε = Angle of Reflection; I = Surface Brightness; $s = -2{,}5 \log I$.

Continentes, Group I.

$\varepsilon = 20^{0}$				$\varepsilon = 30^{0}$				$\varepsilon = 45^{0}$				$\varepsilon = 60^{0}$				$\varepsilon = 75^{0}$			
I	Cosi	s	j	I	Cosi	s	j	I	Cosi	s	j	I	Cosi	s	j	I	Cosi	s	j
$\psi = 120^{0}.7$			+				+				+				+				+
								0.89	0.180	+ 0.12	1.87	0.91	0.152	+ 0.10	2.05	1.16	0.150	— 0.16	2.06
								0.95	0.190	+ 0.05	1.80	1.27	0.266	— 0.26	1.44	1.39	0.239	— 0.35	1.55
												1.66	0.361	— 0.55	1.11	2.10	0.488	— 0.80	0.78
												2.35	0.513	— 0.93	0.72	2.43	0.568	— 0.96	0.61
																2.60	0.770	— 1.04	0.28
$\psi = 83^{0}.6$																			
2.43	0.411	— 0.96	0.97	1.67	0.271	— 0.55	1.42	2.13	0.399	— 0.82	1.00	2.19	0.395	— 0.85	1.01	3.16	0.390	— 1.25	1.02
				2.78	0.568	— 1.11	0.61	2.56	0.714	— 1.02	0.37	3.02	0.612	— 1.20	0.53	3.51	0.840	— 1.36	0.19
												3.13	0.828	— 1.24	0.21				
$\psi = 78^{0}.7$																			
1.93	0.430	-- 0.71	0.92	2.00	0.430	— 0.75	0.92	2.11	0.429	— 0.81	0.92	2.44	0.437	— 0.97	0.90	2.80	0.410	— 1.12	0.97
2.61	0.565	— 1.04	0.62	2.48	0.578	— 0.99	0.59	2.40	0.638	— 0.95	0.49	2.67	0.776	— 1.07	0.28	3.30	0.845	— 1.30	0.18
								2.56	0.780	— 1.02	0.27	3.14	0.832	— 1.24	0.20				
$\psi = 64^{0}.1$																			
2.64	0.740	— 1.05	0.33	1.98	0.549	— 0.74	0.65	2.02	0.555	— 0.76	0.64	2.07	0.515	— 0.79	0.72	2.78	0.444	— 1.11	0.88
				2.53	0.697	— 1.01	0.39	2.36	0.736	— 0.93	0.33	2.69	0.769	— 1.08	0.29	3.37	0.776	— 1.32	0.28
								2.40	0.860	— 0.95	0.16	3.18	0.832	— 1.26	0.20	3.72	0.815	— 1.43	0.22
								2.69	0.932	— 1.08	0.08	3.15	0.966	— 1.24	0.04	3.18	0.946	— 1.26	0.06
$\psi = 43^{0}.1$																			
4.85	0.870	— 1.71	0.15	4.65	0.855	— 1.67	0.17	1.31	0.168	— 0.29	1.94	5.32	0.578	— 1.81	0.59	5.40	0.429	— 1.83	0.92
5.35	0.929	— 1.82	0.08	5.15	0.939	— 1.78	0.07	2.48	0.290	— 0.99	1.34	5.42	0.656	— 1.83	0.46	6.36	0.688	— 2.01	0.41
								5.07	0.674	— 1.76	0.43	6.10	0.825	— 1.96	0.21	7.18	0.791	— 2.14	0.26
								4.90	0.842	— 1.72	0.19	5.86	0.948	— 1.92	0.06				
								4.87	0.955	— 1.72	0.05								
$\psi = 28^{0}.1$																			
7.63	0.957	— 2.21	0.05	6.92	0.845	— 2.10	0.18	3.54	0.342	— 1.37	1.17	2.47	0.159	— 0.98	2.00	7.97	0.662	— 2.25	0.45
				7.57	0.953	— 2.20	0.05	8.28	0.843	— 2.29	0.19	9.84	0.841	— 2.48	0.19				
								7.48	0.950	— 2.18	0.06								

Table 11. Continued.

Maria, Group V.

$\varepsilon = 20^0$				$\varepsilon = 30^0$				$\varepsilon = 45^0$				$\varepsilon = 60^0$				$\varepsilon = 75^0$			
I	Cosi	s	j	I	Cosi	s	j	I	Cosi	s	j	I	Cosi	s	j	I	Cosi	s	j
$\psi = 120^0.7$			+				+				+				+				+
				0.195	0.067	+ 1.78	2.93	0.360	0.070	+ 1.11	2.89	1.35	0.481	— 0.32	0.80	1.82	0.687	— 0.65	0.41
								0.474	0.140	+ 0.81	2.14								
								0.715	0.221	+ 0.37	1.64								
$\psi = 83^0.6$																			
				1.50	0.452	— 0.44	0.86	1.50	0.452	— 0.44	0.86	2.10	0.885	— 0.80	0.13				
				1.61	0.652	— 0.51	0.47	1.93	0.614	— 0.71	0.53								
								2.06	0.726	— 0.78	0.36								
$\psi = 78^0.7$																			
0.930	0.197	0.08	1.76	0.805	0.138	+ 0.24	2.15	0.680	0.080	+ 0.42	2.74	0.875	0.131	+ 0.15	2.21				
				1.40	0.680	— 0.36	0.42	1.40	0.553	— 0.36	0.64	2.02	0.907	— 0.76	0.11				
								2.05	0.795	— 0.78	0.25								
$\psi = 64^0.1$																			
1.22	0.416	— 0.22	0.95	1.60	0.902	— 0.51	0.11	1.19	0.268	— 0.19	1.43	1.32	0.255	— 0.30	1.48				
								2.10	0.679	— 0.80	0.42	2.30	0.966	— 0.90	0.04				
								2.11	0.826	— 0.81	0.21								
								2.19	0.908	— 0.85	0.10								
$\psi = 43^0.1$																			
2.58	0.640	— 1.03	0.49	0.87	0.196	+ 0.15	1.77	1.03	0.202	— 0.03	1.74	3.86	0.941	— 1.47	0.07				
				1.21	0.270	— 0.21	1.42	1.59	0.311	— 0.50	1.27								
				1.53	0.341	— 0.46	1.17	2.13	0.431	— 0.82	0.92								
				1.97	0.443	— 0.73	0.89	4.03	0.951	— 1.51	0.06								
				3.05	0.820	— 1.21	0.22												
				[2.94]	[0.976]	[— 1.17]	[0.03]												
$\psi = 28^0.1$																			
4.15	0.826	— 1.54	0.21	2.20	0.424	— 0.86	0.93	2.32	0.282	— 0.91	1.37	2.13	0.167	— 0.82	1.95				
				3.96	0.648	— 1.49	0.47	2.62	0.376	— 1.05	1.06	4.93	0.846	— 1.73	0.18				
				4.53	0.858	— 1.64	0.17	3.68	0.527	— 1.41	0.70								
				4.33	0.972	— 1.59	0.03	4.78	0.858	— 1.70	0.17								
								5.22	0.927	— 1.79	0.08								

of ε, namely $\varepsilon = 20^0$; 30^0; 45^0; 60^0; 75^0; this was executed with the aid of linear interpolation with ε as argument; each observed value was reduced to the nearest round value of ε; in this way table 11 was obtained. Each "normal value" of this table corresponds to a definite measured value; since in table 10 the average ε are generally very near the round values chosen, and since the dependence of the brightness upon ε proved to be of a linear character, as will be shown afterwards, the data of table 11 may be regarded as representing the observations almost unaltered, only schematized with respect to ε and a little smoothed, because, as the consequence of the method of interpolation used, each value of table 11 was deduced from 2 ar 3 points of table 10; the smoothing was, however, very small — on account of the small range of interpolation — so that some irregularities, produced evidently by differences of the reflecting power of the points, appear still in table 11.

The "normal" values of I and $Cos i$ in table 11 were converted into stellar magnitudes; the corresponding quantities are given in the same table under the headings s and j. The latter quantity represents the intensity of illumination expressed in stellar magnitudes. The definition of s and j is given at the head of table 11.

To find an empirical formula representing the observations, the s of table 11 were plotted with the j as abscissae; straight lines of an inclination depending on the phase-angle seemed to fit the observations well; the general form of these lines may be given by the formula

$$\mathbf{s} = \mathbf{s_0} + \mathbf{kj} \quad . \; . \; . \; (3), \text{ where}$$

s_0 is a function of the phase-angle and angle of reflection, k depends only on the phase-angle, and $j = -2{,}5 \log Cos i$, i being the angle of incidence. Equation (3) is equivalent to the following general form of the law of illumination:

$$I = I_0 \; (Cos i)^k \quad . \; . \; . \; (4), \text{ where}$$
$$I_0 = F \; (\psi, \; \varepsilon) \text{ and}$$
$$k = f \; (\psi).$$

The actual limits for the exponent k came out as

$$0 < k < 1;$$

it may be remarked that with $k = 1$ and $I_0 = \text{const.}$ equation (4) will be transformed into the known law of Lambert.

Table 12.

Representation of the Brightness of the Normal Points with the aid of the formula $s = s_0 + kj$.

ψ	k	ε	s_0	Deviations of Normal Points. Stellar Magnitudes. Computation − Observation.
				Continentes, Group I.
120°.7	0.66	45°	−1.13	−0.01 +0.01
		60	−1.34	−0.11 +0.13 +0.06 −0.07
		75	−1.36	−0.16 −0.01 +0.05 −0.01 +0.13
79.9 1)	0.42	20; 30	−1.22	−0.07 +0.10 −0.10 +0.15 −0.07 +0.06 −0.04
		45	−1.17	−0.03 +0.07 −0.05 −0.01 +0.02
		60	−1.29	+0.10 −0.05 +0.02 −0.08 +0.07 −0.05
		75	−1.48	−0.12 +0.12 −0.07 +0.08
64.1	0.59	20; 30	−1.20	−0.05 +0.08 −0.04
		45	−1.11	−0.03 −0.02 +0.07 −0.02
		60	−1.28	+0.07 +0.03 −0.10 +0.02
		75	−1.50	−0.13 +0.01 −0.06 +0.20
43.1	0.63	20; 30	−1.82	−0.02 +0.05 −0.04 0.00
		45	−1.79	+0.28 −0.04 −0.24 −0.05 +0.04
		60	−2.09	−0.09 −0.03 0.00 +0.13
		75	−2.33	−0.08 +0.06 +0.02
28.1	0.83	20; 30	−2.25	0.00 0.00 −0.01
		45	−2.34	0.00 +0.11 −0.11
		60; 75.	−2.63	+0.01 +0.01 −0.01
				Maria, Group V.
120°.7	0.84	30; 45; 60; 75	−0.99	−0.31 +0.33 0.00 −0.03 0.00 0.00
79.9 1)	0.43	20; 30	−0.66	+0.04 +0.07 −0.03 +0.04 −0.10
		45; 60	−0.80	−0.07 +0.06 +0.06 −0.02 −0.15 +0.11 −0.02 +0.02 +0.01
64.1	0.45	20; 30	−0.60	+0.05 −0.04
		45; 60	−0.92	−0.09 +0.07 −0.02 −0.02 +0.05 0.00
43.1	0.88	20; 30	−1.45	−0.01 −0.05 0.00 +0.03 +0.06 −0.05 [−0.25] 2)
		45; 60	−1.58	−0.03 +0.03 +0.05 −0.02 −0.05
28.1	0.66	20; 30	−1.68	0.00 −0.21 2) +0.12 +0.07 −0.07
		45; 60	−1.82	−0.01 −0.07 +0.05 −0.01 +0.02

Table 12 contains the constants of formula (3) which represent the observations best. The deviations of the normal points from the values computed with the aid of formula (3) are given

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1) Including Neg. 32, $\psi = 83^0.6$; the phase-angle is the weighted mean of ψ for April 4 and May 3.

2) These points were included with small weight in the derivation of k and s_0.

in the same table; the deviations are generally small, which indicates that our formula represents the observations very satisfactorily; from the total of 123 normal points 22 show deviations greater than 0.10, and 8 — greater than 0.20 st. mg., and even these excessive deviations must be attributed rather to real differences in the reflecting power of the points used in deriving the "normal" brightness than to the failure of our empirical formula.

In deriving the constants k and s_0 the data for May 3 (Neg. 32, $\psi = 83^0.6$) and April 4 (Neg. 9 I. 9 II. 10, $\psi = 78^0.7$) were joined together; the reason for doing so was the small difference in the phase-angle, so that a similar distribution of the brightness for both days might be expected; to obtain comparable data systematic corrections were determined, analogous to the corrections contained in table 9; for this purpose the comparison of the measured brightness of the same points could not be used since the angle of incidence showed considerable differences; therefore points with equal *Cosi* were compared, for which purpose preliminary curves connecting s and j for both days were drawn; from the relative shift of the curves the following systematic corrections were adopted:

for May 3, Neg. 32 . . . . $+0.08$ st. mg.
for April 4, Mean of Neg. 9 I. 9 II. 10 . . . . . -0.02 „ „

By applying these corrections the measured brightness was reduced to the mean system of the negatives 32, 9 I, 9 II, 10; the corrections and the mean phase-angle in table 12 were computed attributing to the single days a weight proportional to the number of the photographs.

On Fig. 1 and 2 the data of table 12 are represented graphically. Fig. 3 shows the variation of the factor k with the phase-angle; in drawing the curves, for $\psi = 0$ k was assumed $= 0.00$; this means that for the full Moon uniform brightness over the whole disk is assumed for a physically uniform surface — a distribution of brightness evidently confirmed by direct observation and required also by the law of Lommel-Seeliger; it appears that this law is in agreement with the observations only in the exceptional case $\psi = 0$, whereas for other phase-angles it fails utterly, like other theoretical laws. The point on the curve for the maria, $\psi = 3^0.5$ with $k = 0.04$, was deduced in

Fig. 1. Representation of the Brightness of the Normal Points with the aid of the formula $s = s_0 + kj$. Continents.
× ε = 20°,30°; · ε = 45°; ∘ ε = 60°; ⊙ ε = 75°.

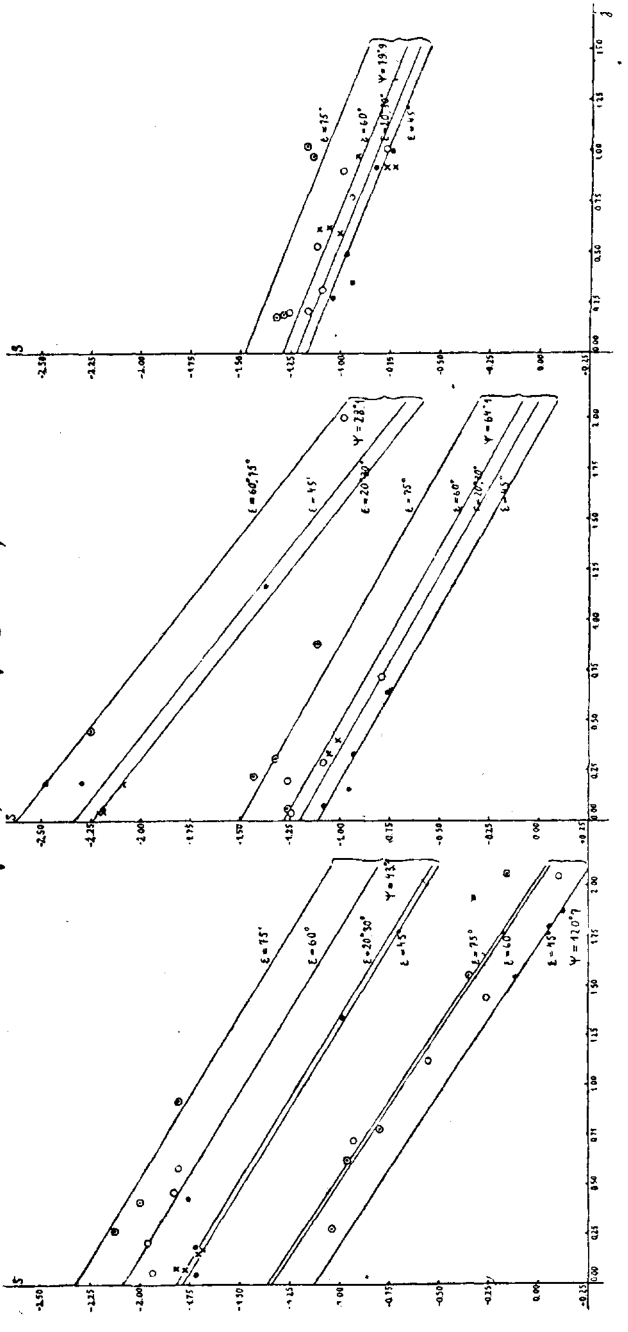
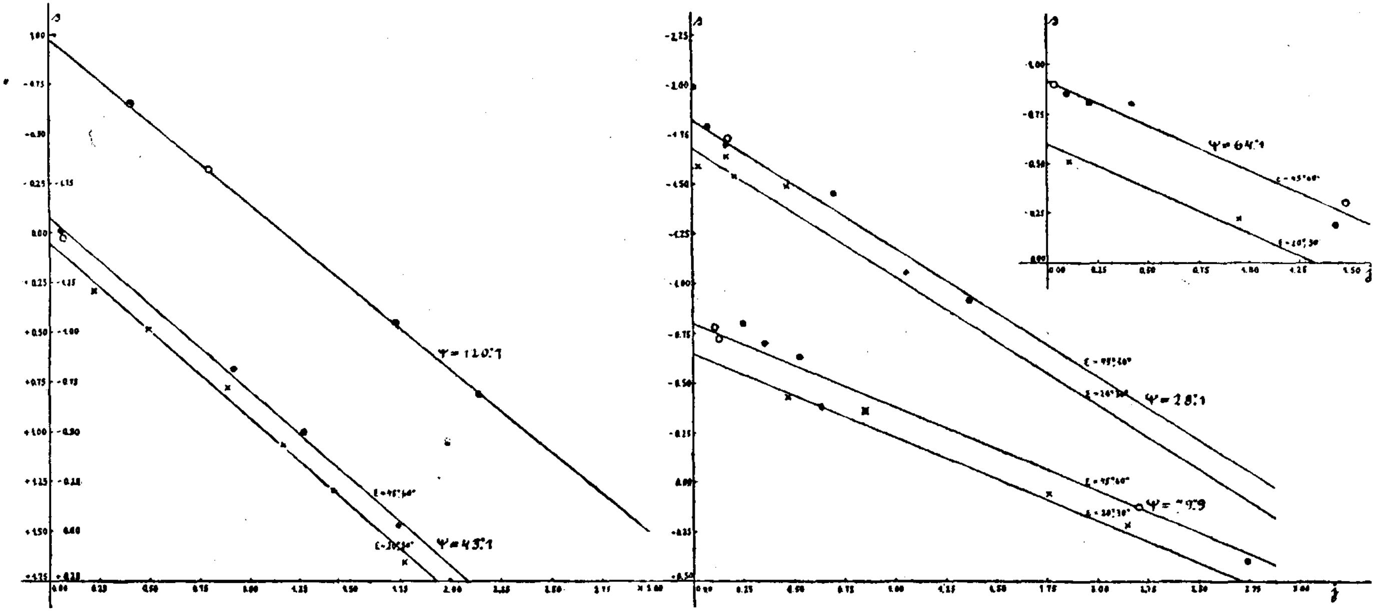


Fig. 2. Representation of the Brightness of the Normal Points with the aid of the formula s = s₀ + kj. Maria.

× ε = 20°30′ • ε = 45°; ○ ε = 60°; ● ε = 75°.



a manner similar to that described here, using the investigation by F. W. P. Goetz on the relative brightness of different points of the full Moon[1]); this point is in good agreement with the general character of the curve; it must be regretted that the data of Dr. Goetz did not allow to determine k for the *continentes*, the points measured on the bright regions being not numerous and chosen almost exclusively on spots which we called here „exceptional“: exceptionally bright areas or points within the lunar craters; but from the general character of his data it appears that for the continentes also the value of k near the full Moon falls close enough to the curve on fig. 3.

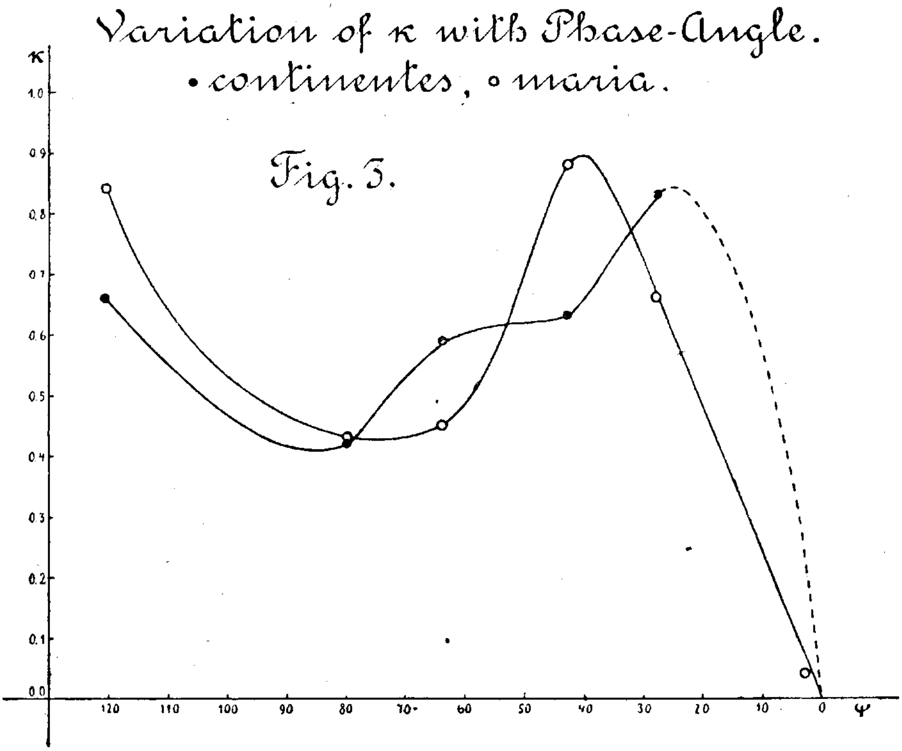

Variation of κ with Phase-Angle.
• continentes, ∘ maria.

Fig. 3.

The fact that $k < 1$ indicates that with decreasing *Cos i* (or intensity of illumination) the decrease of the apparent brightness goes on more slowly than should be expected from Lam-

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[1]) Veröffentlichungen d. Sternwarte Oesterberg zu Tübingen, I Band Heft 2. Photogr. Photometrie d. Mondoberfläche. Teil I. (1919).

bert's law, or, in other words, that *the reflecting power increases with the increasing angle of incidence.*

The general character of variation of k with the phase-angle is similar for the *continentes* and the *maria,* although a certain difference exists between the curves of fig. 3; the *maria* show a more regular curve than the *continentes,* which is due probably to the more regular character of the surface of the former; it must be, however, pointed out that the curves drawn have a considerable hypothetical element about them; the variation of k with the phase-angle is very rapid and of a complicated character, so that the small number of points is not sufficient to draw the curves with certainty; the values of k may be regarded as trustworthy only in the immediate neighbourhood of the phase-angles at which the observations actually took place; for the intermediate points the values of k are rather uncertain.

The influence of the angle of reflection, ε, reveals itself in the different values of s_0; the variation of the latter quantity with ε, as shown by table 12, indicates, as a general rule, *an increase of the surface brightness with increasing angle of reflection*; the increase is equally well shown by the *continentes* and by the *maria;* for the latter the range of variation is somewhat less than for the *continentes;* however, the data of table 12 give little information as to the variation of s_0 with ε, since points with different values of ε were joined together, so that the influence of ε upon the brightness was considerably masked; this question will be discussed later on.

In the character of variation of s_0 with the phase-angle, ψ, the *maria* and *continentes* present also much analogy; from $\psi = 120^0$ to $\psi = 64^0$ s_0 remains practically constant; at $\psi = 43^0$ and 28^0 a sudden increase of s_0 takes place, an increase which probably continues up to the full Moon; however, the *absolute* values of s_0 for different phase-angles are entitled to a lower weight than the *relative* values of s_0 for different ε, the latter depending on differential, the former — on absolute photometric measures.

The near likeness of the optical properties of surfaces so different as the *maria* and the *continentes* are, is worth to be noted; the chief difference seems to be only in the reflecting power, the variation with phase-angle, angle of incidence and angle of reflection being similar although not identical. Many authors have pointed out that the photometric peculiarities of the Moon are due to the roughness and irregularities of her surface; since irregularities attaining the height of several kilometers and those only of a few centimeters can produce the same optical effect, the effect depending only on the shape, not on the size of the irregularities, it is not surprising that the *maria* which are apparently devoid of conspicuous mountains show nevertheless the same optical characteristics as the *continentes.* It appears probable that the solid surface of a planet, subject to various deforming factors, obtains a certain character determined by chance, with a certain distribution of the inclinations of the elementary surfaces, etc.[1]; from this standpoint the likeness of the photometric characteristics of planets devoid of atmosphere becomes probable, a likeness partly confirmed by the analogy of the phase-curves of Mercury and the Moon.

4. Synthetic Calculation of Total Brightness.

From the data of table 12 the brightness of an "ideal" Moon covered with the *continentes* or the *maria* only can be derived. Instead of integrating the surface brightness over the whole illuminated part of the disk, we will choose a less laborious way, although not so accurate, — the method of effective quantities: the average surface brightness of the Moon we shall put equal to the surface brightness of an imaginary point where the intensity of illumination (*Cosi*) and the angle of reflection (ε) represent *effective* values for the whole illuminated crescent. The systematical error produced by such a simplified way of computation may be estimated in the present case as attaining hardly a few per cent, probably less, and will be entirely lost in the uncertainty ot the parameters k and s_0.

The effective surface brightness of the Moon was computed according to the formula

$$s = s_0 + kj,$$

using for j the value $-2.5 \log (\overline{Cos i})$, $\overline{Cos i}$ denoting the effective intensity of illumination according to Lambert's law; as to s_0,

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[1] The most probable structure of the Moon's surface seems to be the structure suggested by H. N. Russel (*Astrophysical Journal, 43, p. 192*): he supposes that „a great part of the Moon's surface is covered by broken fragments of rock, in whose interstices innumerable shadows are formed..."

the weighted mean of the values contained in table 12 was simply taken, the weight being assumed equal to the number of normal points from which the corresponding value was deduced; since the range of variation of s_0 is small, and since the points were distributed over the entire surface of the Moon, the average value so obtained will differ from the true average not more than, say, by 0.01 st. mg. In the following table the process of computation of the integrated brightness is given.

Table 13.

Computation of the Brightness of the "Ideal Moon".

$\psi =$	28°.1	43°.1	64°.1	79°.9	120°.7
Effective $\overline{\cos i}$	0.634	0.595	0.527	0.467	0.288
Effective $\overline{j}$ (st. mg.)	+0.49	+0.56	+0.70	+0.83	+1.35
$k\overline{j}$, *Continentes*	+0.41	+0.35	+0.41	+0.35	+0.89
$k\overline{j}$, *Maria*	+0.32	+0.49	+0.32	+0.36	+1.13
Average $\overline{s_0}$, *Continentes* . . .	−2.41	−1.97	−1.28	−1.27	−1.31
" " , *Maria*	−1.75	−1.51	−0.84	−0.75	−0.99
$\overline{s_0} + k\overline{j} = \overline{s}$ (average surface brightness) *Continentes* . .	−2.00	−1.62	−0.87	−0.92	−0.42
Maria	−1.43	−1.02	−0.52	−0.39	+0.14
Difference *Cont.-Maria*	−0.57	−0.60	−0.35	−0.53	−0.56
Reduction to Integrated Brightness $= -8.57 + 5 \log \cos \frac{\psi}{2}$ . . .	−8.51	−8.41	−8.21	−7.99	−7.04
Integrated Brightn. of the "Ideal Moon", *Continentes*	−10.51	−10.03	−9.08	−8.91	−7.46
Integrated Brightn. of the "Ideal Moon", *Maria*	−9.94	−9.43	−8.73	−8.38	−6.90
Adopted % of *Maria*	50	45	35	30	30
Integr. Brightness of Average Moon.	−10.26	−9.80	−8.97	−8.78	−7.33
Visual Magn. (H. N. Russel)	−11.90	−11.58	−11.16	−10.53	−8.73
Colour Index	+1.64	+1.78	+2.19	+1.77	+1.40
Weight	2	1	1	4	4

Weighted Mean **Colour Index = + 1.66 ± 0.06**

Deviations of the Colour-Index, derived from the single photographs, from its mean value:

Neg.	36	37	35	14	9 I	9 II	10	32	26	27	28 I	28 II
Dev., St. Mg.	—0.06	—0.08	+0.10	+0.60	—0.01	+0.13	+0.30	+0.04	—0.26	—0.10	—0.45	—0.23

Probable error of one photograph = ± 0.18 stellar magnitudes.

In this table the reduction to the integrated brightness was computed from the formula

$$2.5\ \mathit{log}\left[\frac{(f\,tg\,\varrho)^2}{\left(\frac{1}{2}\,d\,\frac{\Delta f}{f}\right)^2}\,\mathit{Cos}\,\frac{2\,\psi}{2}\right] + 0.14, \text{ where}$$

$f = 2745$ mm is the focal length of the objective, $\varrho = 15'32''.6$ — the mean angular semidiameter of the Moon, $d = 39.5$ mm — the diameter of the diaphragm to which the „standard surface brightness“ was reduced, $\Delta f = 31.1$ — the standard distance from focus of the comparison star, and 0.14 — the adopted photographic magnitude of α Lyrae. In other respects the table is self-explanatory.

The visual brightness of the Moon for different phase-angles is taken from a discussion of all available data by H. N. Russel [1]). The average colour-index derived from our measures, + 1.66, is rather too large if compared with the value by Russel-King [2]), + 1.18. There can be the following sources producing the discrepancy: 1) a colour-equation of our measures differing from the colour-equation of King's photographic scale; 2) an error in the zero-point of our scale due to the non-uniform distribution of brightness within the extrafocal image of the comparison star; 3) the fact that the points on the *continentes* were chosen on comparatively even areas and that all points of excessive brightness were excluded should make the average brightness obtained too low by one or two tenths of a stellar magnitude. The two latter factors tend to make the synthetic brightness too low and so their effect was probably summarized in increasing the apparent value of the colour-index. As to the colour-equation of the objective, it cannot be considerable; from an extrafocal comparison the difference in photographic magnitude of $\beta - \alpha$ Geminorum determined with the same objective was found equal to $+0.67 \pm 0.01$ st. magnitudes [3]), which gives with the Harvard visual magnitudes the colour-index of β Gemi-

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1) *Astrophysical Journal, 43,* pp 116 (table VI) and 125.

2) loc. cit. p. 125.

3) Astronomische Nachrichten, 5162 p. 20.

norum equal to $+1.07$, a value very near to the colour-index given by King, $+1.15$.

In any case, the absolute intensities of the moonlight derived from our measures must be entitled to low weight and, as was emphasized at the beginning of this discussion, only the results concerning the distribution of the brightness over the disk of the Moon can be considered as satisfactory.

5. Dependence of the Brightness upon the Angle of Reflection.

The method of joining the observations into normal points gave us the relation between the surface brightness and the angle of incidence, whereas the influence of the angle of reflection could not be determined with precision; to obtain information on the variation of the surface brightness with the angle of reflection, ε, the actually measured points must be treated separately. For every point the value $s_0 = s - kj$, according to formula (3), was determined; s denotes here the measured brightness, and kj is the correction for the intensity of illumination; the reduced brightness, s_0, depends only on the angle of reflection, on the individual reflecting power of the point and on the phase-angle (ψ); for photographs made on the same day ψ remains practically constant; treating the peculiarities of the single points as accidental deviations, the effect of the angle of reflection can be obtained by plotting s_0 with ε as abscissae.

Table 14, under the heading "s_0 observed", contains the brightness, reduced to standard illumination according to the method described. The angles of reflection and illumination for the corresponding points can be found in table 7. On fig. 4 and 5 the observed s_0 are plotted with the angles of reflection as abscissae, separately for the *continentes* and the *maria*.

The points on the figures show a very pronounced dependence of the surface brightness upon the angle of reflection; a good representation of the points can be obtained by a simple linear relation of the form

$$s_0 = a - k'\varepsilon \quad . \; . \; . \quad (5),$$

where a depends only on the phase-angle and the reflecting power (albedo); as to k', this coefficient seems to remain practically constant within the range of ψ observed (28^0—121^0), for both,

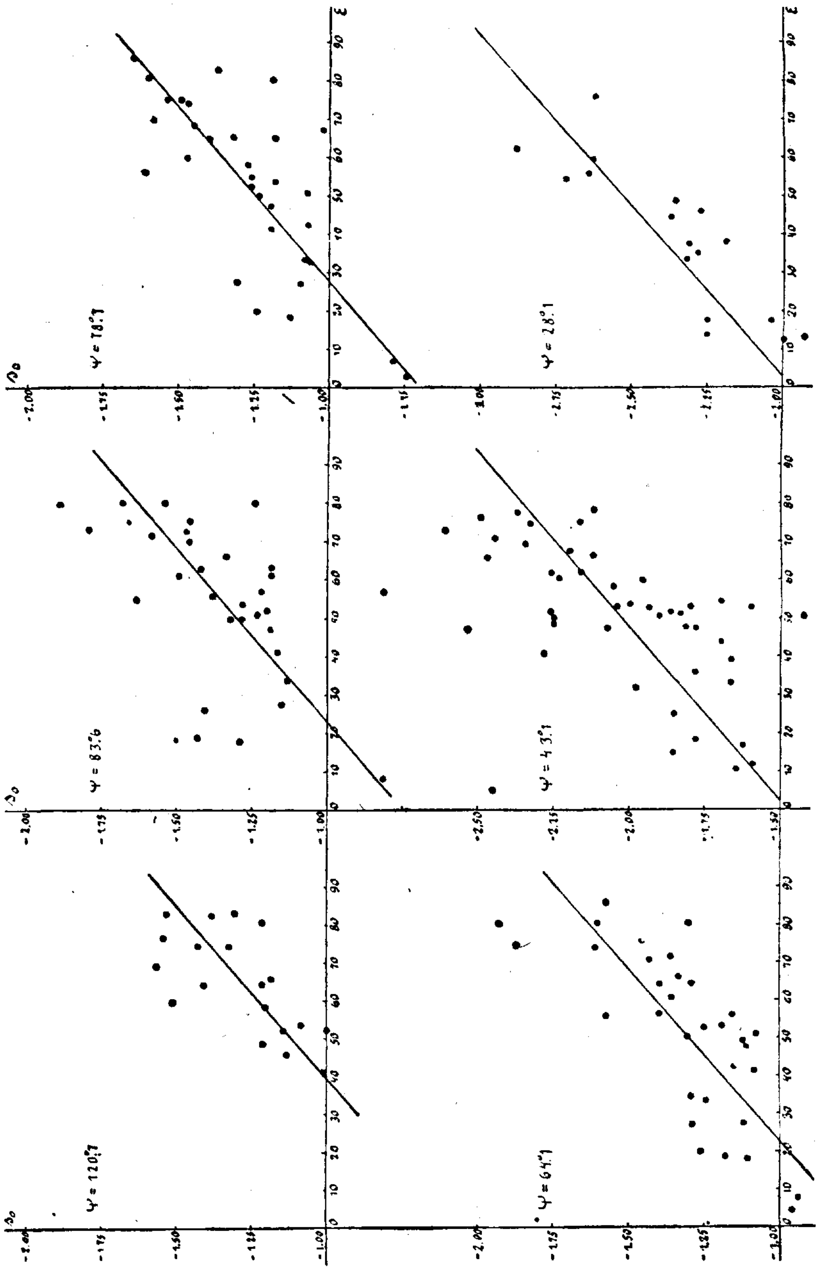

Fig. 4. Influence of the Angle of Reflection Continentes.

$\nu_0 = a - 0.011\,\varepsilon.$

Fig. 5. Influence of the Angle of Reflection. Maria.

$\nu_0 = a - 0.006\,\varepsilon$.

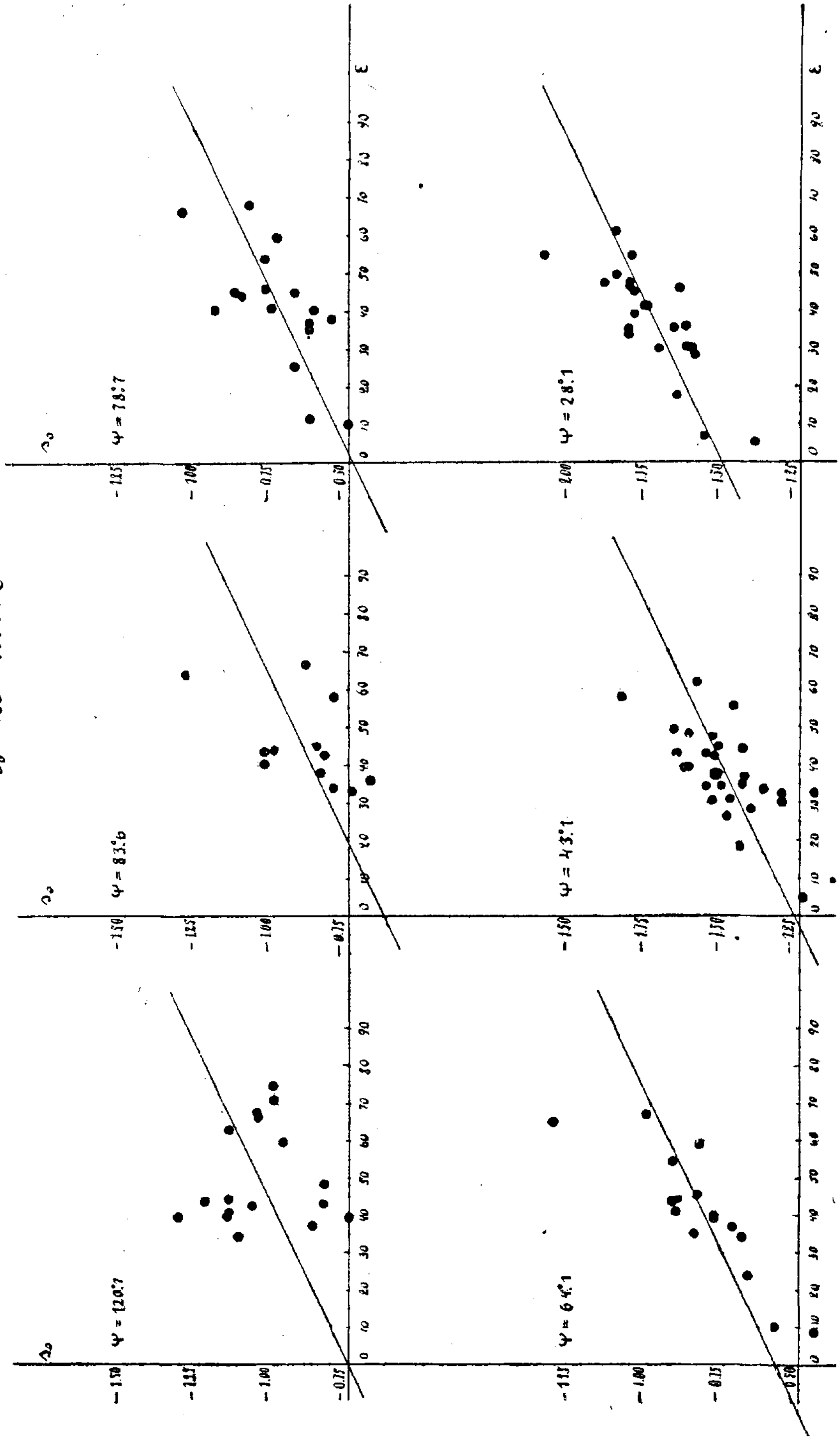

Table 14.

Representation of the observed Brightness of Individual Points with the aid of the formulae

$s = s_0 + kj$, $s_0 = a - k'\varepsilon$ (ε expressed in degrees).

April 30.

$\psi = 120^0.7$

Continentes.

$k=0.66$; $k'=0.011$; $a=-0.57$

Point	kj	Stellar Magnitudes. s_0 Obs.	Stellar Magnitudes. s_0 Comp.	Stellar Magnitudes. O.—C.
5	1.10	—1.37	—1.47	+0.10
6	1.36	—1.54	—1.41	—0.13
8	0.40	—1.30	—1.48	+0.18
9	0.53	—1.32	—1.38	+0.06
10	0.72	—1.18	—1.30	+0.12
11	0.97	—1.19	—1.21	+0.02
12	1.31	—1.00	—1.14	+0.14
13	0.18	—1.22	—1.46	+0.24
15	0.43	—1.21	—1.27	+0.06
16	0.81	—1.08	—1.16	+0.08
25	0.40	—1.43	—1.40	—0.03
26	0.48	—1.56	—1.33	—0.23
27	0.67	—1.51	—1.23	—0.28
28	0.89	—1.14	—1.14	0.00
29	1.15	—1.13	—1.08	—0.05
34	1.19	—1.01	—1.02	+0.01
35	1.28	—1.21	—1.11	—0.10
40 a	1.03	—1.53	—1.48	—0.05
41	1.40	—1.41	—1.27	—0.14

Maria.

$k=0.84$; $k'=0.006$; $a=-0.75$

Point	kj	Stellar Magnitudes. s_0 Obs.	Stellar Magnitudes. s_0 Comp.	Stellar Magnitudes. O.—C.
14	0.55	—1.06	—1.16	+0.10
17	0.69	—0.97	—1.10	+0.13
20	2.23	—0.87	—0.97	[+0.10]
21	2.75	—1.12	—0.96	[—0.16]
31	1.77	—1.16	—0.99	—0.17
32	1.41	—1.23	—1.01	—0.22
33	1.44	—1.15	—1.01	—0.14
1s	0.35	—1.01	—1.20	+0.19
2s	0.42	—1.00	—1.18	+0.18
3s	0.52	—1.06	—1.15	+0.09
4s	0.62	—1.15	—1.13	—0.02
5s	2.28	—1.07	—1.00	[—0.07]
6s	2.41	—1.15	—1.00	[—0.15]
7s	2.68	—1.32	—0.99	[—0.33]
8s	1.19	—0.83	—1.04	+0.21
9s	1.50	—0.83	—1.01	+0.18
10s	1.81	—0.75	—0.99	+0.24

May 3.

$\psi = 83^0.6$

Continentes.

$k=0.42$; $k'=0.011$; $a=-0.75$

Point	kj	Stellar Magnitudes. s_0 Obs.	Stellar Magnitudes. s_0 Comp.	Stellar Magnitudes. O.—C.
5	0.43	—1.88	—1.63	—0.25
6	0.42	—1.79	—1.55	—0.24
8	0.11	—1.54	—1.63	+0.09
9	0.12	—1.58	—1.54	—0.04
10	0.14	- 1.42	—1.44	+0.02
11	0.16	—1.38	—1.37	—0.01
12	0.20	—1.28	—1.30	+0.02
13	0.00	—1.24	—1.63	+0.39
15	0.03	—1.18	—1.44	+0.26
16	0.10	—1.20	—1.32	+0.12
16 a	0.08	—1.63	—1.35	- 0.28
19	0.39	—1.43	—0.96	—0.47
25	0.10	—1.46	—1.57	+0.11
26	0.10	—1.46	—1.52	+0.06
27	0.12	—1.49	—1.42	—0.07
28	0.15	—1.28	—1.34	+0.06
29	0.17	—1.19	—1.27	+0.08
34	0.14	—1.16	—1.21	+0.05
35	0.23	—1.23	—1.31	+0.08
36	0.41	—1.29	—0.95	—0.34
37	0.37	—1.13	—1.12	—0.01
38	0.28	—1.41	—1.05	—0.36
39	0.60	—1.15	—1.06	—0.09
40	0.41	—1.47	—1.55	+0.08
40 a	0.46	—1.68	—1.63	—0.05
41	0.41	—1.33	—1.48	+0.15
42	0.44	—1.22	—1.38	+0.16
43	0.45	—1.32	—1.30	—0.02
49	0.77	—0.81	—0.84	+0.03
50	0.97	—0.45	—0.81	+0.36

Maria.

$k=0.43$; $k'=0.006$; $a=-0.63$

Point	kj	Stellar Magnitudes. s_0 Obs.	Stellar Magnitudes. s_0 Comp.	Stellar Magnitudes. O.—C.
7	0.46	—1.29	—1.01	—0.28
14	0.04	—0.89	—1.03	+0.14
17	0.05	—0.80	—0.98	+0.18
20	0.19	—0.68	—0.85	+0.17
21	0.21	—0.74	—0.83	+0.09
22	0.29	—0.84	—0.86	+0.02
23	0.33	—0.83	—0.89	+0.06
23 a	0.31	—1.03	—0.89	—0.14
24	0.42	—0.80	—0.83	+0.03
31	0.18	—1.03	—0.87	—0.16
32	0.15	—1.00	—0.89	—0.11
33	0.15	—0.86	—0.90	+0.04

April 4.

$\psi = 78^0.7$

Continentes.

$k=0.42$; $k'=0.011$; $a=-0.70$

Point	kj	Stellar Magnitudes. s_0 Obs.	Stellar Magnitudes. s_0 Comp.	Stellar Magnitudes. O.—C.
5	0.42	—1.60	—1.59	—0.01
6	0.40	—1.54	—1.52	—0.02
8	0.11	—1.37	—1.61	+0.24
9	0.11	—1.47	—1.51	+0.04
10	0.12	—1.32	—1.42	+0.10
11	0.14	—1.27	—1.34	+0.07
12	0.16	—1.07	—1.26	+0.19
13	0.00	—1.19	—1.58	+0.39
15	0.02	—1.18	—1.41	+0.23
16	0.07	—1.18	—1.28	+0.10
16 a	0.06	—1.61	—1.32	—0.29
19	0.30	—1.24	—0.92	—0.32
25	0.10	—1.49	—1.52	+0.03
26	0.09	—1.58	—1.47	—0.11
27	0.10	—1.47	—1.36	—0.11
28	0.11	—1.26	—1.28	+0.02
29	0.13	—1.20	—1.22	+0.02
34	0.10	—1.07	—1.17	+0.10
35	0.18	—1.23	—1.25	+0.02
36	0.32	—1.13	—0.90	—0.23
37	0.30	—1.06	—1.06	0.00
38	0.21	—1.30	—1.00	—0.30
39	0.46	—1.09	—0.99	—0.10
40	0.38	—1.45	—1.46	+0.01
40 a	0.45	—1.65	—1.65	0.00
41	0.37	—1.40	—1.41	+0.01
42	0.38	—1.26	—1.31	+0.05
43	0.38	—1.20	—1.23	+0.03
44	0.40	—1.20	—1.15	—0.05
49	0.57	—0.79	—0.78	—0.01
50	0.68	—0.74	—0.73	—0.01
52	0.74	—1.08	—1.06	—0.02

Maria.

$k=0.43$; $k'=0.006$; $a=-0.49$

Point	kj	Stellar Magnitudes. s_0 Obs.	Stellar Magnitudes. s_0 Comp.	Stellar Magnitudes. O.—C.
7	0.42	—1.06	—0.89	—0.17
14	0.03	—0.83	—0.90	+0.07
17	0.03	—0.74	—0.85	+0.11
20	0.14	—0.56	—0.72	+0.16
21	0.15	—0.63	—0.70	+0.07
22	0.23	—0.62	—0.73	+0.11
23	0.27	—0.68	—0.76	+0.08
23 a	0.25	—0.78	—0.77	—0.01
24	0.33	—0.63	—0.71	+0.08
31	0.13	—0.95	—0.73	—0.22
32	0.10	—0.86	—0.75	—0.11
33	0.12	—0.88	—0.76	—0.12
55	0.76	—0.68	—0.56	—0.12
55 a	0.76	—0.50	—0.55	+0.05
58	0.76	—0.68	—0.64	—0.04
59	1.18	—0.76	—0.74	[—0.02
64	1.01	—0.78	—0.81	+0.03

Table 14. Continued.

Point	April 5. $\psi = 64^0.1$ Stellar Magnitudes. kj	s_0 Obs.	s_0 Comp.	$O.—C.$
Continentes.				
k=0.59; k'=0.011; a=−0.75				
5	0.57	−1.93	−1.63	−0.30
6	0.50	−1.87	−1.57	−0.30
8	0.18	−1.60	−1.64	+0.04
9	0.14	−1.61	−1.55	−0.06
10	0.12	−1.40	−1.45	+0.05
11	0.12	−1.40	−1.38	−0.02
12	0.14	−1.30	−1.31	+0.01
13	0.04	−1.30	−1.63	+0.33
15	0.01	−1.29	−1.45	+0.16
16	0.04	−1.25	−1.33	+0.08
16 a	0.04	−1.57	−1.37	−0.20
19	0.22	−1.26	−0.97	−0.29
25	0.15	−1.45	−1.58	+0.13
26	0.12	−1.43	−1.53	+0.10
27	0.09	−1.36	−1.42	+0.06
28	0.10	−1.19	−1.33	+0.14
29	0.10	−1.11	−1.28	+0.17
34	0.05	−1.08	−1.21	+0.13
35	0.17	−1.07	−1.31	+0.24
36	0.24	−1.18	−0.96	−0.22
37	0.25	−1.24	−1.11	−0.13
38	0.15	−1.29	−1.05	−0.24
39	0.38	−1.12	−1.05	−0.07
40	0.50	−1.36	−1.54	+0.18
40 a	0.61	−1.57	−1.70	+0.13
41	0.45	−1.33	−1.48	+0.15
42	0.41	−1.16	−1.37	+0.21
43	0.38	−1.12	−1.29	+0.17
44	0.37	−1.15	−1.22	+0.07
49	0.43	−0.94	−0.84	−0.10
50	0.51	−0.96	−0.79	−0.17
52	0.63	−1.29	−1.13	−0.16
57	0.84	−1.10	−0.95	−0.15
Maria.				
k=0.45; k' 0.006; a=−0.58				
7	0.35	−1.32	−0.97	−0.35
14	0.02	−1.01	−1.01	0.00
17	0.00	−0.83	−0.93	+0.10
20	0.06	−0.72	−0.80	+0.08
21	0.07	−0.69	−0.78	+0.09
22	0.14	−0.77	−0.82	+0.05
23	0.18	−0.92	−0.84	−0.08
24	0.20	−0.85	−0.79	−0.06
31	0.06	−0.91	−0.83	−0.08
32	0.05	−0.90	−0.84	−0.06
33	0.06	−0.84	−0.86	+0.02
55	0.42	−0.63	−0.64	+0.01
55 a	0.42	−0.45	−0.63	+0.18
58	0.44	−0.67	−0.72	+0.05
59	0.63	−0.78	−0.82	+0.04
64	0.66	−0.92	−0.90	−0.02

Point	May 6. $\psi = 43^0.1$ Stellar Magnitudes. kj	s_0 Obs.	s_0 Comp.	$O.—C.$
Continentes.				
k=0.63; k'=0.011; a=−1.48				
1	0.98	−2.61	−2.28	−0.33
2	0.82	−2.47	−2.21	−0.26
3	0.72	−2.16	−2.16	0.00
5	0.62	−2.33	−2.30	−0.03
6	0.49	−2.20	−2.22	+0.02
8	0.26	−2.17	−2.31	+0.14
9	0.18	−2.12	−2.21	+0.09
10	0.13	−2.05	−2.12	+0.07
11	0.10	−1.90	−2.04	+0.14
12	0.09	−1.70	−1.96	+0.26
13	0.14	−2.12	−2.34	+0.22
15	0.04	−1.96	−2.14	+0.18
16	0.02	−1.78	−2.01	+0.23
16 a	0.03	−2.27	−2.04	−0.23
19	0.09	−1.85	−1.64	−0.21
25	0.24	−2.37	−2.34	−0.03
26	0.18	−2.45	−2.26	−0.19
27	0.12	−2.26	−2.16	−0.10
28	0.09	−2.00	−2.07	+0.07
29	0.07	−1.81	−2.01	+0.20
34	0.01	−1.66	−1.91	+0.25
35	0.14	−1.94	−2.06	+0.12
36	0.10	−1.78	−1.68	−0.10
37	0.14	−1.78	−1.88	+0.10
38	0.05	−1.85	−1.75	−0.10
39	0.23	−1.98	−1.83	−0.15
40	0.55	−2.49	−2.32	−0.17
41	0.46	−2.34	−2.24	−0.10
42	0.37	−2.23	−2.14	−0.09
43	0.30	−2.04	−2.06	+0.02
44	0.27	−2.07	−2.00	−0.07
49	0.21	−1.59	−1.61	+0.02
50	0.25	−1.65	−1.59	−0.06
52	0.40	−2.28	−1.93	−0.35
57	0.43	−1.62	−1.67	+0.05
62	0.98	−1.67	−1.84	+0.17
26s	0.59	−2.53	−2.00	−0.53
27s	0.79	−2.25	−2.02	−0.23
28s	0.93	−2.25	−2.03	−0.22
29s	1.13	−2.26	−2.05	−0.21
30s	2.12	−2.81	−2.11	[−0.70]
31s	(∞)	—	—	—
32s	0.79	−1.83	−2.04	+0.21
33s	0.81	−1.87	−2.05	+0.18
34s	0.96	−1.80	−2.06	+0.26
35s	1.14	−1.70	−2.08	+0.38
36s	1.16	−1.60	−2.06	+0.46
37s	1.25	−1.43	−2.04	+0.61
38s	1.36	−1.30	−2.05	+0.75

Point	May 6. $\psi = 43^0.1$ Stellar Magnitudes. kj	s_0 Obs.	s_0 Comp.	$O.—C.$
Maria.				
k=0.88; k'=0.006; a=−1.27				
7	0.56	−1.84	−1.62	−0.22
14	0.09	−1.59	−1.64	+0.05
17	0.03	−1.47	−1.60	+0.13
20	0.03	−1.20	−1.46	+0.26
21	0.03	−1.31	−1.45	+0.14
22	0.12	−1.31	−1.47	+0.16
23	0.19	−1.43	−1.49	+0.06
23 a	0.19	−1.53	−1.50	−0.03
24	0.20	−1.41	−1.44	+0.03
31	0.04	−1.62	−1.51	−0.11
32	0.04	−1.56	−1.53	−0.03
33	0.06	−1.52	−1.54	+0.02
53	1.60	−1.66	−1.53	−0.13
54	1.36	−1.37	−1.47	+0.10
55 a	0.39	−1.24	−1.30	+0.06
58	0.42	−1.45	−1.38	−0.07
59	0.69	−1.56	−1.47	−0.09
60	1.22	−1.44	−1.48	+0.04
61	1.39	−1.62	−1.56	−0.06
64	0.82	−1.67	−1.56	−0.11
11s	1.03	−1.49	−1.43	−0.06
12s	1.28	−1.48	−1.46	−0.02
13s	1.53	−1.51	−1.48	−0.03
14s	1.83	−1.53	−1.50	−0.03
15s	0.90	−1.54	−1.49	−0.05
16s	1.04	−1.52	−1.50	−0.02
17s	1.14	−1.63	−1.51	−0.12
18s	1.35	−1.53	−1.52	−0.01
19s	1.52	−1.44	−1.53	+0.09
20s	1.86	−1.54	−1.56	+0.02

Table 14. Continued.

Point	May 7. $\psi = 28°.1$ Stellar Magnitudes.				Point	May 7. $\psi = 28°.1$ Stellar Magnitudes.			
	k_j	s_0 Obs.	s_0 Comp.	O.—C.		k_j	s_0 Obs.	s_0 Comp.	O.—C.
Continentes.					*Maria.*				
$k = 0.83$; $k' = 0.011$; $a = -1.97$					$k = 0.66$; $k' = 0.006$; $a = -1.51$				
13	0.37	—2.62	—2.80	+0.18	14	0.16	—1.86	—1.88	+0.02
15	0.16	—2.63	—2.62	—0.01	17	0.09	—1.81	—1.84	+0.03
16	0.07	—2.27	—2.48	+0.21	20	0.01	—1.63	—1.70	+0.07
19	0.03	—2.25	—2.12	—0.13	21	0.00	—1.60	—1.68	+0.08
34	0.02	—2.19	—2.39	+0.20	22	0.06	—1.61	—1.69	+0.08
36	0.05	—2.25	—2.17	—0.08	23	0.11	—1.82	—1.72	—0.10
37	0.13	—2.31	—2.39	+0.08	24	0.09	—1.72	—1.69	—0.03
39	0.18	—2.32	—2.33	+0.01	31	0.03	—1.80	—1.74	—0.06
49	0.13	—1.93	—2.12	+0.19	33	0.06	—1.80	—1.78	—0.02
50	0.17	—2.00	—2.11	+0.11	53	0.71	—1.65	—1.79	+0.14
57	0.32	—2.04	—2.16	+0.12	54	0.57	—1.63	—1.73	+0.10
62	0·72	—2.28	—2.35	+0.07	55	0.15	—1.57	—1.55	—0.02
39s	0.95	—2.72	—2.56	—0.16	55 a	0.15	—1.40	—1.54	+0.14
40s	1.38	—2.88	—2.65	—0.23	58	0.18	—1.66	—1.62	—0.04
41s	2.90	—4.00	—2.76	[—1.24]	59	0.34	—1.82	—1.71	—0.11
42s	2.74	—3.73	—2.76	[—0.97]	60	0.55	—1.67	—1.73	+0.06
43s	0.92	—2.37	—2.45	+0.08	61	0.71	—1.86	—1.81	—0.05
44s	1.17	—2.35	—2.51	+0.16	64	0.46	—1.90	—1.79	—0.11
45s	1.66	—2.64	—2.58	—0.06	21s	0.74	—1.72	—1.76	+0.04
					22s	0.96	—1.82	—1.79	—0.03
					23s	0.66	—1.76	—1.76	0.00
					24s	0.86	—1.82	—1.80	—0.02
					25s	1.28	—2.10	—1.84	—0.26

the *maria* and the *continentes*, as shown by the following data obtained for different phase-angles:

ψ	k'	
	Contin.	*Maria*
120°.7	0.010	—
83 .6	0.014	—
78 .7	0.011	0.005
64 .1	0.008	0.008
43 .1	0.012	0.005
28 .1	0.013	0.007
Average	0.011	0.006

The coefficient k' here given corresponds with the angle ε expressed in degrees. For $\psi = 120°.7$ and 83°.6 no reliable value of k' could be obtained for the *maria* because of the small number of points and the small range in ε. The individual values of k'

differ within the limits of the probable error; therefore the average values, $k' = 0.011$ for the continentes and $k' = 0.006$ for the maria, were adopted for all phase-angles, from $\psi = 28^0$ to $\psi = 121^0$. With the aid of the assumed values of k' the constant a was

Table 15.

Relative Reflecting Power of Individual Points. Stellar Magnitudes. *Continentes.* Zero Point = average Reflecting Power of the *Continentes.*

Point	1	2	3	5	6	8	9	10	11	12	13	15	16	16a
Refl. Power	−0.33	−0.26	0.00	−0.10	−0.13	+0.14	+0.02	+0.07	+0.04	+0.12	+0.29	+0.15	+0.14	−0.25
n	1	1	1	5	5	5	5	5	5	5	6	6	6	4
Residuals 1)	—	—	—	+0.20 −0.15 +0 09 −0.20 +0.07	0.00 −0.11 +0.11 −0.17 +0.16	+0.04 −0.05 +0.10 −0.10 0.00	+0.04 −0.06 +0.02 −0.08 +0.07	+0.05 −0.05 +0.03 −0.02 0.00	−0.02 −0.05 +0.03 −0.06 +0.10	+0.02 −0.10 +0.07 −0.11 +0.14	−0.05 +0.10 +0.10 +0.04 −0.07 −0.11	−0.09 +0.11 +0.08 +0.01 +0.03 −0.16	−0.06 −0.02 −0.04 −0.06 +0.09 +0·07	−0.03 −0.04 +0.05 +0.02

Continentes. Continued.

Point	19	25	26	27	28	29	34	35	36	37	38	39	40	40a
Refl. Power	−0.28	+0.04	−0.07	0.10	+0.06	+0.08	+0.12	+0.07	−0.19	+0.01	−0.25	−0.08	+0.02	+0.01
n	5	5	5	5	5	5	6	5	5	5	4	5	4	4
Residuals 1)	−0.19 −0.04 −0.01 +0.07 +0.15	−0.07 +0.07 −0.01 +0.09 −0.07	−0.16 +0.13 −0.04 +0.17 −0.12	−0.18 +0.03 −0.01 +0.16 0.00	−0.06 0.00 −0.04 +0.08 +0.01	−0.13 0.00 −0.06 +0.09 +0.12	−0.11 −0.07 −0.02 +0.01 +0.13 +0.08	−0.17 +0.01 −0.05 +0.17 +0.05	−0.15 −0.04 −0.03 +0.09 +0.11	−0.02 −0.01 −0.14 +0.09 +0.07	−0.11 −0.05 +0.01 +0.15	−0.01 −0.02 +0.01 −0.07 +0.09	+0.06 −0.01 +0.16 −0.19	−0.06 −0.06 −0.01 +0.12

Continentes. Continued.

Point	41	42	43	44	49	50	52	57	62	26_s	27_s	28_s	29_s
Refl. Power	+0.01	+0.08	+0.05	−0.02	+0.03	+0.05	−0.18	+0.01	+0.12	−0.53	−0.23	−0.22	−0.21
n	5	4	4	3	5	5	3	3	2	1	1	1	1
Residuals 1)	−0.15 +0.14 0.00 +0.14 −0.12	+0.08 −0.03 +0.13 −0.17	−0.07 −0.02 +0.12 −0.03	−0.03 +0.09 −0.05	0.00 −0.04 −0.13 −0.01 +0.16	+0.31 −0.06 −0.22 −0.11 +0.06	+0.16 +0.02 −0.17	−0.16 +0.04 +0.11	+0.05 −0.05	—	—	—	—

Continentes. Continued.

Point	32_s	33_s	34_s	35_s	36_s	37_s	38_s	39_s	40_s	43_s	44_s	45_s
Refl. Power	+0.21 2)	+0.18 2)	+0.26 2)	+0.38 2)	+0.46 2)	+0.61 2)	+0.75 2)	−0.16	−0.23	+0.08	+0.16	−0.06
n	1	1	1	1	1	1	1	1	1	1	1	1
Residuals 1)	—	—	—	—	—	—	—	—	—	—	—	—

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1) Residuals given in the order of decreasing phase-angle.

2) With the sun very low; abnormally small surface brightness.

Table 15. Continued.

Maria. Zero Point = average Reflecting Power of the *Maria.*

Point	7	14	17	20	21	22	23	23a	24	31	32
Refl. Power	−0.26	+0.06	+0.11	+0.15	+0.09	+0.08	0.00	−0.06	+0.01	−0.13	−0.11
n	4	6	6	5	5	5	5	3	5	6	5
Residuals [1]	−0.02	+0.04	+0.02	+0.02	0.00	−0.06	+0.06	−0.08	+0.02	−0.04	−0.11
	+0.09	+0.08	+0.07	+0.01	−0.02	+0.03	+0.08	+0.05	+0.07	−0.03	0.00
	−0.09	+0.01	0.00	−0.07	0.00	−0.03	−0.08	+0.03	−0.07	−0.09	0.00
	+0.04	−0.06	−0.01	+0.11	+0.05	+0.08	+0.06		+0.02	+0.05	+0.05
		−0.01	+0.02	−0.08	−0.01	0.00	−0.10		−0.04	+0.02	+0.08
		−0.04	−0.08							+0.07	

Maria. Continued.

Point	33	53	54	55	55a	58	59	60	61	64
Refl. Power	−0.03	0.00	+0.10	−0.04	+0.11	−0.02	−0.05	+0.05	−0.06	−0.05
n	6	2	2	3	4	4	3	2	2	4
Residuals [1]	−0.11	−0.13	0.00	−0.08	−0.06	−0.02	+0.09	−0.01	−0.01	+0.08
	+0.07	+0.14	0.00	+0.05	+0.07	+0.07	−0.04	+0.01	0.00	+0.03
	−0.09			+0.02	−0.05	−0.05	−0.06			−0.06
	+0.05				+0.03	−0.02				−0.06
	+0.05									
	+0.01									

Maria. Continued.

Point	1_s	2_s	3_s	4_s	8_s	9_s	10_s	11_s	12_s	13_s	14_s
Refl. Power	+0.19	+0.18	+0.09	−0.02	+0.21	+0.18	+0.24	−0.06	−0.02	−0.03	−0.03
n	1	1	1	1	1	1	1	1	1	1	1
Residuals	—	—	—	—	—	—	—	—	—	—	—

Maria. Continued.

Point	15_s	16_s	17_s	18_s	19_s	20_s	21_s	22_s	23_s	24_s	25_s
Refl. Power	−0.05	−0.02	−0.12	−0.01	+0.09	+0.02	+0.04	−0.03	0.00	−0.02	−0.26
n	1	1	1	1	1	1	1	1	1	1	1
Residuals	—	—	—	—	—	—	—	—	—	—	—

determined for the *continentes* and the *maria;* the average value of a for *all points* was assumed, no distinction of the groups mentioned in section 3 being made; at the head of each subdivision of table 14 the adopted value of a, together with the two other constants, k and k', is given.

For the *continentes*, $\psi = 28^{0}.1$ (May 7), a was determined in a somewhat different way than at the other occasions; the average value for all measured points was $a = -1.91$; however, since a great number of points on this day was rejected on account of over-exposure, the average a determined from the

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[1] Residuals given in the order of decreasing phase-angle.

remaining points should be too low because of selection; therefore a correction was applied, determined from a comparison with the negative taken on the preceding day, May 6 ($\psi = 43.^{\circ}1$). The points in common to both negatives gave $a = -1.42$, whereas all points measured on May 6 gave $a = -1.48$; hence the correction -0.06 was found; by applying this correction to the mean a for May, 7, the finally adopted value of a for the *continentes*, $a = -1.97$ ($\psi = 28^{\circ}.1$), was obtained.

In calculating the average value of a all points with *Cosi* $\leqq 0.100$ were omitted, since in this case irregularities of the surface, e. g. slight inclinations, produce great differences in the measured brightness; the supplementary points were also entitled to low weight, groups of them being joined into normal points, whereas the points of the regular plan entered into the mean individually.

The values of s_0 computed from (5) with the aid of the a and k' adopted are given in table 14 under the heading "s_0 computed"; they are also represented on fig. 4 and 5 by straight lines. The difference "*O.—C.*" in the table may be regarded as representing the deviation of the individual reflecting power of the measured points from the average albedo of the given category of lunar formations *(continentes* or *maria)*; since the observations indicate a practically identical law of illumination for all points of the *continentes*, or all points of the *maria*, the differences in the albedo found here do not depend on the law of illumination adopted.

In table 15 the mean differential reflecting power of every measured point is given; this value was computed as the mean of the differences *C.—C.* of table 14 for all days on which the corresponding point was measured; measures with *Cosi* $\leqq 0.100$ were not included; in forming the mean, the results of every day received equal weight without regard to the number of plates from which the surface brightness was deduced; this was made so, because the chief source of discordances proved to be not the errors in the differential measures, but real irregularities in the variation of the brightness of single points with the phase-angle; these irregularities are due evidently to the peculiar topography of the points.

In table 15 are also given the residuals of the reflecting power for the single days, in the order of decreasing phase-angle;

a progressive change in the residuals would indicate that for the corresponding point our empirical illumination-law does not hold; it is worth of noting that for the great majority of points no such progressive change can be perceived; the agreement is especially good for the *maria*, whereas the more irregular *continentes* show sometimes considerable discrepancies.

From the data of table 15 the following *mean quadratic residuals* were found:

for the *continentes*. . . . . . ± 0.110 st. mg.
„ „ *maria* . . . . ± 0.067 „ „ .

These quantities were computed from the formula

$$\pm\sqrt{\frac{\Sigma\Delta^2}{n-k}},\ \Delta \text{ denoting the residuals,}$$

n — the total number of residuals, k — the number of points for which the residuals were determined.

From points observed on 2 days or more the following values of the *dispersion of the albedoes* were determined:

for the *continentes* . . . . . ± 0.127 st. mg.
„ „ *maria* . . . . ± 0.095 „ „ .

The dispersion of the albedoes was found from the formula

$$\pm\sqrt{\frac{\Sigma\Delta a^2}{k}},$$

Δa denoting the relative reflecting power taken from table 15, and k — the number of points.

6. Final Form of the Empirical Illumination-Law of the Moon's Surface.

The surface brightness of an elementary area reflecting light diffusely may be represented as the function of three arguments: ψ, the phase-angle; i, the angle of incidence; and ε, the angle of reflection. The influence of i and ε was studied in the present investigation on the basis of differential measures, and can be regarded as satisfactorily determined; as to the variation of the brightness with the phase-angle, its determination depends on absolute photometric measures, in which respect our results are not good; we shall therefore correct our absolute magnitude determinations with the aid of the known variation of the total brightness of the Moon with the phase-angle. We shall suppose that this variation is parallel for the photographic and the visual radiation, so that the colour-index must remain constant; the corrections are then determined by the deviations from the mean colour-index; from the data of table 13 the following corrections to be applied to all measured surface brightnesses were found:

Day	April 30	May 3	April 4	April 5	May 6	May 7
Correction, St. Mg.	+0.26	−0.04	−0.14	−0.60	−0.10	+0.07

Instead of being added to the surface brightness, these corrections may be added to the parameter a of formula (5) or of table 16.

The results of the present discussion may be summarized in the following formula, representing the surface brightness of a point on the Moon's surface:

$$s = a + \Delta a + kj - k'\varepsilon \quad . \; . \; . \; (6),$$

where $j = -2{,}5 \log (\cos i)$.

Here s denotes the surface brightness in stellar magnitudes, the zero surface brightness being ascribed to the image of α Lyrae 31.1 mm out of focus, and the Moon being photographed with an aperture =39.5 mm and the focal length being 2745 mm.; i is the angle of incidence, ε — the angle of reflection (in degrees); a, k and k' are parameters, different for the *maria* and the *continentes*, a and k depending only on the phase-angle, and k' remaining sensibly constant between $\psi = 28^0$ and 121^0; Δa is the *relative albedo*, a constant for a given point. Table 16 contains the parameters found for different phase-angles, the values of a being corrected in the manner described above.

Table 16.

	Continentes			*Maria*		
ψ	a	k	k'	a	k	k'
28°.1	−1.90	0.83	0.011	−1.44	0.66	0.006
43 .1	−1.58	0.63	„	−1.37	0.88	„
64 .1	−1.35	0.59	„	−1.18	0.45	„
78 .7	−0.84	0.42	„	−0.63	0.43	„
83 .6	−0.79	0.42	„	−0.67	0.43	„
120 .7	−0.31	0.66	„	−0.49	0.84	„

As to Δa, the corresponding values for the measured points are contained in table 15.

II. Photographic Measurement of the Intensity of the Earth-Shine.

The method used was in some respect similar to the method described in the preceding investigation; with the same visual objective ($d = 160$ mm, $f = 2745$ mm.) photographs of the earth-shine and extrafocal images of *β Geminorum or α Coronae* were obtained; the same exposure-time, of about 150^s, was used generally for the Moon, and the scale of photographic density of the comparison star was obtained by varying the diameter or the exposure of the extrafocal image. The plates were *Ilford Rapid Chromatics*, backed, $4\frac{1}{2} \times 6$ cm, developed in Methol-Hydrochinon.

The field of the photographs was of 40 mm diameter, the Moon about 25 mm; the extrafocal stellar images were obtained either in the same field near the image of the Moon, or without the field, on the unexposed part of the plate, for which purpose the plate-holder was moved so that the field of the Moon photograph fell on one end, the field of the comparison star — on the other end of the plate. Seven plates were obtained in 1914 and 1915, of which 5 were measured by the writer in 1922 with the microphotometer of the Tartu observatory; the other two plates were rejected, one taken near the last quarter with a highly illuminated background of the sky, the other with the comparison star photographed near the bright limb of the Moon where the background was also very bright; in both cases no reliable results could be expected.

Table 17 contains the data relating to the photographs which were measured. In this table ψ denotes the phase-angle of the Moon; *Secz* represents the air-mass, needed to determine the correction for atmospheric absorption; for the days with

Table 17.

Photographs of the Earth-Shine.

Object and №		Moscow Mean Time	Exposure Seconds	Δf mm	*Sec z*	Remarks
Negative 25. April 28, 1914. $\psi = -143^0$.						
		h m				
Moon		8 40	155	0.0	2.78	Day with intense atmospheric absorption, coefficient of absorption 0.60 stellar magnitudes (specially determined). Earth-shine weak, $\frac{1}{5}$ — $\frac{1}{7}$ of the illuminated background. Images of comparison star superposed on the sky-background.
β Gemin.	1	11 35	155	61.1	2.90	
„	2	11 38	155	73.3	2.95	
„	3	11 42	155	85.3	3.02	
„	4	11 49	155	97.3	3.13	
„	5	11 54	160	115.3	3.22	
Negative 29. April 30, 1914. $\psi = -120^0$.						
		h m				
Moon		9 20	160	0.0	1.96	Normal transparency, coeff. of absorption adopted 0.44 st. mg. Earth-shine about 0,2—0,4 of the background. Comparison star on the illuminated background, edges partially superposed upon one another.
α Coronae	1	13 14	„	57.1	1.14	
„	2	13 09	„	66.1	„	
„	3	13 04	„	76.1	„	
„	4	12 55	„	89.1	„	
„	5	12 31	„	103.2	„	
„	6	12 36	„	115.2	„	
„	7	12 41	„	133.2	„	
„	8	12 46	„	151.2	„	
Negative 41. September 15, 1914. $\psi = +129^0$.						
		h m				
Moon		16 06	150	0.0	2.45	Normal transparency. Earth-shine $\frac{1}{3}$ of the background. Comparison star on the illuminated background.
β Gemin.	1	15 47	„	120.5	1.65	
„	2	15 53	„	106.1	1.62	
„	3	15 57	„	93.1	1.60	
„	4	16 15	„	81.1	1.52	
Negative 47. April 17, 1915. $\psi = -144^0$.						
		h m				
Moon		8 28	140	0.0	2.90	Normal transparency. Earth-shine 0,2—0,3 of the background. Comparison star photographed without the illuminated field.
β Gemin.	1	10 32	26	41.1	1.74	
„	2	10 36	50	41.1	1.76	
„	3	10 40	63	41.1	1.78	
„	4	10 42	140	65.4	1.79	
Negative 48. April 20, 1915. $\psi = -111^0$.						
		h m				
Moon		9 26	150	0.0	1.85	Normal transparency. Earth-shine about $\frac{1}{10}$ of the background. Comparison star photographed without the illuminated field.
β Gemin.	1	9 43	60	41.1	1.48	
„	2	9 45	48	41.1	1.50	
„	3	9 47	37	41.1	1.52	
„	4	9 48	30	41.1	1.54	
„	5	9 49	24	41.1	1.56	
„	6	9 51	19	41.1	1.58	
„	7	9 55	150	63.3	1.64	

"normal transparency" the coefficient of absorption was taken equal to 0.44 stellar magnitudes[1]).

The area measured on the microphotometer corresponded to a circle of 0.20 mm diameter on the plate; the method of measurement and orientation was similar to the method used in the measures of the Moon's surface (see preceding paper).

Table 18 gives a summary of the measures; the "photographic density" is usually the mean of two settings of the microphotometer wedge. The meaning of the different denotations is explained at the end of the table. The way of deriving the provisional intensity, *I*, will be explained later on.

In the reduction of the observations the difference of brightness of α Coronae — β Geminorum was assumed equal to $+0.15$ stellar magnitudes; this would correspond to a colour-index of about $\frac{3}{4}$ the c. i. given by King, a value which seems the most probable for the plates used. Direct determinations of the difference of magnitude of α—β Geminorum and α Coronae — β Geminorum were made on May 3, 1914, on an Ilford RapidChromatic plate; the following result, corrected for atmospherical absorption and reduced with the aid of a provisional value of Schwarzschild's exponent $p = 0.87$, was obtained:

α — β Geminorum $= -0.52$ st. mg.
α Coronae — β Geminorum $= +0.05$ st. mg.
With $p=1.00$ these quantities would become -0.62 and 0.00 respectively.

The data, although uncertain, indicate that our assumption as to the colour-sensitiveness of the plates used is not far from the truth.

An approximate value of the exponent p[2]) was needed, the accuracy desired for this quantity being of about ±10%. Without giving details, the result derived from the extrafocal stellar images is as follows:

from Negative 47, $\beta_{1.2.3.4}$ . . . . . . $p = 0.94$
" " 48, $\beta_{1.2.7}$ . . . . $p = 0.99$

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[1]) For the method of determination of the absorption see *Astronomische Nachrichten*, 5162; the coefficient of absorption for the chromatic plate used was taken 10 % less than for ordinary plates, which corresponds to a colour-index of about 0.8 the colour-index in the Harvard Scale.

[2]) See formula (1) of the preceding paper.

Table 18.

Measurement of the Earth-Shine.

Negative 25.

Orientation:

s, $\xi = 73.57$ and $\eta = 35.95$;
n, $\xi = 51.38$ and $\eta = 45.86$;
centre of disk, $\xi = 62.9$ and $\eta = 41.5$
diameter = 24.1 mm.

№	ξ	η	Density	I	Identification.
1	50.2	43.0	4.76	3.53	b, near n
2	50.5	45.0	4.66	3.31	„ „
3	51.2	47.0	4.57	3.11	„ „
4	52.5	46.2	4.88	3.83	l, NE
5	52.2	49.0	4.49	2.95	b
6	53.8	48.2	4.81	3.66	l, NE
7	53.9	51.0	4.48	2.93	b
8	55.3	49.6	4.74	3.49	l, NE
9	57.1	51.0	4.67	3.33	l, NE
10	60.0	54.0	4.51	2.99	b (i_n)
11	59.0	52.0	4.70	3.40	i_n
12	58.0	50.4	4.75	3.51	i_n
13	57.0	48.8	4.70	3.40	i_n
14	56.0	47.0	4.78	3.58	i_n
15	55.0	45.1	4.80	3.63	i_n
16	53.9	43.3	4.94	3.99	i_n
17	53.0	41.7	4.89	3.85	i_n
18	60.4	52.1	4.70	3.40	l, NE
19	62.3	52.2	4.68	3.35	l, NE
20	64.0	52.0	4.69	3.37	l, NE
21	66.0	51.8	4.74	3.49	l, NE
22	66.7	53.7	4.46	2.89	b
23	68.4	53.2	4.48	2.93	b
24	67.4	51.2	4.88	3.83	l, SE (i_o)
26	65.4	47.3	4.75	3.51	i_o
28	63.2	43.0	4.82	3.68	i_o
30	61.3	39.0	5.10	4.44	i_o
32	59.5	35.0	5.24	4.87	l_o
33	70.1	49.8	4.86	3.78	l, SE
34	71.4	51.2	4.53	3.03	b
35	72.8	49.7	4.56	3.09	b
36	71.4	48.5	4.94	3.99	l, SE
37	72.6	46.6	4.91	3.91	l, SE
38	74.3	47.3	4.64	3.26	b
40	75.7	47.6	4.61	3.20	b
41	74.9	46.0	4.60	3.18	b
42	73.8	43.8	5.00	4.15	l, SE (i_s)
43	73.1	42.0	4.98	4.09	i_s
44	72.4	40.4	5.05	4.29	i_s
45	71.6	38.6	5.10	4.44	i_s
46	71.0	37.0	5.18	4.67	i_s
47	70.2	35.4	5.28	5.00	i_s
48	74.2	41.2	5.03	4.23	l, SE
49	75.9	41.2	4.70	3.40	b, near s
50	75.5	38.8	4.63	3.24	b, near s
51	73.8	39.2	5.02	4.20	l, SE

Table 18. Continued.

Negative 25. Continued.

№	ξ	η	Density	I	Identification
52	73.2	37.2	5.10	4.44	l, SE
53	74.8	36.5	4.70	3.40	b, near s
54	73.6	34.3	4.80	3.63	b, near s
55	76.7	56.4	4.84	3.73	β 1, centre
55a	—	—	4.51	2.99	mean background for β 1
56	69.0	56.0	4.66	3.31	β 2, centre
56a	—	—	4.50	2.97	mean background for β 2
57	63.4	55.6	4.61	3.20	β 3, centre
57a	—	—	4.45	2.87	mean background for β 3
58	57.4	55.6	4.54	3.05	β 4, centre
58a	—	—	4.42	2.82	mean background for β 4
59	free plate		3.32	—	p

Negative 29.

Orientation:

n, $\xi = 73.76$ and $\eta = 34.78$;
centre of disk, $\xi = 62.4$ and $\eta = 40.3$
diameter = 25.2 mm

The greater part of the illuminated crescent without the field.

№	ξ	η	Density	I	Identification
1	75.5	37.2	5.69	2.86	b, near n
2	75.0	35.2	5.08	1.90	"
3	74.2	33.6	4.82	1.60	"
4	73.0	31.8	4.58	1.36	"
5	71.6	30.6	4.54	1.32	"
6	69.3	28.7	4.68	1.45	"
7	72.6	34.6	5.16	2.00	l, NE
8	71.3	33.1	5.07	1.89	l, NE
9	70.0	31.7	5.04	1.85	l, NE
10	68.8	30.6	5.02	1.82	l, NE
11	67.4	29.8	5.08	1.90	l, NE (i_n)
12	68.0	31.1	5.04	1.85	i_n
13	68.6	32.4	5.06	1.88	i_n
14	69.2	33.8	5.12	1.95	i_n
15	69.9	35.0	5.21	2.07	i_n
16	70.6	36.4	5.38	2.32	i_n
17	71.6	38.2	5.60	2.69	i_n
18	67.4	28.0	5.02	1.82	α_8, 0.8 radius from centre
19	64.8	28.7	5.20	2.06	l, NE; superposed α_8, 0.8 rad. from centre
20	62.3	29.1	4.94	1.73	l, NE
21	59.9	29.2	5.02	1.82	l, NE; superposed α_7, 0.6 rad. from centre
22	62.4	27.0	4.92	1.71	α_8, 0.8 rad. from centre
23	60.0	27.2	4.75	1.52	α_7, 0.6 rad. from centre
24	57.8	29.7	5.01	1.81	l, SE; (i_o) superposed α_7, 0.4 rad. from centre
25	58.6	31.2	4.82	1.60	i_o; superposed α_7, 0.8 rad. from centre
26	59.3	32.8	4.64	1.42	i_o
27	60.0	34.5	4.67	1.44	i_o

Table 18. Continued.

Negative 29. Continued.

№	ξ	η	Density	I	Identification
28	60.9	36.2	4.82	1.60	i_0
29	62.0	38.5	5.10	1.92	i_0
30	63.0	41.0	5.40	2.35	i_0
31	63.9	42.6	5.60	2.69	i_0
32	57.9	28.0	4.66	1.44	α_7, centre
33	56.6	27.2	4.58	1.36	α_7, 0.4 rad. from centre
34	56.0	27.2	4.53	1.32	α_7, 0.8 rad. from centre
35	55.5	24.0	4.32	1.14	b
36	53.2	27.3	4.23	1.08	b
37	54.8	28.0	4.48	1.27	α_7, 0.8 rad. from centre
38	56.4	30.6	4.97	1.76	l, SE; superposed α_7, 0.7 rad. from centre
39	55.1	31.8	4.86	1.64	l, SE
40	53.5	30.2	4.53	1.32	α_6, 0.7 rad. from centre
41	53.2	33.1	5.03	1.84	l, SE; superposed α_6, 0.8 rad. from centre
42	51.6	36.0	4.82	1.60	l, SE
43	50.8	34.8	4.28	1.11	b
44	51.0	33.4	4.41	1.21	α_6, 0.7 rad. from centre
45	51.1	31.1	4.48	1.27	α_6, centre
46	50.2	26.9	4.13	1.01	b
47	47.8	28.0	4.08	0.97	b
48	48.2	31.0	4.30	1.13	α_6, 0.85 rad. from centre
49	49.2	33.2	4.64	1.42	superposed α_6, 0.85 rad., and α_5, 0.8 rad. from centre
50	49.8	35.2	4.52	1.31	α_5, 0.8 rad. from centre
51	50.8	37.6	4.80	1.58	l, SE; (i_s)
52	51.8	39.4	4.88	1.66	i_s
53	52.7	41.0	5.02	1.82	i_s
54	53.6	42.5	5.12	1.95	i_s
55	54.4	44.0	4.88	1.66	i_s
56	47.0	31.0	4.11	0.99	b
57	46.4	31.6	4.04	0.95	(b) near boundary of the field; transition
58	45.7	31.8	4.06	0.96	(b) „ [from b to p
59	45.1	32.2	3.68	0.75	„
60	44.4	32.4	3.31	0.58	(p) „
61	43.6	33.6	3.23	0.55	(p) „
63	47.5	35.1	4.52	1.31	α_5, centre
64	44.8	35.4	3.94	0.89	near boundary of the field
65	43.9	35.4	3.28	0.57	p
66	49.3	38.6	4.27	1.10	b
72	50.8	39.3	4.88	1.66	l, SE
73	49.2	40.2	4.34	1.16	b
74	50.8	41.6	4.98	1.78	l, SE
75	49.1	42.1	3.73	0.77	b—p
76	58.1	25.0	5.22	2.08	superposed α_1, 0.6 rad. and α_7, 0.8 rad. from centre
77	58.2	23.8	5.36	2.29	α_1, 0.2 rad. from centre
78	58.0	21.0	4.42	1.22	b
79	60.4	22.9	4.64	1.42	b
80	63.0	20.1	5.22	2.08	α_2, 0.6 rad. from centre
81	63.8	22.2	5.37	2.30	superposed α_2, 0.6 rad., and α_8, 0.7 rad. from centre
82	65.2	25.1	5.09	1.91	α_8, centre
83	68.1	23.7	5.05	1.86	superposed α_4, 0.7 rad., and α_8, 0.7 rad. from centre
84	66.2	22.1	4.89	1.67	α_8, 0.7 rad. from centre
85	65.8	19.8	4.63	1.40	b
86	65.8	18.2	4.56	1.34	b

Table 18. Continued.

Negative 29. Continued.

№	ξ	η	Density	I	Identification
87	65.9	16.3	4.37	1.18	b
88	67.4	19.9	4.98	1.78	α_3, 0.7 rad. from centre
89	68.9	20.0	5.04	1.85	α_3, centre
90	69.4	21.2	5.21	2.07	superposed α_3, 0.7 rad., and α_4, 0.6 rad. from centre
91	69.9	22.9	4.94	1.73	α_4, centre
92	71.6	22.8	4.82	1.60	α_4, 0.6 rad. from centre
93	73.6	23.0	4.39	1.20	b
94	72.2	20.2	4.36	1.18	b
95	74.2	20.9	4.28	1.11	b
96	75.4	20.1	3.94	0.89	b
Free plate			3.14	0.52	p

Negative 41.

Orientation :
s, $\xi = 68.46$ and $\eta = 28.35$
centre of disk, $\xi = 58.2$ and $\eta = 36.3$
diameter = 25.3 mm.

№	ξ	η	Density	I	Identification
1	53.2	49.9	5.98	2.35	b, near n
2	52.8	47.2	6.24	2.80	l, NW
3	51.3	46.1	6.08	2.51	l, NW
4	50.9	48.4	5.62	1.85	b
5	49.2	47.0	5.48	1.68	b
6	50.0	44.6	5.96	2.32	l, NW
7	49.0	43.0	5.86	2.17	l, NW
8	47.2	44.0	5.30	1.49	b
9	48.2	41.1	5.86	2.17	l, NW
10	46.2	39.4	5.20	1.40	b
11	48.0	39.4	5.79	2.07	l, NW (i_n)
12	49.8	41.1	5.76	2.03	i_n
13	51.6	42.7	5.92	2.26	i_n
14	53.4	44.4	6.02	2.42	i_n
15	55.0	45.8	6.21	2.74	i_n
16	56.3	47.0	6.37	3.05	i_n
17	41.1	45.6	4.00	0.63	p
18	42.5	42.0	5.60	1.82	β_4, centre
19	39.8	37.4	4.81	1.08	b, near boundary
17 a	44.1	45.6	5.18	1.38	b
19 a	42.0	37.4	4.95	1.18	b
20	48.1	37.2	5.70	1.95	l, NW
21	48.4	35.0	5.68	1.92	l, NW
22	49.2	32.2	5.46	1.66	l, NW
23	46.4	33.6	5.07	1.28	b
24	41.4	31.4	5.25	1.44	β_2, centre
25	40.9	25.8	4.68	0.99	b
25 a	42.0	27.0	4.74	1.02	b
26	44.1	27.8	4.80	1.07	b
27	49.5	28.2	4.99	1.21	b
28	51.0	29.9	5.52	1.73	l, SW (i_0)
29	52.9	31.4	5.54	1.75	i_0
30	55.0	33.5	5.64	1.88	i_0
31	56.8	35.1	5.87	2.18	i_0
32	58.6	36.8	5.96	2.32	i_0
33	60.2	38.2	6.19	2.70	i_0
34	62.0	39.7	6.40	3.11	i_0
35	63.8	41.4	6.60	3.56	i_0
36	46.4	22.5	4.98	1.21	β_1, centre
37	50.3	19.6	4.66	0.97	b
37 a	52.0	22.0	4.76	1.04	b
38	53.0	28.2	5.56	1.78	l, SW
39	55.4	26.9	5.56	1.78	l, SW
40	54.0	25.2	4.88	1.13	b
41	57.8	26.1	5.70	1.95	l, SW
42	58.6	24.2	4.93	1.17	b
43	60.1	25.9	5.54	1.75	l, SW (i_s)
44	61.8	27.3	5.64	1.88	i_s
45	63.6	29.0	5.76	2.03	i_s
46	65.0	30.3	5.86	2.17	i_s
47	66.4	31.6	6.06	2.48	i_s
48	67.7	32.7	6.25	2.82	i_s
49	69.1	34.0	6.50	3.33	i_s
50	68.7	29.8	6.16	2.65	l, SW
51	71.2	29.8	6.10	2.55	b
52	68.9	27.6	5.61	1.84	b
53	67.5	29.0	6.08	2.51	l, SW
54	66.4	27.9	5.82	2.11	l, SW
55	66.6	25.8	5.32	1.51	b
56	64.6	26.9	5.72	1.98	l, SW
57	62.5	26.2	5.60	1.82	l, SW
58	62.8	24.5	5.06	1.27	b
59	65.1	21.1	4.96	1.19	b
60	61.2	20.6	4.85	1.11	b
62	56.2	20.6	5.32	1.51	β_3, centre
Free plate			4.04	0.64	p

Table 18. Continued.

Negative 47.

Orientation:

s, $\xi = 77.63$ and $\eta = 64.65$
n, $\xi = 70.18$ and $\eta = 42.00$
centre of disk, $\xi = 73.7$ and $\eta = 53.3$
diameter = 24.0 mm.

№	ξ	η	Density	I	Identification	№	ξ	η	Density	I	Identification	
1	69.9	40.9	4.56	0.85	b near n	34	62.1	59.5	4.61	0.88	b	
2	67.4	42.3	4.51	0.82	„	35	63.8	62.1	4.66	0.91	b	
3	63.6	44.7	4.58	0.86	„	36	65.5	60.6	5.04	1.18	l, SE	
4	61.8	47.0	4.54	0.84	„	37	67.4	62.4	5.14	1.26	l, SE	
5	64.0	47.6	4.84	1.03	l, NE (i_n)	38	66.0	64.0	4.81	1.01	b	
6	65.4	46.2	4.90	1.07	l, NE	39	63.7	65.9	4.88	1.06	b	
7	66.8	44.9	4.83	1.02	l, NE	40	69.2	63.7	5.20	1.31	l, SE (i_s)	
8	68.3	43.7	4.84	1.03	l, NE	41	71.0	63.0	5.10	1.22	i_s	
9	69.9	42.9	4.87	1.05	l, NE	42	72.8	62.5	5.18	1.29	i_s	
10	74.9	44.1	5.21	1.32	i_n	43	74.6	61.9	5.28	1.38	i_s	
11	73.2	44.6	5.07	1.20	i_n	44	76.6	61.2	5.36	1.45	i_s	
12	71.0	45.4	4.94	1.10	i_n	45	78.2	60.8	5.40	1.49	i_s	
13	69.4	45.9	4.84	1.03	i_n	46	76.9	63.8	5.43	1.52	l, SE	
14	67.8	46.5	4.78	0:99	i_n	47	75.2	64.0	5.38	1.47	l, SE	
15	66.0	46.9	4.80	1.00	i_n	48	73.0	64.2	5.28	1.38	l, SE	
16	63.2	50.2	4.74	0.96	l, NE	49	71.2	64.1	5.20	1.31	l, SE	
17	60.8	50.0	4.51	0.82	b	50	69.1	65.5	4.78	0.99	b	
18	57.3	49.6	4.54	0.84	b	51	72.8	66.2	4.78	0.99	b	
19	55.4	51.6	4.56	0.85	b	52	75.8	66.0	5.04	1.18	b	
20	60.8	53.4	4.49	0.81	b	53	78.6	66.4	5.12	1.24	b	
21	62.9	52.2	4.72	0.95	l, NE	54	60.8	60.5	4.66	0.91	b, near boundary	
22	62.8	54.4	4.77	0.98	l, NE	55	60.8	40.0	4.53	0.84	p	
23	58.8	58.9	4.53	0.84	b	56	69.3	31.2	4.46	0.80	p	
24	56.5	58.9	4.58	0.86	b	57	69.3	25.0	4.52	0.83	p	
25	61.1	56.8	4.51	0.82	b	58	83.0	13.7	5.62	1.73	β_1, centre	near edge of the plate, background not uniform.
26	63.0	56.8	4.94	1.10	l, SE (i_o)	59	85.0	13.5	5.12	1.24	p, near β_1	
27	65.6	56.0	4.74	0.96	i_o	60	80.2	14.0	5.11	1.23	p, near β_1, β_2	
28	67.8	55.2	4.78	0.99	i_o	61	78.6	14.3	5.82	1.98	β_2, centre	
29	70.6	54.3	4.94	1.10	i_o	62	76.5	15.0	4.88	1.06	p, near β_2, β_3	
30	73.2	53.6	5.11	1.23	i_o	63	74.4	15.6	5.97	2.19	β_3, centre	
31	76.2	52.6	5.33	1.42	i_o	64	71.7	15.5	4.76	0.97	p, near β_3, β_4	
32	78.4	51.7	5.48	1.58	i_o	65	68.5	15.3	5.66	1.78	β_4, centre	
33	64.1	58.8	4.95	1.11	l, SE	66	65.3	15.3	4.80	1.00	p, near β_4	

Negative 48.

Orientation:

s, $\xi = 89.1$ and $\eta = 34.2$
n, $\xi = 66.9$ and $\eta = 44.4$
centre of disk, $\xi = 77.9$ and $\eta = 39.2$
diameter = 24.4 mm.

Only NE quadrant available for measures, the other part being near edge of the plate.

№	ξ	η	Density	I	Identification	№	ξ	η	Density	I	Identification
1	77.0	44.5	6.77	2.80	i_o	4	77.0	47.5	6.56	2.43	i_o
2	77.0	45.5	6.68	2.64	i_o	5	77.0	48.5	6.54	2.40	i_o
3	77.0	46.5	6.68	2.64	i_o	6	77.0	49.5	6.48	2.30	i_o

Table 18. Continued.

Negative 48. Continued.

№	ξ	η	Density	I	Identification	№	ξ	η	Density	I	Identification
7	77.0	50.4	6.42	2.21	l, NE (i_o)	23	78·2	52.1	6.10	1.79	b
8	77.0	52.0	6.23	1.95	b	24	68.6	45.7	6.97	3.20	l, NE (near n)
9	77.0	53.0	6.01	1.68	b	25	67.6	46.6	6.79	2.84	b
10	77.0	54.0	6.12	1.81	b	26	—	—	5.60	1.28	β_1, centre } near edge
11	77.0	55.0	5.98	1.65	b	27	—	—	4.06	0.45	p, between $\beta_1-\beta_2$ }
12	75.0	51.9	6.26	1.99	b	28	—	—	5.25	1.01	β_2, centre
13	75.0	50.0	6.48	2.30	l, NE	29	—	—	3.80	0.39	b, between $\beta_2-\beta_3$
14	73.2	49.4	6.56	2.43	l, NE	30	—	—	4.97	0.84	β_3, centre
15	72.5	51.0	6.38	2.16	b	31	—	—	3.83	0.39	p, between $\beta_3-\beta_4$
16	68.5	53.0	6.19	1.90	b	32	—	—	4.76	0.73	β_4 centre
17	69.3	52.0	6.29	2.03	b	33	—	—	3.72	0.36	p, between $\beta_4-\beta_5$
18	69.9	51.0	6.38	2.16	b	34	—	—	4.54	0.63	β_5, centre
19	70.4	49.8	6.48	2.30	b	35	—	—	3.76	0.37	p, between $\beta_5-\beta_6$
20	71.3	48.3	6.70	2.67	l, NE (i_n)	36	—	—	4.46	0.60	β_6, centre
21	72.0	47.0	6.76	2.78	i_n	37	—	—	3.80	0.39	p, between $\beta_6-\beta_7$
22	78.2	50.5	6.36	2.13	l, NE	38	—	—	5.34	1.08	β_7, centre.

Explanation of table 18.

ξ, η = rectangular coordinates of the measured point, in mm.
density = mean reading of the microphotometer wedge.
I = provisional intensity, corresponding to the density; the surface brightness of the extrafocal image of β Geminorum, having 3 mm diameter, taken = 1.
s = southern } edge of illuminated crescent of the Moon.
n = northern }
β = extrafocal image of β Geminorum.
α = „ „ „ α Coronae.
l = earth-shine near the limb; NE, NW etc. denote the quadrant.
i_o = earth-shine, interior points along the Moon's optical equator.
i_n = earth-shine, interior points in the northern hemisphere.
i_s = earth-shine, interior points in the southern hemisphere.
b = illuminated background of the sky.
p = free plate (unexposed).

The value finally adopted in the reduction was $p = 1.00$; by assuming this value, differing little from the values actually determined, a simplification of the reductions was obtained, since the intensity and the exposure time could be assumed as equivalent.

Table 19 contains the intensity, j, adopted for the extrafocal images of the comparison star; these values were computed on the assumption that the intensity was inversely proportional to the square of the diameter, and were corrected for atmospheric absorption so that the unit of intensity would be the intensity of β Geminorum of 3.0 mm diameter when photographed at *the same zenith distance* and with *the same exposure* as the Moon; the correction for exposure was made, as mentioned above, with $p = 1$ or on the assumption of the proportionality of the effective intensity and exposure.

Table 19.

Effective Intensities (j) of the Extrafocal Images of the Comparison Star. Unit Intensity $= \beta$ Geminorum with $d = 3.0$ mm ($\Delta f = 51.6$ mm), zenith distance and exposure = zenith distance and exposure of the Moon.

Negative 25. **Negative 29.**

	β_1	β_2	β_3	β_4	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
$j =$	0.670	0.453	0.321	0.232	0.995	0.738	0.557	0.405	0.302	0.243	0.182	0.141

Negative 41. **Negative 47.**

	β_1	β_2	β_3	β_4	β_1	β_2	β_3	β_4
=	0.253	0.330	0.432	0.589	0.453	0.879	1.117	0.971

Negative 48.

	β_1	β_2	β_3	β_4	β_5	β_6	β_7
$j =$	0.733	0.582	0.445	0.358	0.284	0.223	0.729

The chief difficulty in photographic measures of the earth-shine is the great intensity of the background of the sky near the Moon and its variation with the distance from the bright limb; the measured densities of the earth-shine and the star could, therefore, not be compared directly, a preliminary study of the distribution of brightness within the field of the photograph being needed. For this purpose a suitable formula of interpolation for converting the photographic density into intensity should be adopted. For the kind of measures here dealt with, when a small additional effect upon a luminous background must be determined, a certain freedom in the choice exists; the following formula was assumed:

$$\log I = K(D - C) \quad \ldots\ldots \quad (7),$$

where D is the measured density, I — the intensity, K — a constant for all plates, C — the density corresponding to $I = 1$, a parameter different for different plates. In this formula the density of the free plate (veil) D_0 is assumed also to be equivalent to a certain intensity, I_0. It may be remarked that for the determination of small intensities superposed on a bright background the parameters K and C are to some extent equivalent to one another: a change in the former can be counterbalanced by a suitable change in the latter, so that the absolute difference of intensities will remain unaltered; it was for this reason that for each plate only one variable parameter, C, was assumed.

As to K, the value $\mathbf{K = 0.290}$, derived from the scale of the comparison star on Negative 48, was adopted. The determination of C was made in the following way.

Let j be the effective intensity of the stellar image (given in table 19), i — the effective intensity of the background, and let D and D_1 be the microphotometer readings on the star and on the free background; then from (7) we have

$$log\ (j+i) = K\ (D - C)\ \ldots\ (8),\ \text{and}$$

$$log\ i = K\ (D_1 - C)\ \ldots\ (9);\ \text{whence}$$

$$log\ (1+\tfrac{j}{i}) = K\ (D - D_1)\ \ldots\ (10).$$

From (10) the ratio $\frac{j}{i}$ and, with j known, i can be found; (9) furnishes us then the value of C; each image of the comparison star gives one separate value; the average of these values can be finally adopted.

Table 20 contains the values of C found for the single images and plates; in deriving them, the density of the background, D_1, corresponding to the centre of the stellar image, was interpolated from measures of the points which were nearest to the star.

Table 20. *Values of C.*

	Negative 25.					**Negative 29.**							**Negative 41.**				
	β_1	β_2	β_3	β_4	Mean	α_1	α_3	α_4	α_5	α_6	α_7	Mean	β_1	β_2	β_3	β_4	Mean
$C =$	2.98	2.43	2.87	2.85	**2.87**	4.11	4.12	4.22	4.00	3,79	4.46	**4.12**	4.39	4.90	4.75	4.57	**4.70**
Relative Weight =	8	2	2	1	—	10	4	3	2	1	1	—	1	3	4	4	—

	Negative 47.					**Negative 48.**							
	β_1	β_2	β_3	β_4	Mean	β_1	β_2	β_3	β_4	β_5	β_6	β_7	Mean
$C =$	4.86	4.78	4.86	4.68	**4.80**	5.34	5.26	5.25	5.23	5.19	5.24	5.21	**5.23**
Relative Weight =	1	1	1	1	—	0	1	1	1	1	1	1	—

With the aid of the K and C adopted the *provisional intensities*, I, of table 18 were computed. It must be remembered that whereas the difference of intensity $I_1—I_2$, corresponding to the densities D_1 and D_2, is probably fairly near the truth when D_1 and D_2 are within the range covered by the comparison star, the absolute values of I cannot be much relied on, and in certain cases have no real meaning, e. g. for the free plate (points "*p*" in table 18) where no light acted on the plate.

The next task was to determine the distribution of brightness in the illuminated background near the Moon; the problem was solved in a purely empirical way, independently for each plate. For this purpose the following points of table 18 were used: 1) direct measures of the background, denoted in the table by the letter *b*; 2) measures of *interior points* on the earthlit part of the Moon's disk; these are denoted in table 18 by i_o, i_n or i_s. The "*b*" points need no explanation, their intensity being taken directly as given in the table. As to the "*i*" points, they were used in deriving the variation of the brightness of the background in the neighbourhood of the sunlit part of the Moon's disk.

The "*i*" points formed ordinarily a sequence of equidistant points on a straight line, approximately parallel to the Moon's optical equator, and as a rule three such sequences were measured: one on the optical equator (i_o), one in the northern (i_n) and one in the southern (i_s) hemisphere; each sequence was continued without the disk by corresponding "*b*" points. The provisional intensity, I, for each sequence was plotted with the distances from the Moon's optical axis as abscissae, and two smooth curves were drawn — one on the Moon's disk, the other through the "*b*" points without the disk; the curves showed a certain "break" at the point where they met the edge of the disk; the "break" or "jump" gave a provisional value of the intensity of the earth-shine for the corresponding sequence; by subtracting this value, the net intensity of the background for the "*i*" points was obtained. The method involves a slight uncertainty from the unknown relative albedo of the different points; the uncertainty is, however, negligible in most occasions, since each sequence comprised either the *maria* or the *continentes* only, so that the surface brightness of the points of one set could be regarded as practically constant; an exception presented the "i_o"

set of the eastern hemisphere (Neg. 25, 29, 47, 48), where the interior points corresponded to the dark Oceanus Procellarum, the points near the limb — to the *continentes;* the latter points were therefore omitted in the derivation of the intensity of the background. The uncertainty in the adopted provisional intensity of the earth-shine has, after all, no real influence on the final result, since in deriving the latter only points on the limb were used, for which any near "*b*"-points with measured brightness of the background were available; the background of the interior points served only to execute a slight interpola-

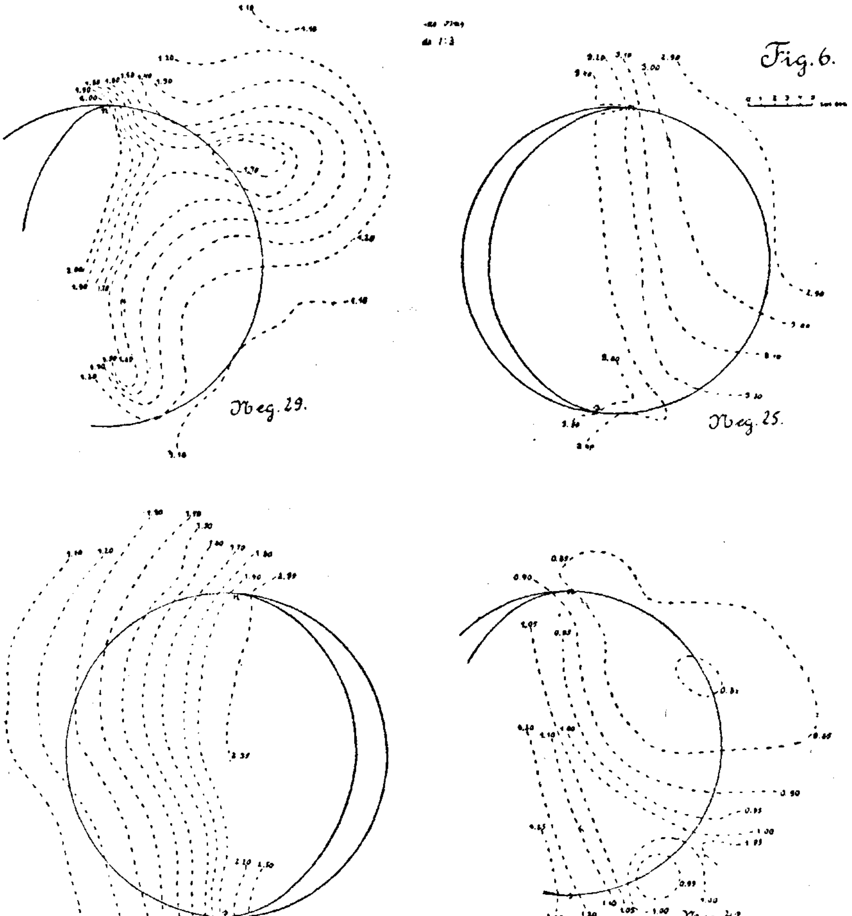
Fig. 6.

tion, namely to reduce the brightness of the neighbouring exterior points to the brightness of the background on the limb.

From the values of the provisional brightness of the background, I_b, for all points, smooth curves of equal brightness (isophotic curves) were drawn on a diagram in rectangular coordinates. Figure 6 gives the diagrams constructed for the negatives 25. 29. 41 and 47. For Neg. 48 the points were too scarce for constructing a diagram, so that numerical interpolation was used.

From the diagrams the intensity of the background for a given point on the Moon's surface could be interpolated; by subtracting this value from the provisional intensity, I, (see table 18) the intensity of the earth-shine on the corresponding point is obtained. Practically the intensity of the earth-shine could be found only for points on the limb (in table 18 denoted by l), about 1 mm within the edge; the interior points presented a *circulus vitiosus,* since the intensity of the background, needed to determine the earth-shine, was derived previously on a certain assumption as to the intensity of the latter.

Table 21 gives under I_e the intensity of the earth-shine for the points on the limb, together with their position angle, P, and the distance from the limb, h; P is reckoned from the northern end of the projection of the Moon's axis. The letters in the column "formation" denote: c — that the point belongs to the *continentes*; m — a normal *mare*; m' — an exceptionally bright *mare (mare Frigoris).* For each quadrant the results are joined into mean values.

Since the measured points belonged to different formations with different reflecting power, the results must be reduced to some standard reflecting power — say, to the mean albedo of the Moon as a whole. For this purpose we shall make use of the investigation by F. W. P. Goetz[1]) on the relative photographic brightness of different points on the Moon's surface at full Moon; our own measures discussed above cannot be used in the present occasion, since they refer to phase-angles greater than 28°, whereas the conditions of observation of the earth-shine correspond to a phase-angle near zero. The zero of the relative intensities given by Goetz corresponds very nearly to the average surface brightness of the full Moon, so that these intensities taken with

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1) *Loc. cit.*

Table 21.

Intensity of the Earth-Shine for points on the limb.
Unit of intensity = surface brightness of extrafocal image of β Geminorum having 3 mm diameter.

Negative 25.

№	I	I_b	I_c	P	h	Formation
			1) NE limb			
4	3.83	3.14	0.69	— 7°	0.028	m'
6	3.66	3.01	0.65	+ 5	0.036	m'
8	3.49	2.97	0.52	+16	0.043	m'
9	3.33	2.95	0.38	+28	0.043	m'
11	3.40	2.93	0.47	+39	0.042	m
18	3.40	2.93	0.47	+47	0.051	m
19	3.35	2.93	0.42	+56	0.056	m
20	3.37	2.92	0.45	+65	0.053	m
21	3.49	2.93	0.56	+76	0.047	m
Mean	3.480	2.968	0.512	—	—	—
			2) SE limb			
24	3.83	2.95	0.88	+85°	0.049	c
33	3.78	3.02	0.76	+101	0.042	c
36	3.99	3.07	0.92	+110	0.041	c
37	3.91	3.12	0.79	+121	0.044	c
42	4.15	3.20	0.95	+137	0.037	c
48	4.23	3.41	0.82	+149	0.034	c
51	4.20	3.56	0.64	+160	0.039	c
52	4.44	3.64	0.80	+172	0.039	c
Mean	4.066	3.246	0.820	—	—	—

Negative 29.

№	I	I_b	I_c	P	h	Formation
			1) NE limb			
7	2.00	1.77	0.23	+ 1°	0.044	m'
8	1.89	1.54	0.35	+10	0.048	m'
9	1.85	1.50	0.35	+20	0.045	m'
10	1.82	1.55	0.27	+29	0.042	m'
11	1.90	1.69	0.21	+36	0.038	m'
19	1.94	1.62	0.32	+50	0.028	m
20	1.73	1.46	0.27	+61	0.054	m
21	1.64	1.28	0.36	+74	0.048	m
Mean	1.846	1.551	0.295	—	—	—
			2) SE limb			
24	1.63	1.20	0.43	+83°	0.040	c
38	1.62	1.17	0.45	+92	0.045	c
39	1.64	1.14	0.50	+102	0.052	c
41	1.64	1.11	0.53	+113	0.032	c
42	1.60	1.14	0.46	+129	0.040	c
51	1.58	1.16	0.42	+137	0.028	c
72	1.66	1.22	0.44	+146	0.038	c
74	1.78	1.28	0.50	+157	0.038	c
Mean	1.644	1.178	0.466	—	—	—

Negative 41.

№	I	I_b	I_c	P	h	Formation
			1) NW limb			
2	2.80	2.16	0.64	+ 2°	0.036	c
3	2.51	1.91	0.60	— 8	0.036	c
6	2.32	1.74	0.58	—16	0.033	c
7	2.17	1.64	0.53	—26	0.032	c
9	2.17	1.55	0.62	—36	0.032	c
11	2.07	1.50	0.57	—44	0.037	c
20	1.95	1.45	0.50	—54	0.042	c
21	1.92	1.40	0.52	—65	0.045	c
22	1.66	1.33	0.33	—79	0.037	c
Mean	2.174	1.631	0.543	—	—	—

Negative 47

№	I	I_b	I_c	P	h	Formation
			1) NE limb			
9	1.05	0.89	0.16	— 6°	0.036	m'
8	1.03	0.84	0.19	+ 4	0.034	m'
7	1.02	0.83	0.19	+14	0.038	m'
6	1.07	0.83	0.24	+24	0.038	m'
5	1.03	0.83	0.20	+34	0.024	m'
16	0.96	0.81	0.15	+47	0.040	m
21	0.95	0.81	0.14	+57	0.044	m
22	0.98	0.82	0.16	+70	0.046	m
Mean	1.011	0.833	0.178	—	—	—

Table 21. Continued.

Negative 41. Continued.

2) SW limb

№	I	I_b	I_c	P	h	Formation
28	1.73	1.29	0.44	−94°	0.044	c
38	1.78	1.24	0.54	−107	0.044	c
39	1.78	1.21	0.57	−120	0.046	c
41	1.95	1.22	0.73	−132	0.040	c
43	1.75	1.27	0.48	−143	0.038	c
57	1.82	1.35	0.47	−155	0.042	c
56	1.98	1.48	0.50	−165	0.038	c
54	2.11	1.66	0.45	−176	0.037	c
53	2.51	1.89	0.62	+177	0.051	c
50	2.65	2.25	0.40	+170	0.036	c
Mean	2.006	1.486	0.520	—	—	—

Negative 47. Continued.

2) SE limb

№	I	I_b	I_c	P	h	Formation
26	1.10	0.84	0.26	+82°	0.032	c
33	1.11	0.88	0.23	+93	0.043	c
36	1.18	0.92	0.26	+105	0.045	c
37	1.26	1.00	0.26	+119	0.038	c
40	1.31	1.00	0.31	+131	0.030	c
49	1.31	0.99	0.32	+141	0.038	c
48	1.38	1.02	0.36	+151	0.038	c
47	1.47	1.12	0.35	+162	0.045	c
46	1.52	1.20	0.32	+172	0.044	c
Mean	1.293	0.997	0.296	—	—	—

Negative 48.

NE limb

№	I	I_b	I_c	P	h	Formation
22	2.13	1.86	0.27	+66°	0.034	m
7	2.21	1.99	0.22	+60	0.034	m
13	2.31	2.13	0.18	+49	0.040	m
14	2.43	2.31	0.12	+40	0.036	m
20	2.67	2.50	0.17	+28	0.035	m′
24	3.20	3.03	0.17	+10	0.030	m′
Mean	2.492	2.303	0.188	—	—	—

I = provisional intensity of background + earth-shine.
I_b = „ „ of background.
I_c = $I - I_b$ = intensity of earth-shine.
P = position angle; h = distance from the limb, fraction of Moon's diameter.
c = *continens; m = mare; m′ = mare* above normal brightness.

the opposite sign can be used as reductions to the mean intensity. From the 55 points measured by Goetz[1]) we choose the following ones as best representing our measured points.

1) Points on the *continentes* near the limb, corresponding to those denoted in table 21 by "c":

№ (Goetz)	28	27	1	55	3	24	35	45	44	38	37
Rel. Intens. (h_o) (St. Mg.)	−0.42	−0.63	−0.19	−0.36	−0.04	−0.20	−0.58	−0.62	−0.08	−0.30	−0.26

Mean $h_o = -0.33$.

Adopted correction for "c" points . . . **+0.33 stellar magnitudes.**

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1) Loc. cit., Tab. XV p. 32.

2) Points on the *Oceanus Procellarum* near the limb, corresponding to those denoted in table 21 by "m":

№ (Goetz)	48	47	51	19	54	46
Rel. Intens. (h_0) (St. Mg.)	+0.49	+0.34	+0.31	+0.40	+0.44	+0.30

Mean $h_0 = +0.38$.

Adopted correction for "m"-points . . . —**0.38 stellar magnitudes.**

3) Points on *Mare Frigoris*, corresponding to the "m′" in table 21, are not represented in Goetz's list; from table 15 of the present publication we find that the relative albedo of Mare Frigoris is by 0.26 st. mg. higher than the average, and that Mare Serenitatis (p. 23, 24) can be regarded as having the mean surface brightness; two points measured by Goetz on Mare Serenitatis give the mean relative intensity as +0.32 st. mg.; applying our difference, —0.26 st. mg., we find: relative intensity of mare Frigoris . . . $h_0 = +0.06$ st. mg.; adopted correction for "m′" points . . . —**0.06 stellar magnitudes.** Table 22 contains the definitive reduction of the measures of the earth-shine. In this table ψ_g denotes the phase-angle of the earth as seen from the Moon; the other data are self-explanatory.

Table 22.

Brightness of the Earth-Shine.

Reduction to Mean Reflecting Power.

Neg.	Quadrant	Number of Points			Observed Brightn.		Red. to mean Albedo St. Mg.	Brightn. of Earth-Shine for mean Albedo St. Mg.	Mean of Plate St. Mg.	Residual St.Mg.	ψ_g
		c	m	m′	I_c	St. Mg.					
						+		+	+		
25	NE	—	5	4	0.512	0.73	—0.24	0.49	0.52	—0.67	37°
25	SE	8	—	—	0.820	0.22	+0.33	0.55			
29	NE	—	3	5	0.295	1.33	—0.18	1.15	1.16	—0.03	60
29	SE	8	—	—	0.466	0.83	+0.33	1.16			
41	NW	9	—	—	0.543	0.66	+0.33	0.99	1.02	—0.17	51
41	SW	10	—	—	0.520	0.71	+0.33	1.04			
47	NE	—	3	5	0.178	1.88	—0.18	1.70	1.68	+0.49	36
47	SE	9	—	—	0.296	1.32	+0.33	1.65			
48	NE	—	4	2	0.188	1.82	—0.27	1.55	1.55	+0.36	69
					Average of all plates			—	+1.19 ± 0.15	—	51°

In forming the mean, equal weight was attributed to each plate; this was made because the plates showed great systematic differences affecting equally all measured points, and for which the only explanation can be found in the unequality of atmospherical absorption for Moon and comparison star; this is not surprising, if the great zenith distances at which the photographs were obtained are taken into account. The measures of one plate show a good internal agreement, as shown by the individual values of I_c in table 21 and by the small differences in the brightness of the earth-shine as derived from the two quadrants (table 22, col. 9); these differences attain only a few hundredths of a stellar magnitude.

The final result of the measures is that the average difference in the surface brightness of earth-shine — β Geminorum, $d = 3$ mm is

$+1.19 \pm 0.15$ (p. e.) stellar magnitudes.

The diameter of the focal image of the Moon at mean distance is 24.9 mm for the objective used; hence we obtain the following difference of brightness:

$$\textbf{Earth-Shine} - \beta \ \textbf{Geminorum} = +1.19 - 5 \log \left(\frac{24.9}{3.0}\right) = = \mathbf{-3.41\ St.\ Magn.}$$

The colour-index of the Moon, according to Russel and King [1]), is $+1.18$, or almost exactly equal to the colour-index of β Geminorum; we may assume therefore the photographic difference of magnitudes of full Moon — β Geminorum to equal the difference of their visual magnitudes, or equal to

-12.55 [2]) $-1.21 = -13.76$ stellar magnitudes.

This gives for the difference in brightness of the earth and the sun, as observed from the Moon at mean distance, the value

$-3.41 + 13.76 = \mathbf{+10.35}$ **st. mg.**, for $\psi_g = 51^0$.

Assuming for the earth the law of variation of brightness with phase-angle to be identical with the variation of Venus, we find the correction for full phase equal to -0.74 st. mg. according to Müller's observations of Venus, and the difference

"full earth" — sun $= +10.35 - 0.74 = \mathbf{+9.61}$ **st. mg.**

This gives for the true albedo of the earth, or the albedo according to Bond's definition [1]), the value

$\mathbf{a} = 1{,}2 \,.\, 2{,}512^{-9{,}61} \,.\, (60{,}27)^2 = \mathbf{0.63 \pm 0.08}$ (60.27 is the mean distance of the Moon in radii of the earth).

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1) Astrophysical Journal **43**, p. 125.

2) Ibidem.

This value of the photographic albedo corresponds to a colour-sensitiveness of the plates giving a colour-index of about $\frac{3}{4}$ of the colour-index by King.

Adopting for the visual albedo the value 0.45 according to H. N. Russel[2]), the reflecting power of the earth for the kind of plates used comes out by 0.38 st. mg. greater than for visual radiation; taking into account the adopted colour-sensitiveness, the difference in colour-index of sun-earth results as $\frac{0{,}38 \cdot 4}{3} =$ $= +0.51$ st. mg. in King's scale, and with the sun's colour-index equal to $+0.79$, the colour-index of the earth in the Harvard Scale must be

$$+0.28 \text{ st. mg.}$$

III. Summary.

1. An empirical law satisfactorily representing the distribution of brightness over the Moon's disk is proposed; formula (6) and table 16 give the analytical expression of the law, and the parameters for different phase-angles, ranging from 28° to 121°, for the *continentes* and the *maria* separately.

2. The relative reflecting power of a number of points on the Moon's surface is determined; table 15 contains the result, the relative albedoes of the points on the *continentes* representing the deviations from the mean albedo of the *continentes*, and the relative albedoes of the points on the *maria* giving the deviations from the mean of all *maria*.

3. From 5 photographs of the earth-shine on Ilford Rapid Chromatic plates the albedo of the earth (Bond's definition) is found equal to 0.63 ± 0.08, if a phase-law identical with the law for Venus is assumed.

4. With a value of the visual albedo = 0.45 the colour-index of the earth in the Harvard Scale is determined as $+0.28 \pm 0.15$ stellar magnitudes.

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1) See H. N. Russel, On the Albedo of the Planets and their Satellites loc. cit. pp. 173—196.

2) Loc. cit. p. 190.