A STATISTICAL METHOD OF COUNTING SHOOTING STARS AND ITS APPLICATION TO THE PERSEID SHOWER OF 1920

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§ 1. If we consider the methods of observing Shooting Stars used at present, we are forced to acknowledge, that from the statistical point of view they are imperfect. The number of meteors is ordinarily recorded by a simple process of counting, where various personal and atmospherical factors are liable to alter the result completely. These factors are: 1) variations of the transparency of air and of the sky-illumination, affecting the result by adding or eliminating numerous faint meteors near the minimum visibile; 2) the acuteness of the observer's eye; 3) the incompleteness of the counting process; if two persons are observing simultaneously the same region in the sky, the meteors recorded by one observer will not all be seen by the second and vice-versa; the percentage of common objects decreases as the brightness diminishes. This indicates that not all meteors are perceived by one and the same observer, the Percentage of Perception depending upon the apparent luminosity; 4) the zenith-distance z of the observed region; neglecting the curvature of the earth and assuming a mean effective height for the meteors, it is easily ascertainable that the horizontal area covered by a constant solid angle varies as the third power of Secz, while the apparent brightness decreases as the second power of Secz; thus, observations at various zenith-distances, for instance — of the same region at different hour-angles —, are quite incomparable; near the horizon we shall count luminous meteors upon a large area, near the zenith — faint meteors upon a small one; if, nevertheless, the results are usually alike, this is explained by the „Luminosity-Curve" of meteors: faint meteors are more numerous than luminous ones, and this roughly counterbalances the effect of decreasing area; such an apparent constancy of meteor-numbers at various zenith-distances conceals a non-homogeneity, contradicting the principal law of statistics.

At the same time the importance of accurate statistical data concerning meteors is obvious; the Radiant furnishes us some of the orbital elements of a meteoric stream; but its period of revolution remains unknown unless statistical data are taken into account: there must appear some periodicity of the number of meteors. For streams like the Leonids or Andromedids exhibiting an enormous maximum of strength the problem is easly solved; but if the stream has no excessive maximum, then more accurate data are needed. For cosmogonical and other problems, where the part played by meteorical matter is to be estimated, the knowledge of the true number of meteors encountered by the earth is of no little importance.

For all these reasons it seemed advisable to apply a more rigorous statistical process to the counting of meteors; the method, which will be described below, may be called „The Double Count Method".

Let us imagine that two observers watching simultaneously, but independently a given region in the sky record data sufficient for identification of their meteors; then the Theory of Probability permits us to calculate the probable true number of meteors which have appeared within the region. Let n_1 and n_2 be the numbers of meteors recorded by the observers, p_1 and p_2 — their Coefficients of Perception, m — the number of common objects; the observations being independent, we may write

$$m = p_1 n_2 = p_2 n_1 \quad (1), \text{ or}$$

$$p_1 = \frac{m}{n_2} \text{ and } p_2 = \frac{m}{n_1} \quad (2).$$

The True Probable Number N will be given by

$$N = \frac{n_1}{p_1} = \frac{n_2}{p_2} = \frac{n_1 n_2}{m} \quad (3).$$

Actual observations have indicated that the coefficients p depend upon the apparent brightness of the meteor and its position within the region; the above equations are therefore correct only for a h o m o g e n e o u s group of meteors, i. e. of approximately the same brightness and appearing within a limited part of the region.

If there are more than two simultaneous observers, the equations will be modified. Let k be the number of observers,

$n_1, n_2, \ldots n_k$ their records, $p_1, p_2, \ldots p_k$ — the Coefficients of Perception, m_{hi} — the number of common meteors of any two observers h and i; then for p_1 we obtain $k-1$ different values: $p_1 = \frac{m_{12}}{n_2}$; $p_1 = \frac{m_{13}}{n_3}$; $\ldots p_1 = \frac{m_{1k}}{n_k}$. Attributing to each value a weight equal to the divisor, the weighed mean results as

$$p_1 = \frac{m_{12} + m_{13} + \ldots + m_{1k}}{n_2 + n_3 + \ldots + n_k} = \frac{\Sigma\, m_{1h}}{\Sigma\, n_h}\Big|_{h=2,3,\ldots k} \quad (4), \text{ or,}$$

generally

$$p_i = \frac{\Sigma\, m_{ih}}{\Sigma\, n_h}\Big|_{h=1,2,\ldots i-1,\, i+1,\ldots k} \quad (4^{I}).$$

The probable error of a value of p will be

$$p.\ e. = \pm 0{,}674 \sqrt{\frac{p_i(1-p_i)}{\Sigma\, n_h}} \quad (4^{II}).$$

Let S denote the number of different meteors, recorded by all observers, N — the true number; then $N-S$ is the number of meteors which have remained unobserved; in virtue of Probability one may write $N-S = N(1-p_1)(1-p_2)\ldots(1-p_k)$, whence

$$N = \frac{S}{1-(1-p_1)(1-p_2)\ldots(1-p_k)} \quad (5).$$

The probable error of N will be

$$p.\ e. = \pm 0{,}674 \frac{N}{\sqrt{S}} \quad (5^{I}).$$

The brightness of meteors is ordinarily recorded in stellar magnitudes; as will be shown later, these estimated magnitudes form a satisfactory photometrical scale — if only some precautions (choice of comparison-stars etc.) are taken. The Statistical Treatement required the reduction of the observed apparent brightnesses to some standard distance; the height of the individual meteors being unknown, it seemed advisable to use the reduction to the zenith. Let the Zenithal Luminosity and Zenithal Magnitude denote the apparent brightness or magnitude of the meteor seen from a distance equal to its mean height[1]).

1) For simplicity's sake we will suppose the whole meteor condensed into a point at the middle of its visible path.

If i is the apparent, i_0 — the Zenithal Luminosity, z — the zenith distance of the middle of the visible path, we may write:

$$i_0 = i.\ \ F^2\, Sec^2\, z \quad (6),$$

where F, a factor depending upon the curvature of the earth, is determined by (7) or (7[I]):

$$F = Cos^2 z \left[\sqrt{\left(\frac{R}{H}\right)^2 + \frac{1 + \frac{2R}{H}}{Cos^2 z}} - \frac{R}{H} \right] (7)$$

$$F = 1 - \frac{H}{2R} tg^2 z + \frac{H^2}{2R^2} Sec^2 z\, tg^2 z - \ldots\ldots \quad (7^I);$$

R here denotes the radius of the earth, H — the height of the middle of the trail; an approximate value of the last quantity will be sufficient.

The Zenithal Luminosity is analogous to the Absolute Luminosity of stars; there exists, however, a difference: the standard distance of the meteors — their height — is not constant; but for meteors of the same shower the conception of Zenithal Luminosity is practically definite enough.

The intensity of a meteoric shower must be reduced to some unity of area. The difficulty is here the same as in the case of Luminosities: the height of the trail is generally unknown. Let $d\sigma$ be an infinitesimal solid angle at the zenith distance z, ds — the corresponding horizontal area at a height H above the earth's surface; then

$$ds = H^2 F^2 Sec^2 z\, d\,\sigma\, F' \quad (8),$$

where F is taken from (7) and F' given by (9):

$$F' = \frac{1 + \frac{H}{R}}{\sqrt{Cs^2 z + \frac{2H}{R} + \frac{H^2}{R^2}}} \quad (9).$$

Let us introduce a somewhat peculiar unity of area — a Zenithal Square Degree; this term will denote a horizontal area at the height H, covering at the zenith a solid angle equal to one square degree; let the symbol „$z^0 \square$" correspond to it. For $H = 100$ kilometers $1\, z^0 \square$ coversapproximately 3 km^2. If we express $d\sigma$ in square degrees and put $H=1$, formula (8) will then give ds expressed in $z^0\square$.

Table 1 contains the values of $F\,Sec\,z$ and F', computed for $H=100$ km, refraction not having been taken into account.

Table 1.

Z	$F\,Sec\,z$	F'	Z	$F\,Sec\,z$	F'
0°	1.000	1.000	78°	4.19	3.71
10	1.016	1.015	80	4.78	4.09
20	1.063	1.062	81	5.13	4.30
30	1.152	1.149	82	5.52	4.51
40	1.298	1.291	83	5.98	4.72
50	1.538	1.524	84	6.49	4.93
60	1.956	1.914	85	7.07	5.14
65	2.288	2.216	86	7.74	5.32
70	2.771	2.640	87	8.49	5.49
72	3.029	2.850	88	9.35	5.62
74	3.340	3.095	89	10.29	5.70
76	3.718	3.380	90	11.36	5.72

§ 2. To test the method observations were organized at the observatory of Tashkent near the maximum of the Perseid shower of 1920. Under the dictation of the observer an assistant wrote down the following data: 1) the time with accuracy to 1 second, from a sharp signal of the observer; 2) the apparent magnitude; in various parts of the region under observation comparison-stars up to the 5-th magnitude were chosen. Immediately after the apparition of the meteor the observer watched the nearest star and made his comparison; the record was thus almost freed from the effect of atmospherical absorption, the latter acting equally upon the meteor and the comparison-star; 3) the position within the region — the nearest star was recorded; 4) the direction of the meteor's path, reckoned from a fixed line (α—γ Andromedae), *to* 45°; 5) the length of path; 6) the duration; 7) the colour. Only the first four data are of importance, the remaining being of a subjective nature.

The middle of the chosen region was at $\alpha = 2^h 40^m$, $\delta = +40^0$ — between β Persei and γ Andromedae; the diameter of the region was approximately 60°.

The persons who took part in the observations were: M-r

Davidovitsch, astronomer of the observatory, M-r Komarevsky, professor of mathematics, M-r Betger, M-r Zuckervanik (observers); M-r Starker, Miss Lein, M-rs Limarev, M-r Pauly (assistants). M-r Davidovitsch (assistant M-r Starker) traced the paths of the meteors on a map; the three other observers together with their assistants were occupied with the Double-Count observations. For brevity's sake we will denote with the following letters the four pairs of observers thus formed: *A* (observer M-r Komarevsky), *B* (M-r Betger), *C* (M-r Zuckervanik) and *D* (M-r Davidovitsch). I am indebted to M-r Davidovitsch for conducting the observations during my absence.

Complete observations took place from August 9-th to Aug. 13-th, between 12^h—15^h Tashkent M. Time; every night these 3 hours were divided into three intervals of time of nearly 50 minutes each with a pause of 10 minutes between. The observers were located 100—200 meters apart from one another; loud signals marked the beginning and end of the observations.

The total number of different meteors employed in the reduction and seen by *A*, *B* or *C*, was 629; many of them were recorded by more than one observer, so that the number of records made use of attained 1000. The number would have been greater but for several meteors rejected, seen without the boundary of the region. There should be added 170 meteors traced upon the map by *D*, not included in the above number.

From meteors observed by two or more observers, which we shall henceforth simply call „Common" meteors, systematical errors might be derived. The most important are systematical differences of the estimated magnitudes. Among the observers M-r Davidovitsch possessed considerable experience in photometrical observations, while the others were amateurs, observing for the first time. Thus it seemed advisable to reduce all estimations to the photometrical system of M-r Davidovitsch or to the „system *D*". The systematical differences as determined from the Common meteors were:

Observer	Difference St. Mg	*n*
D—A	$+0.44 \pm 0.05$	85
D—B	$+0.37 \pm 0.07$	84
D—C	-0.03 ± 0.05	81

On the basis of this table the following reductions of the estimated magnitudes were assumed:

$$\triangle A = +0^{mg}_{.}4;\ \triangle B = +0^{mg}_{.}4;\ \triangle C = 0^{mg}_{.}0.$$

Fifty meteors were observed simultaneously by all four observers, and these allowed of the determination of the probable error of one magnitude-estimation separately for each observer. Let $\triangle_0$, $\triangle_1$, $\triangle_2$ and $\triangle_3$ be the mean square deviations of the observers D, A, B and C respectively, $\triangle_{0-1}$, $\triangle_{0-2}$, $\triangle_{0-3}$ the mean deviations of the differences $D-A$, $D-B$, $D-C$, and $\triangle_{0-1,2,3}$ the mean square deviation between D and the mean from the estimations of the remaining three observers (for meteors observed fourfold); then we obtain the following equations:

$$\triangle_0^2 + \triangle_1^2 = \triangle_{0-1}^2;\ \triangle_0^2 + \triangle_2^2 = \triangle_{0-2}^2;\ \triangle_0^2 + \triangle_3^2 = \triangle_{0-3}^2;$$

$$\triangle_0^2 + \frac{\triangle_1^2 + \triangle_2^2 + \triangle_3^2}{9} = \triangle_{0-1,2,3}^2$$

The quantities on the right-hand side as determined from the Common meteors were; $\triangle_{0-1} = \pm 0.74$; $\triangle_{0-2} = \pm 0.92$; $\triangle_{0-3} = \pm 0.76$; $\triangle_{0-1,2,3} = \pm 0{,}60$; from these the unknown quantities $\triangle_{0,\dots 3}$ and the corresponding probable errors were obtained; they were:

Observer	Pr. error St. Mg.	Weight
D	± 0.29	3
A	± 0.39	2
B	± 0.53	1
C	± 0.40	2

The best estimations are those of D, as might have been expected. Observer B gave his magnitudes without decimals, whence the great probable error. Observers A and C wrote their magnitudes to $0^{mg}_{.}5$, and D — to $0^{mg}_{.}2 - 0^{mg}_{.}3$.

From inspection of the systematical and accidental errors the conclusion may be drawn that visual estimations of the brightness of Shooting Stars possess a degree of accuracy higher than could be expected for such difficult objects, moving and seen only during a fraction of a second.

The errors being considerable, it seemed useful to take into account a sort of „Decimal Equation“: the decimal .0 is recorded more frequently than other decimals. This is clearly seen from the following table, containing the number of records of various magnitude-classes for *A* and *C*. The smoothed numbers are taken from a hand-drawn curve.

Table 2. Number of Meteors.

Recorded Magn.		$\leqslant$0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	Total
A	Recorded	6	16	21	77	36	47	20	13	1	2	239
	Smoothed	6	15	34	52	52	40	25	12	(1)	(2)	239
C	Recorded	5	9	14	31	33	49	32	39	15	3	230
	Smoothed	5	8	18	27	38	44	38	32	18	2	230

The magnitudes of *D* (170 meteors) and the mean magnitudes of the Common meteors (306 objects) did not exhibit any preferred decimal.

To obtain smooth statistical data a part of the meteors noted by the observers *A*, *B* and *C* in round magnitudes should

Table 3.
Equivalent quantities replacing 1 recorded meteor.

Recorded Magn.	Normal Magn. →	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
A	1.0	0.00	0.93	0.07						
	2.0			0.16	0.68	0.16				
	3.0					0.07	0.85	0.08		
	4.0							0.08	0.92	0.00
B	1.0	0.25	0.50	0.25						
	2.0			0.25	0.50	0.25				
	3.0					0.25	0.50	0.25		
	4.0							0.25	0.50	0.25
C	1.0	0.00	0.90	0.10						
	2.0			0.10	0.87	0.03				
	3.0					0.08	0.90	0.02		
	4.0							0.13	0.82	0.05

be transferred to the neighbouring classes (with the decimal .5); for this purpose each meteor with the decimal occurring too frequently was decomposed into three equivalent quantities, referred to the adjacent classes and chosen so as to transform the recorded numbers of table 2 into smoothed numbers. Table 3 contains these quantities. Owing to the fact that the observer *B* did not write decimals, his meteors were equally distributed between the recorded and adjacent magnitudes.

The quantities of table 3 were used only for meteors perceived by a single observer; such meteors, with the decimal 0, formed about $\frac{1}{4}$ of the total number.

The records of position were also subject to systematical errors. The observer marking the nearest star, should reckon from the centre of the visible path of the meteor; but observers *A* and *B* preferred the beginning, observer *C* — the end of the trail; it is shown by the following table, obtained by comparison with meteors simultaneously traced upon the star-map.

Observer	Number of cases, when the star was nearer to: The Beginning	The Middle	The End	Total
A	28	2	11	41
B	36	2	5	43
C	8	4	29	41

The whole region was divided into 13 Sections; table 4 gives the coordinates of their centra, the mean distance from the Perseid-Radiant (ϱ) and the solid angle σ.

The designation of position by means of the nearest star did not prove satisfactory; in several cases there remained some doubt as to the section to which the meteor belonged; in these cases the doubtful meteors were distributed between the neighbouring Sections proportionally to certain factors, depending upon the position and the observer and found empirically by comparison with meteors traced on the map. Had the observers employed from the beginning Sections limited by lines joining definite stars, these difficulties would have been avoided.

As to the direction of the path, this was also controlled by the meteors traced upon the map. The records sometimes

Table 4.

Section	Coord. of Centrum α 1920	δ	ϱ	σ Sq. Degr.	Stars or Constellations Contained In Each Section.
I	$8.^0 3$	$62.^0 6$	20^0	388	Cassiopeja
II	349.5	44.8	35	264	λ Andromedae
III	13.4	45.9	23	197	between Cassiopeja and Andromeda
IV	19.0	36.8	26	256	γ—β Andromedae
V	30.7	25.1	33	359	α—γ Arietis, Triangulum
VI	0.2	27.3	43	223	α Andromedae
VII	49.6	21.7	33	316	η Tauri, δ Arietis
VIII	48.3	37.9	18	269	β, ε, ζ Persei
IX	42.4	53.3	(6)	250	δ, χ Persei
X	80.4	38.0	31	301	Auriga
XI	65.6	43.9	18	222	between Perseus and Auriga
XII	54.1	65.4	10	232	Cameleopardalus
XIII	14.0	12.0	48	500	Pisces

revealed considerable divergences, as may be seen from the following numbers:

Observer	Number of Comparisons with the Map	Errors greater than 45^0	
		Number	Percentage
A	87	3	3
B	87	15	17
C	82	19	23

The best records are those of *A*. The other two observers committed several errors of 180^0, which may be accounted for by momentary loss of orientation.

The Time was read from chronometers, the probable error of one record was $\pm 0,^s7$. Errors of 2^s and more occurred: for *A* in 0% of all cases, for *B* — in 14%, for *C* — in 1% and for *D* — in 3%; such errors are partly due to occasional retardation of the signal.

With the aid of the data recorded the identification of the meteors was executed. It is of interest to estimate the probable number of false identifications. The Probability of a false identification for each record separately was: 1) in the Time divergences up to 3^s were admissible; the mean interval between two meteors was 80^s; thus the Probability of fortuitous coincidences was $\frac{3}{80}=\frac{1}{27}$; 2) for the magnitude the discrepancies did not exceed 2^{mg}; the total interval being practically about 4^{mg}, the corresponding probability results as $\frac{2}{4}=\frac{1}{2}$; 3) the 13 Sections give the Probability of coincidence in position as $\frac{1}{10}$ approximately; 4) if the position is given, the direction of the trail cannot be regarded as an independent quantity for meteors belonging to one shower and shall therefore be omitted here. The total Probability of a false identification will be $\frac{1}{27}\cdot\frac{1}{2}\cdot\frac{1}{10}=\frac{1}{540}$; for the 600 meteors in question that means maximum 1 false identification. Even this must be regarded as exaggerated because if all records would exhibit simultaneously their greatest admissible deviations, the identification would not take place.

§ 3. From a preliminary treatment of the data it appeared that the Coefficient of Perception p may be represented as a product of two factors:

$$p=\chi\pi \quad (10)$$

χ, the same for all observers, being a quantity depending only upon the apparent magnitude, and π — an individual function, depending upon the observer and the position within the region; these quantities we may call: χ — the Magnitude Function and π — the Coefficient of Attention. It was possible to find them from the observational data by successive approximation. A priori it was evident that the Magnitude Function should decrease with increasing magnitude; the preliminary treatment revealed that the decrease began at $m=2^{mg}.7$ approximately; thus it was assumed that for $m\leqslant 2.7\ \chi=\chi_0=1$; for six successive magnitude classes the following notations for χ were assumed:

m St. Magn. System D	$\leqslant 2.7$	3.0	3.5	4.0	4.5	5.0
χ	$\chi_0(=1)$	χ_1	χ_2	χ_3	χ_4	χ_5

The whole region was divided into 3 great parts, called K, L and M; K contained Sections I, III, IV, V, VIII and IX; L — Sections II, VI and XIII; M — Sections VII, X, XI and XII. The average Coefficients of Attention for these parts were denoted as:

Observer / Part of Region	A	B	C
	Coeff. of Attention		
K	π_1	π_2	π_3
L	π_4	π_5	π_6
M	π_7	π_8	π_9

The values of χ and π were regarded as independent unknowns to be determined; the total number of unknowns was thus $5+9=14$. Each observer, class of magnitude and part of region gives one equation of the form (10), where p is determined from the observational data with the aid of formula (4^{I}); the number of equations will then be $3 \times 6 \times 3 = 54$.

The observational data are collected in table 5. *A*, *B* and *C* denote the number of meteors, seen by the corresponding observer alone; *AB* — the number of meteors seen simultaneously by *A* and *B*, *ABC* — by all three observers etc. From

Table 5.

Magn. Syst. *D*	$\leqslant 2.7$	3.0	3.5	4.0	4.5	5.0	$\leqslant 2.7$	3.0	3.5	4.0	4.5	5.0
	Part *K* (341 meteors)						Part *L* (160 meteors)					
A	8	8	11	12	13	12	12	7	6	6	3	1
B	4	10	16	14	13	10	3	2	4	5	7	4
C	8	8	12	14	10	3	4	3	8	9	8	1
AB	9	5	10	8	0	0	3	3	2	1	1	0
AC	7	3	6	4	1	0	3	3	3	2	2	0
BC	3	3	6	8	2	3	4	2	4	2	1	0
ABC	36	22	15	3	1	0	19	5	4	3	0	0
S	75	59	76	63	40	28	48	25	31	28	22	6

Magn. Syst. *D*	$\leqslant 2.7$	3.0	3.5	4.0	4.5	5.0
	Part *M* (128 meteors)					
A	17	9	8	5	2	0
B	3	4	6	5	4	4
C	3	4	5	5	4	0
AB	6	7	1	2	0	0
AC	4	1	2	0	0	0
BC	2	2	0	1	0	0
ABC	5	4	2	1	0	0
S	40	31	24	19	10	4

these quantities the numbers occuring in formulae (4) and (5) are easily found; e. g., $S=A+B+C+AB+AC+BC+ABC$; $n_1=A+AB+AC+ABC$; $m_{12}=AB+ABC$. The numbers A, B and C have been smoothed to avoid the decimal equation.

The equations for the determination of the unknowns assume the form

$$\chi_\alpha \pi_\beta = \frac{r}{s} \quad \text{(P)},$$

where $\alpha=0, 1, 2, \ldots 5$, and $\beta=1, 2, \ldots 9$; r and s are the abbreviated designations of the sums occurring in formula (4^{I}). The weight of each equation was assumed equal to the denominator s, representing the number of observations on which the value of $p=\chi\pi$ is based. The equations themselves are contained in table 6.

Table 6. System of Equations (P).

Magn.		≥ 2.7	3.0	3.5	4.0	4.5	5.0
	Observer	($\chi_0=1$)					
Part of Region K	A	$\pi_1=\frac{88}{106}$	$\pi_1\chi_1=\frac{52}{76}$	$\pi_1\chi_2=\frac{46}{86}$	$\pi_1\chi_3=\frac{18}{62}$	$\pi_1\chi_4=\frac{3}{30}$	$\pi_1\chi_5=\frac{0}{19}$
	B	$\pi_2=\frac{84}{114}$	$\pi_2\chi_1=\frac{52}{74}$	$\pi_2\chi_2=\frac{46}{81}$	$\pi_2\chi_3=\frac{22}{56}$	$\pi_2\chi_4=\frac{4}{29}$	$\pi_2\chi_5=\frac{3}{18}$
	C	$\pi_3=\frac{84}{112}$	$\pi_3\chi_1=\frac{50}{78}$	$\pi_3\chi_2=\frac{42}{89}$	$\pi_3\chi_3=\frac{18}{60}$	$\pi_3\chi_4=\frac{5}{31}$	$\pi_3\chi_5=\frac{3}{25}$
L	A	$\pi_4=\frac{44}{59}$	$\pi_4\chi_1=\frac{16}{25}$	$\pi_4\chi_2=\frac{13}{33}$	$\pi_4\chi_3=\frac{9}{27}$	$\pi_4\chi_4=\frac{3}{20}$	$\pi_4\chi_5=\frac{0}{5}$
	B	$\pi_5=\frac{45}{67}$	$\pi_5\chi_1=\frac{15}{31}$	$\pi_5\chi_2=\frac{14}{34}$	$\pi_5\chi_3=\frac{9}{28}$	$\pi_5\chi_4=\frac{2}{17}$	$\pi_5\chi_5=\frac{0}{2}$
	C	$\pi_6=\frac{45}{66}$	$\pi_6\chi_1=\frac{15}{30}$	$\pi_6\chi_2=\frac{15}{29}$	$\pi_6\chi_3=\frac{10}{23}$	$\pi_6\chi_4=\frac{3}{15}$	$\pi_6\chi_5=\frac{0}{5}$
M	A	$\pi_7=\frac{18}{28}$	$\pi_7\chi_1=\frac{16}{28}$	$\pi_7\chi_2=\frac{7}{18}$	$\pi_7\chi_3=\frac{4}{16}$	$\pi_7\chi_4=\frac{0}{8}$	$\pi_7\chi_5=\frac{0}{4}$
	B	$\pi_8=\frac{16}{44}$	$\pi_8\chi_1=\frac{17}{32}$	$\pi_8\chi_2=\frac{5}{22}$	$\pi_8\chi_3=\frac{5}{15}$	$\pi_8\chi_4=\frac{0}{6}$	$\pi_8\chi_5=\frac{0}{0}$
	C	$\pi_9=\frac{14}{46}$	$\pi_9\chi_1=\frac{11}{38}$	$\pi_9\chi_2=\frac{6}{22}$	$\pi_9\chi_3=\frac{3}{17}$	$\pi_9\chi_4=\frac{0}{6}$	$\pi_9\chi_5=\frac{0}{4}$

The system of equations (P) may be solved in the following way. If approximate values of all π_β are known, the next approximation of the χ is given by the equation

$$\chi_\alpha = \frac{\Sigma r}{\Sigma \pi_\beta s} \quad \Big| \quad \alpha = \text{const.},$$

where the sum is formed only from those equations (P), which contain χ_α; similarly the π are determined:

$$\pi_\beta = \frac{\Sigma r}{\Sigma \chi_\alpha s} \quad \Big| \quad \beta = \text{const.}$$

If the approximation is good, differential formulae may be applied:

$$\triangle \chi_\alpha = -\chi_\alpha \frac{\Sigma \triangle \pi_\beta \cdot s}{\Sigma \pi_\beta \cdot s}$$

$$\triangle \pi_\beta = -\pi_\beta \frac{\Sigma \triangle \chi_\alpha \cdot s}{\Sigma \chi_\alpha \cdot s},$$

where $\triangle \pi_\beta$ and $\triangle x_\alpha$ mean small corrections of the quantities π_β and χ_α.

The first approximation of the values of π is given directly in the 1-st column of table 6; the successive approximations were:

π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8	π_9	χ_1	χ_2	χ_3	χ_4	χ_5
1-st Approximation													
0.83	0.74	0.75	0.75	0.67	0.68	0.64	0.36	0.300	0.880	0.666	0.460	0.174	0.100
2-st Approximation													
0.778	0.793	0.734	0.712	0.639	0.702	0.610	0.454	0.329	0.878	0.669	0.462	0.175	0.101

Thus the second approximation gives no sensible change in the x; definitively the following values of the Magnitude Function were assumed:

Magnitude Syst. D	$= \leqslant 2.7$	3.0	3.5	4.0	4.5	5.0	(5.5)
χ	$= 1.000$	0.878	0.669	0.462	0.175	0.101	(0.)
Probable error	$=$ —	± 0.014	± 0.020	± 0.024	± 0.025	± 0.027	—

Thus an ideal observer (Coefficient of Attention = 1) records 46% of meteors of the 4-th magnitude and only 10% of the 5-th magnitude!

The values of χ are plotted on fig. 1; the curve resembles two straight lines crossing at $m = 2.7$, $\chi = 1$. Deviations from the straight line occur at $m = 4.5$ and 5.0, but their reality is questionable.

The fact that three observers revealed identical curves of χ is probably not fortuitous; at bottom there must exist some general psycho-physical law.

The values of χ having been found the Coefficients of Attention (π) were determined for each Section separately. They were:

Section-	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
Observer	Coefficient of Attention												
A	0.58 ±0.05	0.53 ±0.08	0.69 ±0.08	0.80 ±0.04	0.85 ±0.03	0.61 ±0.06	0.70 ±0.06	0.78 ±0.05	0.78 ±0.05	0.49 ±0.09	0.63 ±0.09	0.62 ±0.08	0.80 ±0.03
B	0.60 ±0.05	0.66 ±0.08	0.71 ±0.09	0.91 ±0.04	0.92 ±0.02	0.52 ±0.07	0.55 ±0.06	0.76 ±0.05	0.75 ±0.05	0.31 ±0.07	0.44 ±0.08	0.54 ±0.08	0.62 ±0.04
C	0.64 ±0.05	0.61 ±0.09	0.73 ±0.08	0.65 ±0.05	0.81 ±0.03	0.67 ±0.06	0.56 ±0.06	0.82 ±0.04	0.50 ±0.05	0.22 ±0.06	0.28 ±0.07	0.24 ±0.06	0.69 ±0.04

The general feature of these quantities is their maximum value near the centre and the decrease towards the boundaries of the region.

The Magnitude Function depends upon the apparent magnitude or the absolute quantity of light received by the observer's eye; but the magnitudes recorded are not purely apparent: thanks to the differential method of estimation they were freed from the effect of absorption. Thus variations of atmospherical transparency would produce a shift of the whole curve on fig. 1 along the horizontal axis; the shift was employed to determine the differential absorption on various days. According to the atmospherical conditions the days were divided into two groups: 1) August 9-th and 10-th of normal transparency; mean magnitude of faintest stars visible in the region 5.7 (observer *D*); 2) August 11-th to 13-th, with intensified absorption due probably to the dust from the desert; mean magnitude of faintest stars $= 5.3$. The Magnitude Function was determined for both periods separately; the following shifts, interpreted as variations of absorption, were obtained:

Magnitude	Shift (Deviation from the Mean) St. Magn. Aug. 9 and 10	Aug. 11 to 13
3.5	-0.14 ± 0.11	$+0.04 \pm 0.07$
4.0	-0.04 ± 0.12	$+0.02 \pm 0.10$
4.5	-0.41 ± 0.18	$+0.60 \pm 0.21$
Weighted mean	-0.14 ± 0.07	$+0.07 \pm 0.06$

The total difference is $0.07 + 0.14 = 0.21 \pm 0.09$ st. magnitudes. The mean Secant of the zenith distance of the region was $= 1.4$; thus the difference of absorption-coefficients results as 0.15 st. mg. Assuming for Aug. 9-th and 10-th an absorption-coefficient $= 0.18$ st. mg. (elevation of Tashkent above sea level 440 m.) for the days from Aug. 11-th to Aug. 13-th we obtain $0{,}18 + 0{,}15 = 0{,}33$ st. mg. These values were assumed in the reduction; they produce an essential change in the Coefficients of Perception; it was assumed, that the curve of fig. 1 corresponds to a mean absorption-coefficient $= 0{,}28$ st. magn. per unit of air mass.

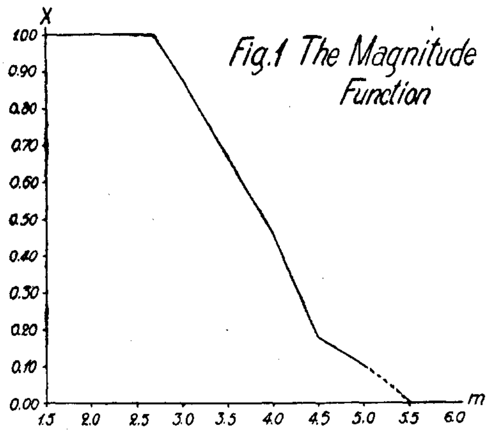


Fig. 1 The Magnitude Function

The transition from observed to probable meteor numbers is executed through form. (5); table 7 contains the factor

$$z = \frac{1}{1 - (1 - p_1)(1 - p_2)(1 - p_3)}$$

for our three observers, which might be called the „Extrapolation Factor"; owing to the unequal transparency of air the values were computed separately for the two groups of days.

Table 7. Values of z.

Section	Aug. 9-th to Aug. 10-th Magnitude						Aug. 11-th to Aug. 13-th Magnitude					
	$\leqslant 2.7$	3.0	3.5	4.0	4.5	5.0	$\leqslant 2.7$	3.0	3.5	4.0	4.5	5.0
I	1.06	1.09	1.21	1.47	2.56	4.85	1.06	1.13	1.31	1.68	3.65	6.71
II	1.07	1.09	1.21	1.48	2.59	4.85	1.07	1.13	1.31	1.69	3.67	6.71
III	1.03	1.04	1.13	1.34	2.25	4.24	1.03	1.07	1.20	1.50	3.21	5.69
IV	1.01	1.02	1.08	1.26	2.08	3.82	1.01	1.04	1.14	1.40	2.90	5.13
V	1.00	1.01	1.05	1.20	1.94	3.55	1.00	1.02	1.10	1.33	2.70	4.79
VI	1.07	1.09	1.21	1.48	2.59	4.85	1.07	1.13	1.31	1.69	3.67	6.71
VII	1.06	1.09	1.21	1.47	2.56	4.85	1.06	1.13	1.31	1.68	3.65	6.71
VIII	1.01	1.02	1.08	1.26	2.08	3.82	1.01	1.04	1.14	1.40	2.90	5.13
IX	1.03	1.04	1.14	1.37	2.34	4.37	1.03	1.08	1.22	1.55	3.31	5.96
X	1.38	1.44	1.72	2.24	4.28	8.26	1.38	1.54	1.90	2.68	6.25	11.6
XI	1.18	1.22	1.42	1.80	3.32	6.43	1.18	1.29	1.55	2.09	4.81	8.91
XII	1.15	1.19	1.38	1.75	3.19	6.17	1.15	1.26	1.51	2.02	4.61	8.55
XIII	1.03	1.04	1.13	1.35	2.28	4.28	1.03	1.07	1.21	1.52	3.24	5.76

§ 4. Further treatment required the separation of Perseids from meteors not belonging to that stream; the term „Perseids“ means here meteors having their Radiant near η Persei — say not more than some 5^0 distant from the mean Radiant of the great August shower. Even an exact position of the meteoric trail, observed from a single point, solves the problem only with some degree of probability; it is clear that the unexact records of position and direction employed increased the vagueness of the solution. According to the accuracy of the observations, meteors whose direction varied within 45^0 were considered as parallel; if the direction was in accord with the Radiant of the Perseids, the meteor was called an „Apparent Perseid“; let the ratio of Apparent Perseids to the total number of meteors be P', of true Perseids — P; the fraction $\frac{45^0}{360^0}=\frac{1}{8}$ of meteors not belonging to the stream is contained among the Apparent Perseids; on the contrary, a fraction e of the true Perseids is lost, e being the frequency of errors in direction greater than 45^0; thus we may write the following equation between these effective quantities:

$$P' = P + \frac{1}{8}(1-P) - Pe, \quad \text{whence}$$

$$P = \frac{P' - \frac{1}{8}}{\frac{7}{8} - e} \quad (11).$$

For different groups of meteors the following data were obtained:

	Common Meteors	Meteors Observed by a Single Person		
		A	*B*	*C*
P'	0.820 ± 0.017	0.721 ± 0.027	0.624 ± 0.032	0.654 ± 0.032
e	0.00	0.03	0.17	0.23
P	0.794	0.705	0.708	0.820

The values of e for *A*, *B* and *C* are those already found above from comparison with the map (§ 2); the records for the Common meteors controlling one another, it was assumed $e = o$ for this group.

The data obtained above are a little discordant; there seemed to exist some difference between the bright and faint meteors, the Perseids being more frequent among the former. According to the values of e the data were distributed in two groups: 1) the Common meteors and meteors recorded by the observer *A*, with a mean $e = 0{,}01$; 2) the meteors observed by a single observer *B* or *C*. For the first group the values of P were computed separately for faint and bright meteors; with the aid of these quantities the corresponding values of the error-frequency e for the second group were found [from equation (11), P and P' given]. The result is given in the following table.

	First Group		Second Group			Second Group from Comparison with the Map.
	P'	P	P'	e Computed		e
Fainter than 3.7 mg	0.670 ± 0.033	0.630	0.631 ± 0.033	0.072 ± 0.040	Fainter than 3.4 mg	0.150 ± 0.040
Brighter than 3.8 mg	0.824 ± 0.017	0.808	0.655 ± 0.032	0.219 ± 0.032	Brighter than 3.5 mg	0.200 ± 0.025
All Magnitudes	0.786 ± 0.015	0.764	0.639 ± 0.023	0.201 ± 0.028	All Magnitudes	0.182 ± 0.030

The values of e for the second group are obtained by two independent ways: from formula (11) and from comparison with meteors traced upon the map; both values are in accordance within the limits of the probable error; they indicate an i n c r e a s e of the frequency of errors with increasing brightness.

Other arguments except the brightness did not influence the value of e; e. g., if we treat Aug. 11-th — the day of maximum intensity of the shower — separately, we obtain an insensible difference:

Days:	Mean e (Second Group)	
Aug. 9-th, 10-th, 12-th and 13-th	0.180 ± 0.025	All Magnitudes
Aug. 11-th	0.200 ± 0.025	

The strange improvement of the observations for faint meteors may be explained in the following way: bright meteors are frequently observed under bad conditions of seeing (lateral sight) and attention; whilst the fainter meteors are recorded only if the attention is fixed upon them and the seeing direct. That is, however, only a hypothesis.

The difference of the e between faint and bright meteors as determined by comparison with the map would probably be greater but for faint meteors almost not being traced upon the chart; thus greater weight was attributed to the smaller value of e (0.072) and finally the following mean values of frequency of errors were assumed for the observers B and C:

meteors fainter than 3.7 magn. $e = 0{,}10 \pm 0.03$
„ brighter than 3.8 „ $e = 0{,}21 \pm 0{,}02$.

Because of these errors a considerable proportion of meteors recorded by one of the above mentioned observers as Non-Perseids will belong to the stream; this proportion is approximately equal to $\frac{P - P'}{1 - P'}$; thus each meteor, recorded by a single observer B or C alone, as Non-Perseid, was assumed equal to the following equivalent quantities, computed with the values of e found above.

One meteor, recorded as Non-Perseid, is equivalent to:

	Perseids	Non-Perseids
If Brighter than 3.8 mg	0,45	0,55
If Fainter than 3.7 mg	0,06	0,94

In other cases (meteors recorded as Perseids and all of the first group) no corrections were applied owing to the smallness of the change produced.

In a few cases where the direction was omitted, the record of one meteor was assumed equal to the following effective quantities:

	If Brighter than 3.7		If Fainter than 3.8	
	Perseids	Non-Perseids	Perseids	Non-Perseids
Aug. 9-th, 10-th, 12-th and 13-th	0.7	0.3	0.5	0.5
Aug. 11-th	0.9	0.1	0.7	0.3

§ 5. The effective numbers were arranged in a table, separately for each Section and Interval of observation. For example a part of this table is here given:

Table of Effective Numbers.

Aug. 11-th 2-nd Interval

	Sections:															
	I $s = 529\, z^0 \square$						II $s = 272 z^0 \square$		III $s = 251\, z^0 \square$			IV $s = 364\, z^0 \square$				
$m =$	1.0	1.5	3.5	4.0	4.5	5.0	3.0	3.5	3.0	3.5	4.0	3.0	3.5	4.0	4.5	5.0
$m_0 =$	0.8	1.3	3.3	3.8	4.3	4.8	3.0	3.5	2.8	3.3	3.8	2.8	3.3	3.8	4.3	4.8
$S =$	0.90	0.10	4.26	1.89	0.60	0.25	1.00	1.00	0.50	1.00	0.50	1.25	2.63	2.07	0.05	1.00
$P =$	0.90	0.10	3.71	1.89	0.60	0.25	1.00	0.06	0.36	0.73	0.36	1.23	2.46	0.28	0.00	1.00
$N = Sz =$	0.95	0.11	5.60	3.17	2.19	1.68	1.13	3.67	0.54	1.20	0.75	1.30	3.00	2.90	0.14	5.13
$R = Pz =$	0.95	0.11	4.38	3.17	2.19	1.68	1.13	0.21	0.39	0.88	0.54	1.28	2.80	0.39	0.00	5.13

etc.

The designations mean: 1) m — the mean apparent magnitude reduced to system D and rounded off to 0.5; 2) m_0 — the Zenithal Magnitude; 3) S — the number of different meteors observed; fractions are introduced by a correction for the decimal equation (see Table 3); the sum of S for each Section is an even number and represents the number of really observed objects; 4) P — the effective observed number of Perseids; 5) N — the Probable True Number of all meteors; 6) R — the Probable True Number of Perseids alone; 7) at the head s represents the Effective Area of the corresponding Section for the given Interval in Zenithal Square Degrees.

The separate Intervals of observation during the five consecutive nights were:

Date and Interval

	August														
	the 9-th			the 10-th			the 11-th			the 12-th			the 13-th		
	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
Middle of Interval (Tashkent M. T.)	$12^h 48^m$	13 56	14 52	12 20	13 32	14 32	12 27	13 32	14 32	12 26	13 31	14 31	12 27	13 32	14 32
Duration Minutes	57.3	58.6	30.3	72.7	49.1	48.7	60.0	49.9	49.7	59.9	49.7	49.6	60.0	50.1	49.4

The Effective Area of the Sections was computed for the zenith distance of the centre of the Section at the middle of the corresponding Interval according to formula (8), where the infinitesimal quantities ds and $d\sigma$ were substituted by the Effective Area s and the solid angle σ of the Section (from table 4); to form an opinion as to the order of the quantities, areas for August 11-th are here given. For the other days the data were very similar.

August 11-th
Effective Areas . z^0 □.

Section	Interval		
	I	II	III
I	632	529	487
II	301	272	268
III	330	251	209
IV	554	364	292
V	1 737	856	576
VI	344	267	244
VII	6 700	2 015	973
VIII	1 923	893	531
IX	894	556	402
X	29 700	6 110	2 240
XI	3 460	1 368	718
XII	918	633	482
XIII	1 811	1 075	820

These Effective Areas suited their purpose well except at very great zenith distances (e. g. Section X) where some vagueness appeared.

The Effective Numbers found allowed of answering some questions as to the photometrical meaning of the magnitudes recorded; namely, there might be expected systematical influences from two sources: 1) the magnitude of a fixed star measures its intensity of light — per unit of time and area; but has the estimated magnitude of the moving meteor the same meaning? There might be expected an underestimation of the brightness, due to the motion of the meteor: its action is distributed over a greater area upon the retina than the light of a fixed star; the diminution of apparent brightness at great angular velocities is easily observed in fixed stars with the aid of a telescope moved by hand. The question arises whether the said „Effect of Motion" is sensible enough for angular velocities of the order of 10^0—20^0 per second;

2) the other effect, which we will call the „Phase-Effect", is a suspected dependence of the apparent brightness of meteors from their angular distance from the Radiant; this distance, equal to the angle between the line of sight and the direction of the path, we will call the Phase-Angle.

To investigate both effects it was necessary to invent some method of photometrical comparison of meteors at variable conditions: distance and Phase-Angle. The method of simultaneous magnitude-estimations from different observing-points would practically lead to no result, systematical differences being too great and the number of common meteors too small. Thus there was no other way but to make use of the fact that meteors observed in various parts of the region differred in distance and Phase-Angle. A statistical method, called the „Method of Equivalent Groups", was applied. Let us imagine that the shower is homogeneous, so that the probable numbers of meteors physically similar, falling upon equal areas, are equal. Let N and N_1 be the number of meteors falling upon areas of the size s and s_1 respectively; the density of the shower $\frac{N}{s}$ is a function of the magnitude m; thus $\frac{N}{s} = f(m)$ and $\frac{N_1}{s_1} = f_1(m)$; for a homogeneous shower both functions will be similar, with a certain constant shift in the magnitudes $= x$, due to the different distances and Phase-Angles, so that we may write:

$$f(m) = f_1(m + x) \quad (12).$$

The value of x can be found graphically; if the $f(m)$ and

$f_1(m)$ are plotted as ordinates with the magnitudes as abscissae (fig. 2), then the mean shift of the curves (AB, $A'B'$) will give the difference of magnitudes. The function $f(m)$ is analogous to the Luminosity Curve for stars, and the problem of photometrical comparison of different meteoric groups resembles the method used for the estimation of the probable distance of star-clusters from the apparent magnitudes of physically similar stars.

Practically it seemed advisable to use instead of the function $f(m)$ a function $F(m) = \sum_{m=-\infty}^{m} f(m)$: this is the total number of meteors from the brightest to the given magnitude m. The relation (12) remains unaltered, with the substitution of F instead of f. The values of x were computed numerically with the aid of interpolation.

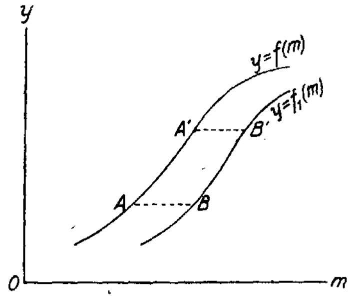


Fig. 2

The question arises as to the homogeneity of the shower; it must be pointed out that the homogeneity here required need not be absolute: a transversal uniformity will be sufficient. The meteors falling upon two different Sections of the region during the 15 hours of observation are contained in the volume of two cylinders with a basis of some thousands square kilometers and length of nearly 3 millions kilometers, whose axes are only 100—300 kilometers distant from one another: the density of the meteoric matter within volumes separated by such a small distance may be subjected only to casual, but not to systematical variations. — The accuracy of the method depends upon the number of meteors actually observed; from the probable error of the number N (see formula (5^{I})) the probable error of the determined magnitude-difference can be found.

The presupposed Effect of Motion may be revealed in the following way. With reliable assumptions as to the properties of our eye the apparent brightness of a moving object may be represented as $i = \frac{c}{d(k+d)}$, where d is the distance from the observer, and k — a constant proportional to the angular velocity at unit distance; for $k = o$, $i = \frac{c}{d^2}$; if k is great in comparison to d, $i=\frac{e}{kd}$. Thus the apparent brightness varies as d^{-a}, a being an effective exponent, varying between 2 and 1. If m and m_1 are the observed apparent magnitudes, d and d_1 — the distances, then

$$a=\frac{0{,}4\,(m-m_1)}{lgd-lgd_1}.$$

A value of a less than 2 will indicate a sensible Effect of Motion. For meteors d is proportional to $F\,Sec\,z$, this quantity being taken from table (1).

Observations of the first Interval of all 5 days were combined, the early hour revealing the greatest differences of $Sec\,z$; Sections with nearly equal zenith-distance were joined into groups (Sections IV, X and XI have not been included because of the small number of meteors observed); for these groups, with the aid of the N from the Table of Effective Numbers, the following data were obtained:

Group a = Sections I + II + III + VI
mean $Cos\,z = 0{,}877$; $\bar{\varrho} = 30^0$

m =	$\leqslant 0.7$	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$F(m)$ =	0.0	6.1	6.7	43.7	121.4	185.6	283.1	368.2

Group b = Sections V + IX + XII + XIII
mean $Cos\,z = 0{,}622$; $\bar{\varrho} = 24^0$

m =	$\leqslant 0.7$	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$F(m)$ =	0.0	3.6	9.1	24.4	39.1	83.2	141.5	—

Section VIII
Mean $Cos\,z = 0{,}503$; $\varrho = 18^0$

m =	$\leqslant 0.7$	1.0	1.5	2.0	2.5	3.0	3.5
$F(m)$ =	0.0	4.8	4.8	15.3	18.6	35.7	59.5

Section VII
Mean $Cos\,z = 0{,}335$; $\varrho = 33^0$

m =	$\leqslant 0.7$	1.0	1.5	2.0	2.5	3.0	3.5
$F(m)$ =	0.0	0.0	3.6	5.3	6.3	15.4	—

Here the values of $F(m)$ are reduced to a standard area of $10\,000\,z^0$ □. For purposes of the determination of the magnitude-difference only the values based upon more than 5 meteors actually observed were used; these values are surrounded by double lines. The data are closed at a magnitude equivalent to the magnitude 4.2 at the zenith.

The single differences as determined by the Equivalent-Group Method, and the corresponding values of a are given below; as a first approximation the Phase-Effect has been neglected.

Groups	m	m_b	$m - m_b$ Obs.	Absorption	$m - m_b$ Corr.	a
$a - b$	2.56	3.50	−0.94	0.00	−0.94	2.6
"	2.50	3.33	−0.83	"	−0.83	2.3
"	2.00	2.55	−0.55	"	−0.55	1.5
VIII − b	3.00	2.38	+0.62	"	+0.62	2.1
"	3.50	2.73	+0.77	"	+0.77	2.6
VII − b	3.00	1.71	+1.29	0.29	+1.00	1.6
						Mean 2.12 ± 0.13

All groups were compared with group b; m and m_b denote the magnitudes with equal values of $F(m)$; the differential correction for atmospherical absoption was applied only to Section VII, where no comparison-stars were available. The mean value of a obtained deviates from 2 in the sense opposite to the expected and within limits of the probable error. Thus the Effect of Motion is quite insensible for the angular velocities of the meteors observed, and the legitimacy of formula (6) is established.

Taking into account the atmospherical absorption, the transition from the apparent to the Zenithal Magnitude is executed through the formula:

$$m_0 = m - 5\, lg\,(F\, Sec\, z) - a\, (Sec\, z - Sec\, z') \quad (13),$$

where z and z' are the zenith distances of the middle of the meteor's trail and the star of comparison, a — the coefficient of absorption of the corresponding day.

The absorption was sensible only for some Sections where no comparison-stars were available, namely:

Section	Section of Comparison-Stars
II	IV
VII	V and VIII
X (fainter than 2.5)	VIII and IX
XI	VIII and IX

In search of the Phase-Effect the Sections were combined into groups according to the Phase-Angle ϱ; these groups were:

Group	Sections	Mean ϱ
a	IX, XII	8^0
b	I, III, VIII, XI	20
c	II, IV, V, VII	31
d	VI, XIII	46
$M = a + b + c + d$	(All Meteors)	—

Section X was omitted because of its great zenith distance.

For each of the three Intervals separately the observations of the 5 nights were combined and the deviations of the Zenithal Magnitude for each group from the mean of all meteors (group M) was found with the aid of the Method of Equivalent Groups. These deviations, designed $\triangle a$, $\triangle b$, $\triangle c$ and $\triangle d$, with their probable errors determined a priori, are given below:

Deviations of Zenithal Magnitude Depending Upon Phase-Angle.

	Stellar Magnitudes			
	$\triangle a$	$\triangle b$	$\triangle c$	$\triangle d$
1-st Interval	-0.05 ± 0.08	$+0.20 \pm 0.07$	-0.15 ± 0.07	-0.08 ± 0.07
2-nd Interval	$+0.43 \pm 0.10$	-0.20 ± 0.07	$+0.25 + 0.07$	-0.31 ± 0.09
3-d Interval	-0.22 ± 0.12	$+0.03 \pm 0.09$	$+0.14 + 0.09$	-0.02 ± 0.10
Mean	$+0.06 \pm 0.06$	$+0.01 \pm 0.04$	$+0.07 \pm 0.04$	-0.13 ± 0.05

The mean deviations indicate no sensible systematical change; assuming for the deviation the form $\triangle m = k \, Sin \, \varrho + c$, the result gives for the coefficient of Phase-Effect k the value $+0{,}03$, which is practically zero Thus we arrive at the conclusion, that the meteor radiates uniformly in all directions; owing to the fact that systematical irregularities of form must doubtless occur, it seems that the following explanation will account for the observed phenomenon; during the motion through the upper atmospherical layers the incandescent body of the meteor evaporates, producing a gaseous shell or a sort of an „instantaneous microatmosphere“; this shell, extremely rare and diaphane, thanks to its relatively great dimensions emits the greatest portion of the observed radiation; because of its diaphanity the shell radiates as a whole, the radiation of every particle passing through the shell without sensible absorption; in this case, whatever the form of the shell might be, its radiation will be independent of the direction.

Remark. The word „shell“ is here used for lack of an appropriate term; the gases are without doubt left behind, but that does not prevent them from producing a combined effect upon our eye.

§ 6. In the following lines we shall propose some general considerations as to the nature of meteors, especially their masses. Only meteors having a common Radiant were considered; all numerical data refer to the Perseids. It must be pointed out, that the computations given below are only approximate, executed with the chief purpose of determining the general features of the phenomenon; the estimation of absolute masses pretends only to the order of the quantities.

The radiation of the meteor has its origin in the energy of motion; as the result of the resistance of the medium the Kinetic Energy of the meteor is converted into other forms of energy: radiation, heat, ionization, mechanical energy (movements of the atmosphere); only the visible radiation is within the reach of our rough estimations. If we make the plausible assumption, that the energy converted into aether-waves will form a constant fraction of the whole amount, a means for the estimation of relative masses will be available.

Let the mass of the meteor be μ, the total amount of the radiative energy lost j; then we have $\mu \backsim j$. If i_0 is the mean Zenithal Brightness, τ the duration of visibility, then, assuming as a first approximation a constant mean height of the meteors, we may put $j \backsim i_0 \tau$; with a similar degree of approximation τ may be assumed proportional to the absolute length of the path L; thus finally we obtain

$$\mu \backsim i_0 L \quad \text{(a).}$$

The length of the path will be a function of the mass or the luminosity. The general character of this function may be determined as follows. The duration of visibility of the meteor depends upon the speed of evaporation of its matter; if a uniform evaporation is assumed, the length of the path will be proportional to the radius of the meteor or

$$L \backsim \mu^{\frac{1}{3}} \quad (b);$$

substituting μ from (a) we obtain

$$L \backsim i_0^{\frac{1}{2}} \quad (c);$$

we will generally put

$$L \backsim i_0^{x} \quad (c^{I}); \text{ for (c) } x = \frac{1}{2}.$$

While the meteor gradually penetrates into the denser layers, the evaporation must go on with acceleration, and the exponent x will be less than the value found above; on the other hand, at $x = 0$ the length of the path will be independent of the size of the meteor — but that contradicts observational evidences; thus we have

$$0 < x < \frac{1}{2}.$$

A provisional value of x was determined from the data available. For 81 Perseids[1]) traced upon the map by M-r Davidovitsch the length of the path was computed with the aid of an average assumed height of the visible trail $H = 100$ km. The relation (c^{I}) is equivalent to the equation

$$lg\, L = -0{,}4\, m_0 x + c \quad (14);$$

plotting the Zenithal Magnitudes as abscissae with the $lg\, L$ as ordinates, we may obtain x from the inclination of the straight line which suits the plotted points best. The average lengths of path obtained for various Zenithal Magnitudes are given below.

Zen. Magn.		Average L kilom.	$lg\ L$	n
Interval	Average			
$\leqslant 0.7$	0.0	90.0	1.954	8
0.8 — 1.7	1.2	57.7	1.761	20
1.8 — 2.7	2.2	51.7	1.714	36
2.8 — 3.7	3.2	39.0	1.591	17

Fig. 3 represents these data graphically; the inclination of the straight line gives

$$x = 0{,}28.$$

1) To obtain homogeneous data only meteors agreeing well with the Radiant of the day were included.

Combining (a) and (c^I), we have $\mu \backsim i_0^{1+x}$, or, with the value found,

$$\mu = c\, i_0^{1{,}28} \quad (15).$$

The factor of proportionality C remains unknown.

Table (8) contains the Zenithal Luminosities (i_0) and the corresponding relative masses (μ) of the Perseids, those for $m_0 = 2.0$ assumed as the unit. The formulae for computation were:

$$\left.\begin{aligned} lg\, i_0 &= -0{,}4\,(m_0 - 2) \\ lg\, \mu &= -0{,}512\,(m_0 - 2) \end{aligned}\right\} (16).$$

Estimations of the order of meteorical masses have been made several times; the ordinary criterion was the apparent brightness, in connection with a certain assumption as to the effective temperature of the meteor; from these data the apparent radius and the volume were computed. But the chief assumption on which such estimations are based — that the light is emitted by a solid body — is doubtful; if the effect of the radiating shell, mentioned in the previous paragraph, is great, then the values of the masses so obtained will have been over-estimated.

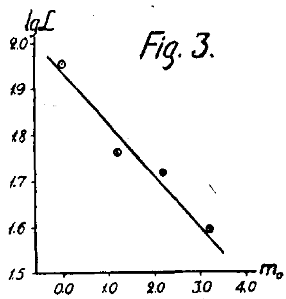


Fig. 3.

Table 8.

m_0	i_0	μ	m_0	i_0	μ
5.0	0.06	0.029	0.5	3.98	5.9
4.5	0.10	0.052	0.0	6.31	10.6
4.0	0.16	0.095	—0.5	10.0	19.0
3.5	0.25	0.17	—1.0	15.8	34.4
3.0	0.40	0.31	—1.5	25.1	61.9
2.5	0.63	0.55	—2.0	39.8	112.
2.0	1.00	1.00	—2.5	63.1	201.
1.5	1.58	1.80	—3.0	100.0	363.
1.0	2.51	3.25	—3.5	158.	655.

Let the radiation due to the solid nucleus of the meteor be i_1, the radiation of the gaseous shell i_2, the total amount being $i_1 + i_2 = i$. Then we have $i_1 < i$ and thus all computations based on the assumption $i_1 = i$ give an upper limit of the mass.

The question arises as to the effective temperature. Roughly estimated it is of the same order as the sun's. In this case the colour of the meteors may give us some information. That 3000—4000° is too low, seems without doubt: the corresponding reddish hue does not occur in the case of the Perseids; these quickly moving meteors are for the most part white, sometimes with a greenish coloration.

Our observers recorded the colour of the brighter meteors in an arbitrary scale ranging from 1 to 8, 1 meaning bluish-white, 8 — reddish-orange; the records were made in words, afterwards converted into scale numbers. Systematical differences determined from the Common meteors were:

$$A - D = +0.6$$
$$B - D = +1.6$$
$$C - D = +2.2$$

With these differences the colour-estimations — of the Perseids only — were reduced to the scale of D; the result was:

Observer	Colour of the Perseids		n
	Recorded	Reduced	
A	2.7	2.1	96
B	3.6	2.0	116
C	4.4	2.2	53
D	1.7	1.7	65
		Mean 2.0	

As standard for the scale-number 2.0 observer D chose α Persei (type F_5); thus these colour-estimations indicate an effective temperature of about 7000°.

Taking the magnitude of the sun as —26.7 the following data for a Perseid-meteor of the 2-nd Zen. Magnitude were obtained (H assumed $= 100$ km);

Temperature	Diameter	Maximum mass (Density = 4)
6000°	1,7 mm	10,4 milligram
7000°	1,3 „	4,4 „

To evaluate the minimum mass there is at our disposal a method more reliable. The total amount of energy lost by radiation — j — cannot exceed the Kinetic Energy of the meteor; thus we obtain

$$\frac{\mu v^2}{2} > j, \text{ or } \mu > \frac{j}{2 v^2} \quad \text{(e)},$$

where v is the cosmical velocity. To compute j, the duration of visibility τ is needed; assuming a mean velocity within the atmosphere equal to the cosmical (56 km/sec), and taking the length of the path for a 2-nd Zen. Mg. meteor from fig. 3 as $L = 51{,}5$ km, we obtain $\tau = 0{,}92$ seconds. The reason why the cosmical velocity was adopted will be explained later on.

The following assumptions were next made: 1) the effective temperature of the meteor — from 6000^0 to 7000^0 (the difference does not matter); 2) stellar magnitude of the sun $= -26.7$; 3) mean height of the trail $= 100$ km.

With these data it is easy to obtain:

1) the total amount of radiation emitted $j = 139$ gr. calories and, 2) the energy of motion for $v = 56$ km/sec being 376 000 gr. calories per gram of mass,

the minimum mass as about 0,3 milligram.

The assumed temperature has only a small influence upon the result; the difference can only be produced by the variation of the ratio of visual to total radiation, this ratio at temperatures near 6000^0 varying slowly. For other temperatures we would obtain greater values:

Eff. Temperature	3000^0	12000^0
Minimum Mass Milligr.	1.7	0.6

Taking into account the rapid increase of emission with increase of temperature it seems probable, that a considerable — if not the greatest part of the energy is lost through radiation, and the computed minimum mass will thus represent the best approximation to the true value. Taking roundly 1 milligram as the mass of a Perseid of the Second Zenithal Magnitude, we shall probably not be far from the truth. At a temperature of 7000^0 it means, that 0.37 of the radiation is due to the solid (or fluid) body and 0,63 — to the gaseous particles carried back.

For the computation of the duration of visibility the cosmical velocity was assumed. There were two reasons:

1) the value of τ found (0.^{s}92) is considerable enough; if the velocity had been lower, we should obtain an incredibly great value of τ: everyone who has observed the Perseids knows, that even 0.^{s}9 seems to be too great. 2) It seems probable that, contrary to the general opinion, the retardation of the meteoric nucleus due to resistance of the medium, is very small; the meteor evaporates before it reaches the denser layers; this may be stated theoretically in the following way: let q be the amount of Kinetic Energy per unit of mass of the nucleus, the mass of the latter being μ, and let $d\mu$ be some loss of the mass due to evaporation; the heat evidently is taken at the expense of the Kinetic Energy; if c is the total heat needed to transform 1 gr. of the meteoric matter into vapour at the given effective temperature, and assuming, that a fraction $= 1 - r$ of the heat originating from friction is lost in the resisting medium, we may write:

$$c\, d\mu = r\, \mu\, dq, \text{ whence } q = \frac{c}{r} \int \frac{d\mu}{\mu};$$

that gives

$$q = q_0 - \frac{c}{r\, lg\, e}\, lg\, \frac{\mu_0}{\mu} \quad (17),$$

where q_0 and μ_0 are the initial energy and mass.

Assuming a mean capacity of heat $= 0{,}20$ (including the latent heat), we obtain for a temperature $= 7000^0$ the value $c = 1400$ calories per gram. The fraction r we will put $= \frac{1}{3}$ (it is probably greater); substituting these data, we have

$$q = q_0 - 10\,000\, lg\, \frac{\mu_0}{\mu} \quad (17^I).$$

If the velocity v is expressed in kilometers per second, the value of q will be given by

$$q = 120\, v^2 \text{ cal./gr.} \quad (18).$$

From formulae (a) and (c^I), previously found, we obtain

$$\mu \backsim L^{\frac{1+x}{x}} \quad (m),$$

L being the total length of path of the mass μ; the empirical value of x is 0,28, hence $\frac{1+x}{x} = 4{,}57$; we shall assume the round number $4{,}5 = \frac{9}{2}$. If r is the radius of the meteor, $\mu \backsim r^3$, and from (m), substituting the assumed value of x, we obtain

$$r \backsim L^{\frac{3}{2}} \quad \text{(n)}.$$

The linear speed of evaporation. measured by the rate of diminution of the radius of the meteoric body, is determined from (n):

$$\frac{dr}{dL} \backsim \frac{3}{2} L^{\frac{1}{2}} \quad \text{(p)}.$$

The previous relations from (a) to (n) being true only for the total length of path of different meteors, (p) may equally be applied to parts of the trajectory of one meteor, if L is regarded as a coordinate determining the absolute depth of the given atmospherical layer, and if the schematizing assumption is made that all meteors grow visible at the same height. Integrating (p) from $L = o$, $r = r_0$ to $L = L_0$, $r = o$ we obtain (n); but integrating from $L = L$, $r = r$ to $L = L_0$, $r = o$, we obtain

$$r \backsim (L_0^{\frac{3}{2}} - L^{\frac{3}{2}}), \text{ or } L \backsim (L_0^{\frac{3}{2}} - r)^{\frac{2}{3}} \quad \text{(q)};$$

but owing to the fact that the velocity throughout the major part of the visible path is practically constant (data are given below), we have $L \backsim \Theta$, where Θ is the time reckoned from the beginning of evaporation; with $r \backsim \mu^{\frac{1}{3}}$ and $L_0^{\frac{3}{2}} \backsim r_0 \backsim \mu_0^{\frac{1}{3}}$ [from (n)], (q) is transformed into

$$\Theta = (\mu_0^{\frac{1}{3}} - \mu^{\frac{1}{3}})^{\frac{2}{3}} \quad \text{(19)}.$$

This formula gives the time Θ in certain units, during which the initial mass μ_0 decreases to the value μ.

From (19) the intensity of evaporation $-\frac{d\mu}{d\Theta}$ is found:

$$J = -\frac{d\mu}{d\Theta} \backsim (\mu_0^{\frac{1}{3}} - \mu^{\frac{1}{3}})^{\frac{1}{3}} \mu^{\frac{2}{3}} \quad \text{(20)}.$$

This quantity determines the hypothetical variation of brightness of the meteor during the flight: the quantities of vaporized matter expand, forming the gaseous shell previously mentioned and, in consequence of the great area of resistance, are instantaneously stopped, transforming their Kinetic Energy into radiation.

From (17^{I}), (19) and (20), with $\mu_0 = 1$, table 9 was computed and the results plotted on fig. 4 with the time Θ as abscissae. For $\mu = 0$, Θ is 1; but theoretically this value is not attained; the velocity is entirely lost before this point is reached; but, computing from (17^{I}) the corresponding mass for $q = o$, we obtain the value $\mu = \frac{1}{4.10^{37}} \mu_0$ ($q_0 = 376\,000$); this value has no physical meaning. For a remaining mass of the order of an atom (17^{I}) gives $v = 30 - 40$ km per second; thus the body of a meteor with a high velocity will maintain its velocity practically unaltered during the whole time it is seen; for a short time there will still remain a nucleus rapidly vanishing but moving with cosmical velocity.

Table 9.

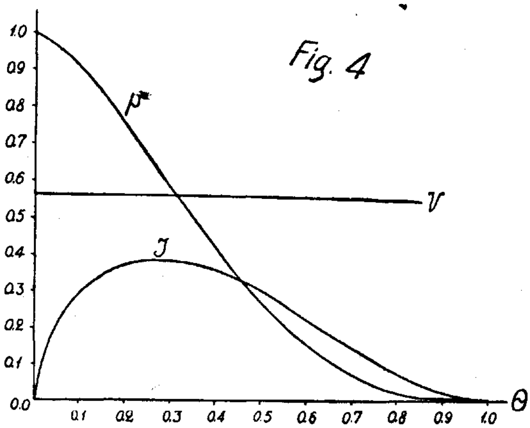
μ	Θ	J	v kilom.
1.0	0.000	0.000	56.0
0.9	0.105	0.302	
0.8	0.171	0.357	
0.6	0.291	0.384	
0.4	0.411	0.348	
0.2	0.557	0.255	
0.1	0.658	0.175	55.2
0.05	0.738	0.116	
0.01	0.852	0.043	54.4
0.00	1.000	0.000	

It may be added, that for smaller initial velocities the result will be different. For $v_0 = 10$ km/sec and $c = 700$ cal. per gram, corresponding to an effective temperature $= 3500^0$ the following data were obtained:

$\mu =$	1.0	0.6	0.1	0.01	0.004
$\Theta =$	0.00	0.29	0.66	0.85	0.89
$v =$	10.0	9.5	7.6	4.1	0.0

The velocity here decreases considerably and the remaining mass $= 0{,}004$ is not negligeable.

§ 7. From the Table of Effective Numbers (see § 5) quantities determining the intensity of the shower were obtained. For 0.5 magn. classes of the Zenithal Magnitude and for each Interval of observation the numbers were summed up and reduced to the standard horizontal area $= 10\,000\ z^0 \square$ (about 30 000 km²) and standard interval $= 1$ hour by the multiplication with $\frac{10\,000.60}{\Sigma s \,.\, t}$, where Σs denotes the sum of the Effective Areas of all Sections employed, t — the duration of the Interval in minutes of time. The number n_h so obtained was called the Standard Horizontal Number of meteors of the given Zenithal Magnitude. Only those Sections were employed where the apparent magnitude of the meteors of the given category was not fainter than 4.3 mg. stars at the Zenith, and thus the faintest category of Zenithal Magnitude was 4.0, including the interval 3.8—4.2. This limit was assumed to avoid great extrapolation factors. The Zenithal Magnitudes (m_0) of the Table of Effective Numbers were not rounded off, but were distributed amongst the neighbouring categories; e. g., for the magnitude 3.8 40% were added to the category 3.5 and 60% — to category 4.0.



The quantity $N_h = \sum_{-\infty}^{m_0} n_h$, called the Horizontal Intensity, gives the total Probable Number of meteors up to a certain limiting magnitude m_0; in the tables given below, $m_0 = 4{,}2$.

For meteors belonging to a definite stream, it is advisable to use the Standard Normal Number n_0 and the Normal Intensity $N_0 = \sum_{-\infty}^{m_0} n_0$, where $n_0 = \frac{n_h}{Cos\, z_r}$, $Cos\, z_r$ being the mean cosinus of the zenith distance of the Radiant during the Interval.

But the Intensity is insufficient to characterize the shower; the luminosity and massiveness of the meteors is of importance.

Therefore the following quantities were introduced:

the Horizontal Horary Luminosity $J_h = \sum_{-\infty}^{m_0} i_0 n_h$ and

the Normal Horary Luminosity $J_0 = \sum_{-\infty}^{m_0} i_0 n_0$,

the i_0 being taken from table 8; the same table gives the relative masses of the Perseids, and thus for meteors belonging to the shower

the Normal Horary Mass $M_0 = \sum_{-\infty}^{m_0} \mu n_0$ was computed.

The reason why these quantities were introduced was, besides, that systematical errors influence them in a different way; the Intensity (N_h or N_0) highly depends upon the numerous faint meteors for which the extrapolation factors are great; the Horary Luminosity and Mass, however, are much influenced by occasional bright meteors, while the probable deviations caused by the faint meteors are negligeable.

At the end of this paper the results are given for every day and Interval of observation; the Perseids and the meteors not belonging to the stream were treated separately, for the former the Normal Numbers, for the latter only the Horizontal Numbers being computed. The probable errors are found according to (5^I). The number of Non-Perseids was taken from the Table of Effective Numbers as the difference $N - R$. $\lambda_\odot$ means the apparent longitude of the sun at the middle of the given Interval.

All statistical data of the preceding investigation were based on the observations of A, B and C. August 14-th the Double-Count was continued by observer C (with an assistant) and D, working alone; the latter, writing down himself the records needed, succeeded to trace upon the map some of the meteors observed the best. The data of this day being too scarce to allow of a detailed treatment, a simplified method of reduction was used. It was assumed: 1) that the Magnitude Function χ remained the same; for the Coefficients of Attention were assumed preliminary values π'_C and π'_D, π'_C being equal to the values found for the previous days for the same observer, and $\pi'_D = \frac{n_D}{n_C}\pi'_C$, where n_C and n_D denote the number of meteors recorded by the corresponding observers within a given Section; 3) that the true Coefficients of Attention are proportional to their preliminary values; thus

$$\left.\begin{aligned}\pi_C &= C\,\pi'_C \\ \pi_D &= D\,\pi'_D\end{aligned}\right\},$$

C and D being the unknown factors of proportionality. These factors as determined from the observations were:

$$C = 0{,}749 \pm 0{,}070\,;\quad D = 0{,}948 \pm 0{,}036.$$

In this way the Horary Numbers for August 14-th, given below, were obtained.

The Horary Numbers give us information as to the variation of the intensity from day to day; following mean values for each night were found (table 10) and the results plotted on fig. 5.

Table 10. Mean Horary Numbers. Limiting Zenithal Magnitude 4,2.

	Date. August 1920.					
	9-th	10-th	11-th	12-th	13-th	14-th
Perseids. Unit of Luminosity and Mass a 2-nd Zen. Magn. Perseid.						
Mean N_0	40.0 ± 4.4	78.0 ± 7.2	196.7 ± 10.5	77.8 ± 6.5	44.7 ± 4.5	35.3 ± 4.7
„ J_0	24.5 ± 3.3	37.2 ± 3.1	172.0 ± 13.7	47.7 ± 3.9	31.0 ± 3.3	33.0 ± 5.1
„ M_0	27.4 ± 4.6	36.7 ± 4.0	259.7 ± 47.0	48.3 ± 4.7	34.3 ± 4.8	41.2 ± 8.2
Non-Perseids. Unit of Luminosity a 2-nd Zen. Magn. Meteor.						
Mean N_h	7.5 ± 2.0	29.9 ± 3.9	31.4 ± 4.2	16.9 ± 2.8	24.1 ± 3.3	31.8 ± 7.1
„ J_h	3.3 ± 1.2	18.6 ± 5.4	20.0 ± 2.5	7.8 ± 1.3	7.0 ± 0.9	7.6 ± 1.4

The upper curves of the figure may be regarded as representing the cross-section of the Perseid-shower for 1920; they are schematized from necessity, the observations of consecutive nights being separated by a considerable interval of 21 hours, during which the variation of intensity is unknown. To obtain a detailed picture of the cross-section, observations from different points on the earth's surface ought to be made; the cooperation of observers located at four observing-points, 90° of longitude one from another, would be sufficient.

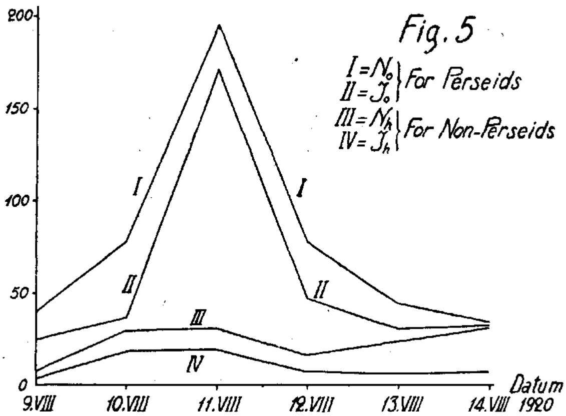


Table 11 contains the average Horary Numbers for each magnitude separately and their probable errors expressed in %.

The n_0 and n_h of this table represent the Luminosity Curves of the meteors. The logarithms of these numbers are plotted

Table 11.

Average Horary Numbers. All days (Aug. 9-th to Aug. 14-th).

$m_0 =$	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0	−0.5	−1.0	−1.5	−2.0 and Brighter	Sum
							Perseids							
n_0	22.3	18.2	16.2	11.6	9.5	4.6	3.28	1.17	0.56	0.16	0.07	0.04	0.04	87.7 ± 3.1
P. E. %	±10.0	8.1	6.7	6.7	7.0	9.4	11	17	21	38	45	52	± 52	—
$i_0 n_0$	3.6	4.5	6.4	7.1	9.5	7.3	8.2	4.7	3.5	1.5	1.1	1.0	4.0	62.6 ± 3.1
μn_0	2.1	3.6	5.0	6.2	9.5	8.2	10.7	6.8	5.9	2.9	2.4	2.4	15.8	81.3 ± 9.6
							Non-Perseids							
n_h	10.8	4.5	2.65	1.51	1.19	0.53	0.23	0.25	0.15	0.16	0.00	0.01	0.00	22.0 ± 1.5
P. E. %	± 12	13	14	18	18	23	35	29	39	± 50	—	—	—	—
$i_0 n_h$	1.73	1.12	1.07	0.95	1.19	0.83	0.59	1.01	0.94	1.61	0.00	0.29	0.00	11.3 ± 1.2

on fig. 6. The Luminosity Curve of the Perseids is satisfactorily represented by the formula

$$lg\, n_0 = 1.357 + 0{,}036\,(m - 4) - 0{,}0928\,(m - 4)^2 \quad (21).$$

The curve for the meteors not belonging to the stream has quite a different character.

If formula (21) is to be extrapolated over the fainter meteors, it means, that about 95% of the total mass of the Perseids belongs to meteors brighter than the 4-th Zenithal Magnitude; that gives us a means to estimate the order of the total mass of the Perseids. Assuming, 1) that the cross-section of the stream is a circle of the diameter = 12,5 mill. kilometers (the 5-days path of the earth); 2) that the intensity of the stream is constant throughout its revolution and equal to the average observed in 1920, giving a Horary Mass of 81 meteors of the 2-nd Zen. Magn. upon 30 000 sq. km. of the cross-section; and 3) that the period of revolution is 100 years, — we obtain a mass $= 3{,}3.10^{17}$ meteors of the 2-nd Zen. Magnitude, or, taking into account that some of our data are probably underestimated, we assume the round number 10^{18}; with the mass of a second-magnitude Perseid = 1 milligram that means, that the total mass of the Perseid shower is of the order of

$$10^9 \text{ tons.}$$

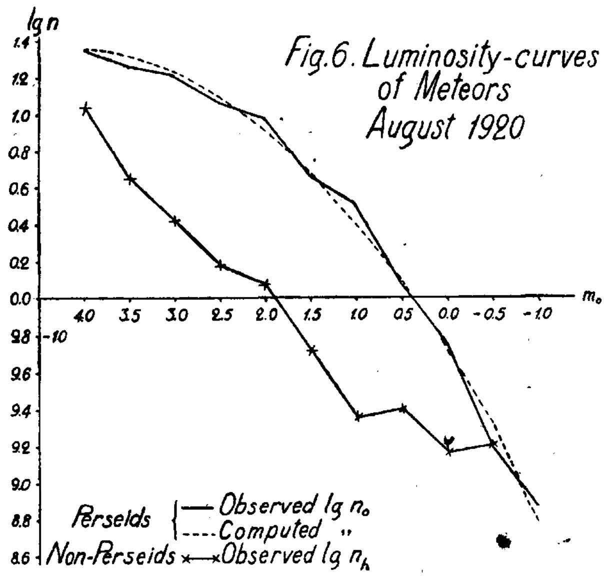


Fig. 6. Luminosity-curves of Meteors August 1920

Thus the Perseids if condensed into one body would form a globe having a diameter of somewhat less than 1 kilometer.

Summary. General results.

1. A new method to determine the absolute number of shooting stars has been invented and applied.

2. The reliability of naked-eye magnitude estimations of meteors has been ascertained. The magnitudes recorded are not affected by the angular motion in any appreciable degree.

3. In the „Equivalent Group Method“ a means for statistical photometrical comparisons of meteors belonging to a definite shower has been found; the method enables us to investigate the influence of various conditions upon the brightness of meteors.

4. As indicated by the above said method the meteor radiates uniformly in all directions.

5. From theoretical considerations, confirmed by scarce observational data being at our disposal, it appears that during its visibility the nucleus of the meteor maintains its initial velocity almost unaltered, the retardation being the less the higher the velocity; the main loss of Kinetic Energy takes place only after vaporization.

Results obtained for the Perseid-shower of August 1920, observed at Tashkent.

1) Data representing the absolute intensity of the shower for consecutive nights were found. These data are directly comparable with future observations if made according to the method described and may thus forward the detailed study of the structure of the shower.

2) The order of the integrated mass of the shower has been estimated.

3) A preliminary formula representing the relative mass of meteors as a function of their luminosity and length of path is proposed.

4) The Luminosity-Curve for meteors belonging to the shower has been found.

Before concluding I should like to express my sincere thanks to M-r V. N. Milovanoff, director of the Observatory of Tashkent, as well as to the Members of the Observatory and all other persons who partook in the difficult task to furnish the observational data discussed in this paper.

Tartu (Dorpat) 22-nd December 1921.

Normal Horary Numbers For Perseids. 1920.

August 9.

I Interval. $\lambda_{\odot} = 136^0\ 53'\ 12''$ 1). $\overline{Cos\ z_r} = 0{,}658$ 2)

$m_0 =$	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0	−0.5	−1.0	−1.5 and brighter	Sum
n_0	9.1	17.0	5.1	4.5	2.0	1.7	1.1	0.21	0.55	0.29	0.24	0.00	$N_0 = 41.8 \pm 7.7$
$i_0\,n_0$	1.5	4.2	2.0	2.8	2.0	2.7	2.8	0.8	3.5	2.9	3.8	0.	$J_0 = 29.0 \pm 5.6$
$\mu\,n_0$	0.9	2.9	1.6	2.5	2.0	3.1	3.6	1.2	5.8	5.5	8.3	0.	$M_0 = 37.3 \pm 9.9$

II Interval. $\lambda_{\odot} = 136^0\ 55'\ 55''$. $\overline{Cos\ z_r} = 0{,}769$

$m_0 =$	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0	−0.5	−1.0	−1.5 and brighter	Sum
n_0	3.2	3.9	5.6	6.2	2.9	0.10	0.60	0.92	0.	0.	0.	0.	$N_0 = 23.4 \pm 4.8$
$i_0\,n_0$	0.5	1.0	2.2	3.9	2.9	0.2	1.5	3.6	0.	0.	0.	0.	$J_0 = 15.8 \pm 3.5$
$\mu\,n_0$	0.3	0.7	1.7	3.4	2.9	0.2	2.0	5.4	0.	0.	0.	0.	$M_0 = 16.5 \pm 4.5$

III Interval. $\lambda_{\odot} \doteq 136^0\ 58'\ 09''$. $\overline{Cos\ z_r} = 0{,}846$

$m_0 =$	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0	−0.5	−1.0	−1.5 and brighter	Sum
n_0	16.9	13.3	11.7	4.5	2.5	2.1	3.7	0.	0.	0.	0.	0.	$N_0 = 54.7 \pm 9.5$
$i_0\,n_0$	2.7	3.3	4.7	2.8	2.5	3.3	9.3	0.	0.	0.	0.	0.	$J_0 = 28.6 \pm 7.0$
$\mu\,n_0$	1.6	2.3	3.6	2.5	2.5	3.8	12.0	0.	0.	0.	0.	0.	$M_0 = 28.3 \pm 8.3$

August 10.

I Interval. $\lambda_{\odot} = 137^0\ 49'\ 40''$. $\overline{Cs\ z_r} = 0{,}617$

$m_0 =$	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0	−0.5 and brighter	Sum
n_0	35.8	27.5	21.4	5.1	6.8	2.7	0.90	0.36	0.24	0.	$N_0 = 100.8 \pm 14.8$
$i_0\,n_0$	5.7	6.9	8.6	3.2	6.8	4.3	2.3	1.4	1.5	0.	$J_0 = 40.7 \pm 4.6$
$\mu\,n_0$	3.4	4.7	6.6	2.8	6.8	4.9	2.9	2.1	2.5	0.	$M_0 = 36.7 \pm 4.8$

II Interval. $\lambda_{\odot} = 137^0\ 52'\ 38''$. $\overline{Cs\ z_r} = 0{,}738$

$m_0 =$	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0	−0.5 and brighter	Sum
n_0	38.3	10.2	12.7	7.3	11.2	3.9	0.77	0.28	1.07	0.	$N_0 = 85.6 \pm 14.0$
$i_0\,n_0$	6.1	2.5	5.1	4.6	11.2	6.2	1.9	1.1	6.7	0.	$J_0 = 45.4 \pm 6.9$
$\mu\,n_0$	3.6	1.7	3.9	4.0	11.2	7.0	2.5	1.6	11.3	0.	$M_0 = 46.8 \pm 8.9$

III Interval. $\lambda_{\odot} = 137^0\ 44'\ 57''$. $\overline{Cs\ z_r} = 0{,}826$

$m_0 =$	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0	−0.5 and brighter	Sum
n_0	9.7	10.3	13.2	4.7	4.8	2.2	2.2	0.34	0.	0.	$N_0 = 47.4 \pm 7.2$
$i_0\,n_0$	1.3	2.1	4.4	2.4	4.8	3.5	5.5	1.4	0.	0.	$J_0 = 25.4 \pm 4.4$
$\mu\,n_0$	0.9	1.7	4.1	2.1	4.8	4.0	7.1	2.0	0.	0.	$M_0 = 26.7 \pm 6.5$

1) $\lambda_{\odot} =$ apparent geocentric longitude of the sun for the middle of the Interval.

2) $\overline{Cos\ z_r} =$ mean cosinus of the zenith distance of the Radiant.

Normal Horary Numbers For Perseids. 1920.

August 11.

$m_0=$	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0	—0.5	—1.0	—1.5	—2.0	—3.5	Sum
I Interval. $\lambda_{\odot} = 138^0 47' 33''$. $\overline{Cos\, z_r} = 0.636$															
n_0	54.5	32.7	32.2	39.7	24.7	6.3	5.4	5.9	3.0	1.36	0.42	0.58	0.31	0.31	$N_0 = 207.4 \pm 20.4$
$i_0 n_0$	8.7	8.2	12.9	25.0	24.7	10.0	13.5	23.5	18.9	13.6	6.6	14.6	12.3	48.0	$J_0 = 240.5 \pm 36.8$
μn_0	5.2	5.5	10.0	21.8	24.7	11.3	17.6	34.5	31.8	25.9	14.5	36.0	34.7	203.0	$M_0 = 476.5 \pm 139.0$
II Interval. $\lambda_{\odot} = 138^0 50' 09''$. $\overline{Cos\, z_r} = 0{,}744$															
n_0	44.0	40.3	38.6	26.1	22.7	6.5	6.9	3.7	2.9	0.57	0.	0.	0.	0.	$N_0 = 192.3 \pm 18.7$
$i_0 n_0$	7.0	10.1	15.4	16.4	22.7	10.3	17.3	14.7	18.3	5.7	0.	0.	0.	0.	$J_0 = 137.9 \pm 12.5$
μn_0	4.2	6.8	12.0	14.3	22.7	11.7	22.4	21.7	30.7	10.9	0.	0.	0.	0.	$M_0 = 157.4 \pm 18.0$
III Interval. $\lambda_{\odot} = 138_0 52' 33''$. $\overline{Cos\, z_r} = 0{,}831$															
n_0	49.8	35.6	27.2	20.0	19.2	21.6	16.6	0.52	0.	0.	0.	0.	0.	0.	$N_0 = 190.5 \pm 15.2$
$i_0 n_0$	8.0	8.9	10.9	12.6	19.2	34.1	41.7	2.1	0.	0.	0.	0.	0.	0.	$J_0 = 237.5 \pm 12.8$
μn_0	4.8	6.0	8.4	11.0	19.2	38.9	53.9	3.0	0.	0.	0.	0.	0.	0.	$M_0 = 145.2 \pm 15.3$

Remark. One casual bright meteor of the Zenithal Magnitude — 3.5 has considerably influenced the Horary Mass of the 1-st Interval, whence the excessive value of M_0 and its great probable error.

August 12.

$m_0=$	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0 and brighter	Sum
I Interval. $\lambda_{\odot} = 139^0 45' 09''$. $\overline{Cs\, z_r} = 0{,}641$										
n_0	15.5	5.0	7.0	11.5	10.6	1.9	1.44	0.39	0.	$N_0 = 53.3 \pm 10.6$
$i_0 n_0$	2.5	1.2	2.8	7.2	10.6	3.0	3.6	1.6	0.	$J_0 = 32.5 \pm 4.9$
μn_0	1.5	0.8	2.2	6.3	10.6	3.4	4.7	2.3	0.	$M_0 = 31.8 \pm 5.0$
II Interval. $\lambda_{\odot} = 139^0 47' 45''$. $\overline{Cs\, z_r} = 0.749$										
n_0	24.0	24.5	25.5	9.1	10.1	7.5	2.11	0.11	0.	$N_0 = 102.9 \pm 13.6$
$i_0 n_0$	3.8	6.1	10.2	5.7	10.1	11.8	5.3	0.4	0.	$J_0 = 53.4 \pm 5.9$
μn_0	2.3	4.2	7.9	5.0	10.1	13.5	6.8	0.6	0.	$M_0 = 50.4 \pm 6.2$
III Interval. $\lambda_{\odot} = 139^0 50' 13''$. $\overline{Cs\, z_r} = 0.837$										
n_0	14.1	18.7	13.2	11.8	7.6	5.3	3.1	3.5	0.	$N_0 = 77.3 \pm 9.2$
$i_0 n_0$	2.3	4.7	5.3	7.3	7.6	8.4	7.8	13.9	0.	$J_0 = 57.3 \pm 8.8$
μn_0	1.3	3.2	4.1	6.5	7.6	9.5	10.1	20.5	0.	$M_0 = 62.8 \pm 11.5$

Normal Horary Numbers For Perseids. 1920.

August 13.

$m_0=$	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0	—0.5	—1.0	—1.5 and brighter	Sum
					I Interval. $\lambda_{\odot} = 140^0\,42'\,50''$. $\overline{Cos\,z_r} = 0{,}650$								
n_0	2.0	11.7	16.8	11.4	4.9	1.33	0.17	0.17	0.62	0.08	0.43	0.	$N_0 = 49.6 \pm 8.9$
$i_0\,n_0$	0.3	2.9	6.7	7.2	4.9	2.1	0.4	0.7	3.9	0.8	6.8	0.	$J_0 = 36.7 \pm 6.8$
$\mu\,n_0$	0.2	1.9	5.4	6.3	4.9	2.4	0.6	1.0	6.6	1.4	13.9	0.	$M_0 = 44.6 \pm 11.3$
					II Interval. $\lambda_{\odot} = 140^0\,45'\,26''$. $\overline{Cos\,z_r} = 0{,}759$								
n_0	6.4	10.7	5.5	6.2	5.5	2.5	1.28	1.19	0.	0.	0.	0.	$N_0 = 39.3 \pm 7.4$
$i_0\,n_0$	1.1	2.7	2.2	3.9	5.5	4.0	3.2	4.7	0.	0.	0.	0.	$J_0 = 27.3 \pm 4.7$
$\mu\,n_0$	0.6	1.8	1.7	3.4	5.5	4.5	4.2	7.0	0.	0.	0.	0.	$M_0 = 28.7 \pm 5.8$
					III Interval. $\lambda_{\odot} = 140^0\,47'\,50''$. $\overline{Cos\,z_r} = 0{,}841$								
n_0	11.3	10.5	6.9	2.9	7.6	3.1	3.0	0.	0.	0.	0.	0.	$N_0 = 45.3 \pm 7.0$
$i_0\,n_0$	1.8	2.6	2.8	1.8	7.6	4.9	7.5	0.	0.	0.	0.	0.	$J_0 = 29.0 \pm 5.5$
$\mu\,n_0$	1.1	1.8	2.1	1.6	7.6	5.6	9.8	0.	0.	0.	0.	0.	$M_0 = 29.6 \pm 6.5$

August 14.

$m_0=$	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0	—0.5	—1.0 and brigther	Sum
					I Interval. $\lambda_{\odot} = 141^0\,42'\,07''$. $\overline{Cs\,z_r} = 0{,}722$							
n_0	0.0	7.6	7.3	15.5	1.0	1.6	0.0	0.0	3.0	0.70	0.	$N_0 = 36.7 \pm 7.1$
$i_0\,n_0$	0.0	1.9	2.9	9.8	1.0	2.5	0.0	0.0	18.9	7.0	0.	$J_0 = 44.0 \pm 8.8$
$\mu\,n_0$	0.0	1.3	2.3	8.5	1.0	2.9	0.0	0.0	31.8	13.3	0.	$M_0 = 61.1 \pm 15.3$
					II Interval. $\lambda_{\odot} = 141^0\,45'\,19''$. $\overline{Cs\,z_r} = 0{,}841$							
n_0	0.0	6.5	13.1	7.6	4.3	0.0	2.4	0.	0.	0.	0.	$N_0 = 33.9 \pm 6.2$
$i_0\,n_0$	0.0	1.6	5.2	4.8	4.3	0.0	6.0	0.	0.	0.	0.	$J_0 = 21.9 \pm 5.1$
$\mu\,n_0$	0.0	1.1	4.0	4.2	4.3	0.0	7.8	0.	0.	0.	0.	$M_0 = 21.4 \pm 6.0$

Horizontal Horary Numbers For Non-Perseids. 1920.

August 9.

m_0 =	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5 and brighter	Sum
					I Interval				
n_h	0.0	0.0	0.1	1.3	0.6	0.0	0.63	0.	$N_h = 2.6 \pm 1.1$
$i_0 n_h$	0.	0.	0.0	0.8	0.6	0.0	1.6	0.	$J_h = 3.1 \pm 1.3$
					II Interval				
n_h	2.5	1.1	1.3	1.0	0.2	0.1	0.	0.	$N_h = 6.2 \pm 2.9$
$i_0 n_h$	0.4	0.3	0.5	0.6	0.2	0.1	0.	0.	$J_h = 2.2 \pm 0.9$
					III Interval				
n_h	4.8	5.6	0.8	1.0	1.5	0.	0.	0.	$N_h = 13.7 \pm 5.1$
$i_0 n_h$	0.8	1.4	0.3	0.6	1.5	0.	0.	0.	$J_h = 4.6 \pm 1.8$

August 10.

m_0 =	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0	—0.5	—1.0 and brighter	Sum
							I Interval					
n_h	14.7	11.9	1.6	0.7	1.5	0.36	0.52	0.49	0.36	0.	0.	$N_h = 32.1 \pm 7.0$
$i_0 n_h$	2.3	3.0	0.6	0.4	1.5	0.6	1.3	2.0	2.3	0.	0.	$J_h = 14.0 \pm 2.6$
							II Interval					
n_h	16.6	8.3	3.4	2.1	0.6	0.	0.	0.	0.	0.	0.	$N_h = 31.0 \pm 7.7$
$i_0 n_h$	2.7	2.1	1.4	1.3	0.6	0.	0.	0.	0.	0.	0.	$J_h = 8.1 \pm 1.7$
							III Interval					
n_h	13.2	3.4	3.4	0.6	1.1	1.4	0.4	0.70	0.28	2.00	0.	$N_h = 26.5 \pm 5.5$
$i_0 n_h$	2.1	0.8	1.4	0.4	1.1	2.2	1.0	2.8	1.8	20.0	0.	$J_h = 33.6 \pm 12.4$

August 11.

m_0 =	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0	—0.5	—1.0	—1.5	Sum
							I Interval						
n_h	14.3	4.2	3.9	2.2	0.7	0.61	0.71	0.97	0.97	0.41	0.14	0.08	$N_h = 29.2 \pm 6.9$
$i_0 n_h$	2.3	1.0	1.6	1.4	0.7	1.0	1.8	3.8	6.1	4.1	2.3	2.0	$J_h = 28.1 \pm 5.5$
							II Interval						
n_h	25.0	8.3	2.7	2.6	2.4	2.3	0.47	0.70	0.31	0.	0.	0.	$N_h = 44.8 \pm 9.3$
$i_0 n_h$	4.0	2.1	1.1	1.6	2.4	3.7	1.2	2.8	1.9	0.	0.	0.	$J_h = 20.8 \pm 3.9$
							III Interval						
n_h	7.3	3.3	2.4	3.2	1.8	1.6	0.7	0.	0.	0.	0.	0.	$N_h = 20.3 \pm 4.5$
$i_0 n_h$	1.2	0.8	1.0	2.0	1.8	2.5	1.8	0.	0.	0.	0.	0.	$J_h = 11.1 \pm 2.9$

Horizontal Horary Numbers For Non-Perseids. 1920.

August 12.

$m_0=$	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0	−0.5 and brighter	Sum
						I Interval					
n_h	1.2	0.0	3.9	1.3	1.6	0.8	0.0	0.46	0.	0.	$N_h = 9.3 \pm 2.6$
$i_0 n_h$	0.2	0.0	1.6	0.8	1.6	1.3	0.0	1.8	0.	0.	$J_h = 7.3 \pm 1.9$
						II Interval					
n_h	9.0	2.3	2.1	0.3	0.0	0.32	0.0	0.47	0.32	0.	$N_h = 14.8 \pm 5.6$
$i_0 n_h$	1.4	0.6	0.8	0.2	0.0	0.5	0.0	1.9	2.0	0.	$J_h = 7.4 \pm 2.9$
						III Interval					
n_h	15.7	3.4	2.8	2.0	2.4	0.3	0.	0.	0.	0.	$N_h = 26.6 \pm 5.8$
$i_0 n_h$	2.5	0.8	1.1	1.3	2.4	0.5	0.	0.	0.	0.	$J_h = 8.6 \pm 2.0$

August 13.

$m_0=$	4.0	3.5	3.0	2.5	2.0	1.5	1.0 and brighter	Sum
				I Interval				
n_h	18.5	5.7	4.9	2.1	2.6	0.06	0.	$N_h = 33.9 \pm 8.2$
$i_0 n_h$	3.0	1.4	2.0	1.3	2.6	0.1	0.	$J_h = 10.4 \pm 2.0$
				II Interval				
n_h	12.9	6.0	5.4	1.7	0.1	0.	0.	$N_h = 26.1 \pm 4.1$
$i_0 n_h$	2.1	1.5	2.2	1.1	0.1	0.	0.	$J_h = 7.0 \pm 1.3$
				III Interval				
n_h	5.9	4.1	1.0	0.6	0.8	0.	0.	$N_h = 12.4 \pm 3.7$
$i_0 n_h$	0.9	1.0	0.4	0.4	0.8	0.	0.	$J_h = 3.5 \pm 1.1$

August 14.

$m_0=$	4.0	3.5	3.0	2.5	2.0 and brighter	Sum
			I Interval			
n_h	17.5	13.7	6.8	0.	0.	$N_h = 38.0 \pm 12.6$
$i_0 n_h$	2.8	3.4	2.7	0.	0.	$J_h = 7.9 \pm 2.3$
			II Interval			
n_h	11.2	6.7	4.9	2.8	0.	$N_h = 25.6 \pm 6.5$
$i_0 n_h$	1.8	1.7	2.0	1.8	0.	$J_h = 7.3 \pm 1.6$

Supplement.

Instruction For Statistical Observations of Shooting Stars.

The Double-Count Method.

At least two independent observers are needed, located 80—200 meters apart from one another; the distance must be chosen so that ordinary speech of one observer could not be heard by the other, nor shall the observers see one another. It is desirable that each observer be accompanied by an assistant, recording the time and other data under the dictation of the observer; this enables the observer to watch for meteors uninterruptedly. If the intensity of the shower is moderate — say, not more than 20 per hour — an experienced observer may work alone, without assistant; in this case, however, meteors appearing in groups will be systematically omitted.

The observers watch during one and the same interval of time a definitely limited region in the sky, of a diameter not exceeding 60^0; the beginning and end of the observations may be marked by some loud signal. A preliminary acquaintance with the region must antecede the observations.

During the observation the eye is to be moved uniformly throughout the region; the observer may imagine a circle at a median distance between the centre and the boundaries of the region, and move the eye uniformly along this circle, substituting sometimes this movement by a transverse one.

Care shall be taken to avoid the influence of external illumination upon the observer's eye; the lamp must be carefully screened; if the moon is above the horizon, it may be advisable to use a transportable screen, placed so as to shadow the eye from direct illumination. The attitude of the observer must be comfortable enough; every source of fatigue shall be avoided.

Data concerning the quality of sight of the observer must be given together with the observational records.

The Observational Records.

At the beginning and the end of the observations the condition of the sky must be noted as well as every change occurring in the sky: the appearance of haze, clouds etc. To note the condition the following scale may be used (a normal acuteness of the observer's eye is supposed):

1 . . . . . hazy; stars up to the 2-nd magnitude
2 . . . . „ „ „ „ 3-d — 4-th magn.
3 . . . . „ „ „ „ 5-th „
4 . . . . „ „ „ „ 5—6-th „
5 . . . . faintest stars visible.

The atmospherical absorption plays an important part in the reduction of the observations; with an accuracy sufficient for our purposes the coefficient of absorption may be determined in the following way: the observer choses several stars of the 2-nd magnitude and brighter; these stars, when at an altitude from 4^0 to 15^0 above the horizon, are to be compared with stars at an altitude not less than 30^0 and photometrically about 0,5—2,0 magnitudes fainter than the former; the time of comparison must be carefully noted; the comparison may be made according to the method of Pickering: two comparison-stars are chosen, one brighter, the other fainter than the star near the horizon; the brightness of the latter is evaluated in tenths of the interval between the comparison-stars. Such comparisons must be made before, after and between the observations.

The records to be noted are: 1) the number; 2) the moment of time; 3) the magnitude; 4) the position; 5) the direction of the path; 6) the length of the path; 7) the duration of flight; 8) the colour; 9) remarks. The last four data (6—9) are of a secondary importance and may be omitted, if the intensity of the shower is too considerable to allow of a complete record.

The time is recorded by the assistant with accuracy to 1 second from a short signal of the observer. The chronometers or clocks of the observers must be compared before and after the observations. It must be pointed out that accurate knowledge of the absolute time is not needed: for purposes of identification of the meteors the relative differences between the clocks will be sufficient; these differences may be found even from the common meteors observed. The absolute correction of the clocks must be known to 0,5 minutes of time.

The beginning and end of the observations is to be noted to 0,1 minute of time.

The magnitude of the meteor is to be recorded to $0^{mg}.5$. Definite stars of comparison, whose magnitudes rounded off to $0,^{mg}5$ must be remembered, are chosen within the region, and immediately after the apparition of the meteor the eye must be directed on the corresponding comparison-star; estimations by memory must be avoided; such estimations may be applied of necessity only for very bright meteors; in this case one may make use of the following scale:

Object	Magnitude	Object	Magnitude
α Tauri	+1	Jupiter	—2.5
α Aurigae, α Lyrae, α Bootis	0	Venus	from —3 to —4
Sirius	—1.5	The Moon, I or III quarter	—9
		Full Moon	—12

The stars of comparison may be chosen in two different ways: 1) from 4 to 6 standard stars, ranging from the 1-st to the 5-th magnitude, are taken near the centre of the region and the comparisons are made only with these stars; 2) from 20 to 30 stars, uniformly distributed over the whole region, are chosen and the meteor compared with the nearest star of the suitable magnitude. The advantages of both methods are: in the first case an uniformity and exactness of the scale of comparison stars is attained; in the second — the effect of atmospherical absorption will be nearly eliminated. In the schemes given below the first proceeding has been preferred.

The position of the meteor is of great importance for purposes of identification and reduction. The whole region must be divided into sections, limited by lines joining certain stars. The diameter of the sections may range from 10^0 to 20^0; but their vertical diameter shall not exceed one third of the mean altitude above the horizon during the whole time of observation.

The observer records the section where the middle of the meteor's visible path appeared; the various sections may be denoted by numbers or by neighbouring stars.

Meteors for which the middle of their path would be located without the boundary of the region, must be omitted or may be recorded only with a special remark: „without the region“.

The direction of motion is reckoned from a line joining two conspicuous stars within the region; the observer may imagine, that a geographical map is placed before him, where the line mentioned above is taken for a meridian or a parallel; then the direction will be recorded as the direction of an imaginary wind; e. g., *NE* would mean, that the meteor moved from *NE* to *SW*.

The length of the path may be estimated in degrees; for comparison standard distances between certain stars may serve.

The duration can be estimated only if considerable enough — say not less than $\frac{3}{4}$ seconds of time.

The colour may be described in words, e. g. — white, yellow-white, yellow etc. Standard stars must be chosen, as, for instance:

Star	Colour
α Lyrae	White
α Aurigae	Yellow
α Tauri	Orange.

Colour-estimations can be obtained only for the brighter meteors.

It is desirable that the Double-Count observations would be accompanied by simultaneous tracing of the meteor-trails upon a map, executed by a third independent observer; such parallel observations allow of the determination of various systematical errors. The observer must avoid the for-conceived selection of meteors belonging to a certain Radiant.

To facilitate the reduction a homogeneity of the observations is required; the observations must begin and end at the same hour during the consecutive nights. Between every 50 minutes of observation it is advisable to place a 10-minute interval.

Below are given schemes for the Double-Count observations.

I. The North Polar Region.

This region suits best General Statistical Investigations to be made the whole year.

Limits of the Region:

ε, γ Cassiopejae; ι, α, ϑ Cephei; δ, ζ, η, ι, α Draconis; α, h, o Ursae Majoris; β Camelopardali; ε Cassiopejae.

Limits of the Sections:

Section	Limits
I	α Ursae Minoris, B. D. 81° 302 Draconis ($\alpha = 9^h 23^m$, $\delta = 81^\circ 46'$ $m = 4.4$), β, γ Ursae Minoris. κ, γ Cephei, α Urs. Minoris.
II	β Urs. Min.; B. D. 81° 302; ϱ Ursae Majoris, λ, κ Draconis; β Ursae Minoris.
III	α Ursae Min.; B. D. 81° 302; ϱ Ursae Majoris; γ Camelopardali, α Ursae Minoris.
IV	α Ursae Min., γ Camelopard., A Cassiopejae, γ Cephei, α Ursae Minoris.
V	κ, γ, β Cephei, ε Draconis, κ Cephei.
VI	ε, δ, ζ, η Draconis, γ Ursae Min., κ Cephei, ε Draconis.
VII	β, γ Ursae Min., η, ι, α, κ Draconis, β Ursae Minoris.
VIII	α, κ, λ Draconis; α Ursae Maj.; α Draconis.
IX	λ Draconis; ϱ, o, h Ursae Maj., λ Draconis.
X	ϱ, o Ursae Maj., β, γ Camelopard. ϱ Ursae Majoris.
XI	ε, A Cassiopejae, γ, β Camelop., ε Cassiopejae.
XII	A, ε, γ Cassiopejae; ι, o Cephei; A Cassiopejae.
XIII	A Cassiopejae; o, ι, β, γ Cephei; A Cassiopejae.
XIV	ι, α, ϑ Cephei; δ, ε Draconis; β, ι Cephei.

Comparison-stars:

Star	Magn. P. D.	Adopted Magnitude
α Ursae Minoris	2.3	2.5
γ Cephei	3.4	3.5
ε Ursae Min.	4.5	4.5
B. D. 85° 383	5.4	5.5

($\alpha = 22^h 21^m$ $\delta = 85^\circ 36'$ 1900)

Stars for the Determination of Atmospherical Absorption:

Star	Magnitude P. D.
α Lyrae	0.4
α Aurigae	0.4
α Persei	2.2
γ Cassiopejae	2.5
α Cygni	1.6
η Ursae Majoris	2.3
ε " "	2.2

These stars must be compared with the preceding comparison-stars.

Colour-Standards: α Ursae Minoris . . . . . yellowish-white.
β " " . . . . . orange.

Fixed Line for Records of Direction: α — β Ursae Minoris.

Standard Distances for the Estimation of the Length of Path:

α Ursae min. — γ Cassiopejae $= 29^0$
" " " — β Ursae min. $= 17$
" " " — ε " " $= 8^1/_2$

The time of the observations must be arranged symmetrically before and after Midnight.

II. Scheme for Double-Count Observations of the Lyrids (from April 15 to April 25).

Limits of the Region:

β, χ, δ, ϑ, ι, κ Cygni; c, ξ, ν, μ, 17 Draconis; τ, φ, χ Herculis; ζ, ϑ, β, α Coronae; β Serpentis; ω, h Herculis; κ, β, 72 Ophiuchi; 110, 113 Herculis; β Cygni.

Limits of the Sections:

Section	Limits
I	ι, τ Herculis, 17, μ, ν, ξ, γ Draconis; ι Herculis.
II	π, ρ, ι, τ, σ, π Herculis.
III	ε, π, σ, τ, φ, χ Herculis; ζ, κ, σ, ν Coronae; ζ, ε Herculis.
IV	γ, β, ζ Herculis; ν, σ, κ, ζ, ϑ, β, α Coronae; β Serpentis; ω, γ Herculis.
V	ι Herculis; γ, ξ, c Draconis; κ Cygni; R, α Lyrae; ι Herculis.
VI	α, κ Lyrae; ν, ξ, ρ, ι Herculis; α Lyrae.

Section	Limits
VII	ξ, μ, δ, ε, π, ϱ, ξ Herculis.
VIII	β, ς, ε, δ, α, β Herculis.
IX	γ, β, α Herculis, ϰ Ophiuchi; *h*, ω, γ Herculis.
X	γ, β, α. *R* Lyrae; ϰ, ι, ϑ, δ Cygni; ϑ, γ Lyrae.
XI	α, β Lyrae; 95, ξ, ν Herculis; ϰ, α Lyrae.
XII	ξ, μ, δ, α Herculis; α Ophiuchi; 95, ξ Herculis.
XIII	α, β, ϰ Ophiuchi; α Herculis; α Ophiuchi.
XIV	δ, χ, β Cygni; γ, ϑ Lyrae; δ Cygni.
XV	β Cygni; γ, β Lyrae; 95, 110, 113 Herculis; β Cygni.
XVI	α, β, 72 Ophiuchi; 110, 95 Herculis; α Ophiuchi.

Comparison-Stars:

Star	Magnitude P. D.	Magnitude Adopted
α Lyrae	0.4	0.5
α Ophiuchi	2.5	2.5
δ Herculis	3.5	3.5
λ "	4.6	4.5
d "	5.6	5.5

Stars for the Determination of Atmospherical Absorption:

Star	Magn. D. P.
α Aquilae	1.1
α Cygni	1.6

Colour-Standards: α Lyrae, α Ophiuchi . . . . . white
γ Draconis . . . . orange.

Fixed Line for Records of Direction: α Lyrae — β Herculis.

Standard Distances for the Estimation of the Length of Path:

$$\alpha\text{—}\pi \text{ Herculis} = 22^1/_2{}^0$$
$$\zeta\text{—}\eta \quad " \quad = 7^1/_2$$
$$\pi\text{—}\varrho \quad " \quad = 2$$

The Observations may begin at 11^h M. T. The Sidereal Time of the beginning and the end of the observations must be the same during the whole period.

III. Scheme for Double-Count Observations of the Perseids (from Aug. 8 to Aug. 14) and of the Andromedids (November 14 to Nov. 20).

Limits of the Region:

α, ι, λ Andromedae; τ, ϰ, ψ, ι Cassiopejae; β Camelopardali; α, η, ι Aurigae; φ, *A*, *f* Tauri; μ Ceti; ξ Arietis; η Piscium; η, ζ, α Andromedae.

Limits of the Sections:

Section	Limit
I	η, γ, α, μ Persei; α Aurigae; β Camelopard.; *i* Cassiopejae; η Persei.
II	ϑ, γ, ϰ, ψ, ι Cassiopejae; η, φ Persei; ϑ Cassiopejae.
III	λ Andromedae; τ, ϰ, γ, ϑ, π Cassiopejae; λ Andromedae.
IV	φ, η, γ, α, ϰ, β Persei; γ Andromedae; φ Persei.
V	β, μ, ν Andromedae; π, ϑ Cassiopejae; φ Persei; γ, β Andromedae.
VI	α, ι, λ Andromedae; π Cassiopejae; ν, μ, β, δ, α Andromedae.
VII	β, ε, μ, α, ϰ, β Persei.
VIII	β, ε, ξ, ζ, ο Persei; *c* Arietis; ϱ, β Persei.
IX	β, ϱ Persei; *c*, α Arietis; γ Trianguli; γ Andromedae; β Persei.
X	β, γ Andromedae; γ Trianguli; α, β Arietis; β Andromedae.
XI	α, δ, β Andromedae; β Arietis; η Piscium; η, ζ, α Andromedae.
XII	ξ, ε, μ Persei; α, η Aurigae; ξ Persei.
XIII	ζ, ξ Persei; ν, ι Aurigae; φ Tauri; ζ Persei.
XIV	ε, *c* Arietis; ο, ζ Persei; φ, *A*, η Tauri; ε Arietis.
XV	η Piscium; β, α, *c*, ε, π Arietis; μ Ceti; ξ Arietis; η Piscium.
XVI	μ Ceti; π, ε Arietis; η, *A*, *f* Tauri; μ Ceti.

Comparison-Stars:

Star	Magnitude P. D.	Magnitude Adopted
α Aurigae	0.4	0.5
α Persei	2.2	2.0
γ "	3.2	3.0
i "	4.2	4.0
c Andromedae	5.6	5.5

Stars for the Determination of Atmospherical Absorption.

Star	Magn. P. D.
β Tauri	2.0
β Orionis	0.6
α Geminorum	1.9
β "	1.5
α Canis Minoris	0.7

Colour-Standards: α Persei . . . . . . yellowish-white
β Andromedae . . reddish-orange.

Fixed Line for Records of Direction: γ—α Andromedae.

Standard Distances for the Estimation of the Length of Path:

α Andromedae — *α* Cassiopejae . . .	27^0
α—*β* Persei	10^0
β—*ϱ* „	$2^1/_4{}^0$

The observations of the August meteors may begin at $11^h\,30^m$ M. T.

The observations of the Andromedids — during the whole night.

The Sidereal Time of the beginning and the end of the observations must be the same for the whole period of observations of one shower.