RESEARCHES ON THE PHYSICAL THEORY OF METEOR PHENOMENA

III

BASIS OF THE PHYSICAL THEORY OF METEOR PHENOMENA

BY

ERNST ÖPIK

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List of Standard Notations and Abbreviations.

A = Paper A of this series, cf. Ref. [1].
I = Paper I „ „ „ , cf. Ref. [2].
II = Paper II „ „ „ , cf. Ref. [3].
C. G. S. units used except when explicitly stated otherwise.
μ = molecular weight.
δ = density of nucleus (solid, or liquid).
ϱ = density of the atmosphere; $\varrho = \varrho_0$ for $H = 0$.
γ = density of coma.
c_1, c_2 = specific heats in solid, resp. liquid state.
f = latent heat of fusion.
h = latent heat of vaporization.
q_1 = heat up to, and including fusion, erg/gr.
q_2 = heat from fusion to vaporization.
$q = q_1 + q_2$.
P = pressure of vapours.
D = density of vapours.
ξ = intensity of vaporization (mass vaporized per cm^2 and sec).
p, p_n = total aërodynamic pressure, and its normal component.
p_s = average aërodynamic resistance per cm^2 of cross-section.
α = angle of incidence of the air molecules.
$\varkappa$, $\varkappa$ = fraction of the relative kinetic energy of the air molecules, available for heating the nucleus.
$\varkappa_0$ = value of $\varkappa$ for direct (unshielded) impact into the meteor nucleus.
d = thickness of a layer of vapours, or air, measured in units of the kinetic half-energy range.
E = heat generated by aërodynamic work and transported per second toward the nucleus.
F = total heat generated by aërodynamic work and absorbed per cm^2 cross-section of the nucleus.
M = air mass traversed by cm^2 of the cross-section of the meteor.

B = pressure in the atmosphere (barometric pressure).
g = acceleration of gravity.
z_r = zenithal angle of incidence of meteor trajectory (zenith distance of the radiant point).
H = height.
H_0 = difference in height for an atmospheric density ratio $e:1$.
H_{10} = same, for a density ratio $10:1$.
L = length of visible trail.
R = radius, R_0 = initial radius of a spherical nucleus.
r = radius of the drop-fragment, detached by fissure or spraying.
Q = intensity of black body radiation.
T = absolute temperature.
k = Stefan's constant; also different coefficients of proportionality.
w = velocity of the meteor; w_0 = initial velocity.
u_0 = velocity of sound.
U_e = equatorial velocity of the rotation of the nucleus.
u = proportion of air admixed to coma.
ω = angular velocity of rotation.
σ = surface tension of liquid.
ν = coefficient of viscosity of the liquid.
k_t = conductivity for heat.
η = liquefied fraction of the mass of the nucleus.
λ_0 = kinetic half-energy range.
a, a_m = air mass accumulated in air cap, gr/cm^2.
m = apparent magnitude of meteor; m_0 = absolute magnitude; m_z = zenithal magnitude.
β = visual efficiency, or fraction of kinetic energy converted into visible radiation.
s_c = fraction of secondary atomic collisions taking place within the coma.
I = absolute luminosity (visual).

Basis of the Physical Theory of Meteor Phenomena [1]).

Abstract.

The present paper prepares a foundation for the general theory of meteor phenomena.

Probable values of the physical constants for the meteor substance are adopted. Formulae, valid for low atmospheric densities when no air cap is formed, are given for the aërodynamic pressure and work.

A first approximation theory is applied for the purpose of obtaining estimates of the physical conditions in meteors of different size, velocity, and composition. The first approximation formulae may be used for a study of observational data referring to the dependence of the mean height of meteors upon velocity and magnitude, and to estimates of the density gradient of the atmosphere; absolute estimates of the density of the atmosphere, and the calculation of the range in height for meteors cannot be made with sufficient reliability on the basis of the first approximation.

Fissure of the molten meteor as well as the spraying of the liquid during the process of fusion are found to play an important rôle in meteor phenomena; conditions of fissure and spraying are investigated, the viscosity of the liquid being one of the chief parameters. Rotation influencing spraying determines the character of the meteor display.

Conditions of the formation of an air cap, of the heat transfer within the nucleus under different conditions, and

1) Continued from [1], [2], and [3].

various other phenomena are investigated. The upper limit of size for meteor dust caught by the terrestrial atmosphere without getting vaporized is estimated.

The conditions of radiation by atomic collisions are investigated; for most naked-eye meteors except the very brightest the degree of dilution of the coma is found to be such that secondary collisions mostly happen in the free atmosphere instead of taking place between the atoms of the vaporized meteor substance; this result is essential for quantitative estimates of the radiation from meteors.

The shielding effect of meteor vapours with respect to the flow of heat toward the nucleus is investigated; the shielding effect increases with the velocity and size of the meteor; for actual conditions the shielding effect is always only slight.

Three typical problems, for numerical treatment in a second approximation theory, are outlined (cf. Synopsis) on the basis of the scope of our estimates.

Introduction.

The interpretation of meteor observations requires as a basis a sound physical theory of meteor phenomena taking place during the flight of the meteor in our atmosphere. With such a purpose in view the present research was undertaken.

The problem is rather complicated and must be approached by consecutive approximations. The first step is an approximate theory where the radiation losses of the meteor nucleus, phenomena connected with capillarity, viscosity, and conductivity for heat, as well as rotation, are neglected; this framework theory, simple enough for direct analytical treatment, furnishes preliminary estimates of the physical conditions at different points of the meteor trajectory, estimates which are necessary for deciding such questions as whether a liquid drop breaks into pieces, or not, etc.; these estimates serve, so to speak, as traffic signals in developing the detailed numerical theory which forms the next step of our task. Some principles of our framework theory have been already published and applied elsewhere[1], and in many cases the approximate theory works well. On the other hand, such a problem as a satisfactory theory of the average light curve of a meteor can generally be solved only by numerical methods.

The physical theory of meteor phenomena may be divided into the following three major problems.

Problem 1: the theory of the flight of the meteor in the upper atmosphere without formation of an "air cap"; the meteor nucleus is bombarded by single air molecules directly, because their mean free path is large as compared with the depth of the air layer accumulated in front of the nucleus; practically all "naked eye" meteors up to very bright fireballs[1]) remain within the limits of this problem, never reaching portions of the atmosphere dense enough for the formation of an air cap; in this case the deceleration of the meteor even near the end point of its trajectory is small and may be neglected in most cases.

Problem 2: the theory of the flight of the meteor with the formation of an air cap. The latter shields the meteor from excessive heating and favours the penetration of the meteor into the atmosphere, in some cases making it possible for remnants of the nucleus to reach the ground; deceleration may be very considerable in this case; this problem refers to fire-balls of the brightness of the moon and brighter, and particularly to meteorite falls.

Problem 3: the theory of the impact of a meteorite on solid ground[2]. This problem is concerned chiefly with the formation of meteor craters, although the theory of small meteors needs some of the results of the theory of impact, namely those referring to rotation[3].

With our special purpose in view, in the present paper we shall confine ourselves to Problem 1 exclusively.

A brief review of earlier work on the theory of naked eye meteors has been given in *A*[2]). Of the former theories, Sparrow's work[4] must be regarded as the most successful. A recent contribution by the late Dr. Fisher[5] deals chiefly with Problem 2.

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1) cf. Section 3. *c* below.

2) cf. [1], pp. 5—6.

A complete theory of observable meteor phenomena can be built up only on the basis of a knowledge of the structure of the upper atmosphere. Now, this structure is yet unknown to us; we rather expect to learn something about the structure from meteor observations. Our meteor theory therefore must be adjustable to a considerable extent; it must not be bound to some particular model of the upper atmosphere. Actually we describe below a kind of standard theory, referring to an arbitrary constant logarithmic density gradient of the atmosphere; with the aid of this standard theory it may be possible, at least with a good degree of approximation, to build up a theory for an atmosphere of arbitrary structure. An iron sphere was taken as the principal basic model in our calculations, the physical properties of iron at elevated temperatures being known better than those of meteor stones. The probable difference in the behaviour of stone meteorites is sketched in general outline. As to the probable non-sphericity of actual meteors, no fundamental qualitative difference can be produced in our theory by this circumstance. However, the absolute height of meteors may be considerably influenced by their shape, a circumstance which must be kept in mind when dealing with meteor streams such as the Perseids, or Leonids[3].

Section 1.

General Physical Data and Formulae.

The data were taken mostly from Landolt-Börnstein's Tables, also from the International Critical Tables and Smithsonian Physical Tables; they are also partly guessed from general physical considerations. CGS units and absolute temperatures everywhere used.

a. Adopted numerical data for iron.

Density $\delta = 7.8$; molecular weight $\mu = 56 =$ atomic weight.

Melting point 1800^0; boiling point 3508^0; average specific heat of solid $c_1 = 6 \times 10^6$ (erg/gr. deg), of liquid $c_2 = 7.5 \times 10^6$; latent heat of fusion $f = 2.7 \times 10^9$, of vaporization $h = 6.0 \times 10^{10}$ (erg/gr).

Table I.

Pressure (P), density (D), maximum flow (ξ) of saturated iron vapours, and black body radiation (Q).

T	1800°	2000°	2200°	2400°	2600°	2800°
$P\ \frac{\text{dynes}}{\text{cm}^2}$	40.7	321	1766	7140	23 400	64 600
$D\ \frac{\text{gr}}{\text{cm}^3}$	1.55×10^{-8}	1.10×10^{-7}	5.50×10^{-7}	2.04×10^{-6}	6.17×10^{-6}	1.58×10^{-5}
$\xi\ \frac{\text{gr}}{\text{cm}^2\cdot\text{sec}}$	6.24×10^{-4}	4.67×10^{-3}	2.46×10^{-2}	9.50×10^{-2}	0.298	0.794
$Q\ \frac{\text{erg}}{\text{cm}^2\cdot\text{sec}}$	5.99×10^{8}	9.12×10^{8}	1.34×10^{9}	1.88×10^{9}	2.60×10^{9}	3.50×10^{9}

T	3000°	3200°	3400°	3600°	3800°
P	1.59×10^{5}	3.54×10^{5}	6.85×10^{5}	1.29×10^{6}	2.26×10^{6}
D	3.63×10^{-5}	7.59×10^{-5}	1.38×10^{-4}	2.46×10^{-4}	4.07×10^{-4}
ξ	1.890	4.07	7.65	14.0	23.8
Q	4.61×10^{9}	6.00×10^{9}	7.64×10^{9}	9.60×10^{9}	1.19×10^{10}

The maximum flow, ξ, is the maximum amount vaporized per cm² of surface; this quantity is computed from the adiabatic formula

$$\xi = 0.654\, u_0 D = 950 \sqrt{T} D = \frac{6.5\times10^{-4} P}{\sqrt{T}} \quad . \quad . \quad . \quad (1),$$

where u_0 is the velocity of sound. The amount lost by vaporization is always equal to ξ when the ratio of vapour pressure to external pressure, $\frac{P_v}{P_e}$, exceeds 2.05; for smaller ratios the vaporized amount is roughly proportional to $\frac{P_e}{P_v}\sqrt{\frac{P_v}{P_e}-1}$, thus it varies slowly[1]). For meteors, we may assume the amount vaporized per cm² and second equal to ξ with a fair degree of approximation, when $\frac{P_v}{P_e} > 1$, and equal to zero, when $\frac{P_v}{P_e} \leqslant 1$.

1) with $\frac{P_v}{P_e} = 1.1$ only, the flow still amounts to about 0.55 ξ.

Effective surface tension, $\sigma = 950$ (dyne/cm) (observed 950 to 997) near the temperature of fusion; at effective temperatures of vaporization of meteors σ may be smaller, about 850.

Viscosity of the liquid at vaporization temperatures small, of the order of $\nu = 0.01 \left(\frac{\text{dyne} \times \text{sec}}{\text{cm}^2}\right)$.

Conductivity for heat $k_t = 4 \times 10^6$.

b. Adopted numerical data for meteoric stone.

Density 3.4. Average atomic weight = 28 [1]). Average composition (Noddack) (including 5.5% Troïlite):

O = 39.8%; Mg = 15.1%; Si = 20.3%; Fe = 15.4%; S = 3.8%; Al = 1.5%; Ca = 1.8%; Na = 0.7%; all remaining elements = 1.6%.

The different minerals during vaporization are supposed to be dissociated into simple oxides; iron oxides, with their low energies of binding, must be completely dissociated; with the average composition by weight as given above this gives the following schematical composition of the vapours: SiO_2=43.5%; MgO = 25.1%; CaO = 2.5%; Fe = 15.4%; S_2 = 3.8%; O_2 = 3.8%; Al_2O_3 = 3.2%; Na_2O = 0.9%; rest 1.8%. These figures lead to an average molecular weight of the vapours $\mu = 52$.

Melting point of Olivine from 1600° to 2000°, of Troïlite 1470°; of different other minerals occurring in meteors from 1450° to 1800°. The effective melting point of a stone meteor apparently does not differ much from the melting point of iron, 1800°.

The effective boiling point (at 760 mm pressure) is determined by the boiling points of the simple constituents into which the minerals probably are decomposed during the process of vaporization, and before it: SiO_2, b. p. 2500°; MgO, b. p. 3070°; CaO, b. p. 3120°; Fe, b. p. 3508°; the effective boiling point may be assumed about 3000°, thus lower than for iron. The curve of vapour pressure probably runs flatter than for iron because of the various constituents which are volatilized step by step as the temperature rises.

Average specific heat of solid $c_1 = 1.1 \times 10^7$; of liquid $c_2 = 1.3 \times 10^7$; for the latent heat of fusion no data are available;

1) The average is computed weighting by relative proportion of weight, and not by relative number of atoms.

assuming it equal to the heat of fusion of Li_2SiO_3, we get as a guess $f = 3.4 \times 10^9$ (erg/gr). The latent heat of vaporization can be estimated from Trouton's law with more certainty; applying this law separately to the constituents SiO_2, MgO, etc., and adding about $200 \frac{\mathrm{cal}}{\mathrm{gr}}$ for the heat of dissociation of silicates (a quantity estimated from the known thermochemical data of a few silicates), we get $h = 5.6 \times 10^{10}$ (with an uncertainty of perhaps $\pm$ 10 per cent).

For surface tension $\sigma = 400 \left(\frac{\mathrm{dyne}}{\mathrm{cm}}\right)$ may be a fair guess at fusion, and $\sigma = 360$ at temperatures of volatilization.

The viscosity of molten silicates is known to be very large; in Landolt-Börnstein we find for artificial Diopside (group of Pyroxenes, common in meteorites) at $T = 1553^0$, $\nu = 106$ and at $T = 1573^0$, $\nu = 33$; these values refer to temperatures close to the melting point; unfortunately we are unable to make any reliable estimate of the viscosity at the temperatures of volatilization, about 2500^0; it is not probable that the viscosity continues decreasing as fast as it does near the melting point; we may set the following probable limits for the effective viscosity of the liquid at volatilization: $10 > \nu > 0.1$. Conductivity for heat $k_t = 2 \times 10^5$.

c. *Radiation from the nucleus.*

The total loss by radiation from the nucleus (solid or liquid) is given by Stefan's formula

$$Q = kT^4, \text{ where } k = 5.7 \times 10^{-5} \frac{\mathrm{erg.}}{\mathrm{cm^2.sec}} \quad . \quad . \quad . \quad . \quad (2).$$

Values of Q are given in Table I.

We notice that the radiation from the nucleus is of little significance in producing observable radiation [1]); nevertheless, radiation losses must be considered in a detailed meteor theory because of their influence upon the balance of heat of the nucleus.

d. *Aërodynamic resistance and coefficient of accomodation.*

In the absence of an air cap, i. e. of hydrodynamic stream-lines, each molecule of the air hits the nucleus of the meteor directly and rebounds independently of other air molecules; the

1) cf. [1] and earlier authors.

theory of air resistance in such a case leads invariably to the square law of velocity, whatever the velocity is.

Let p denote the total aërodynamic pressure, p_n its component normal to the surface (thus p_n is the radial component for a spherical meteor), p_s the average resistance[1]) per cm² cross section of a spherical body, ϱ the density of the atmosphere, w the velocity, α the angle of incidence, and $\varkappa$ the fraction of relative kinetic energy converted into heat available for the heating of the meteor nucleus. $\varkappa$ is practically very close to the so called "coefficient of accomodation". Three typical cases may be considered.

1) Perfectly elastic impacts, when the particles rebound according to the laws of ideal reflection; evidently $\varkappa = 0$; in this case the pressure is normal to the surface,

$$p = p_n = 2\varrho\, w^2 \cos^2 \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

and

$$p_s = \varrho\, w^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4);$$

the heating effect upon the nucleus is zero, $\bar{\varkappa} = 0$.

2) Inelastic impacts of the first type, when the impinging particles lose their normal component, but preserve their tangential component of velocity; thus $\varkappa = \cos^2 \alpha$. The pressure is again normal to the surface,

$$p = p_n = \varrho\, w^2 \cos^2 \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5),$$

and

$$p_s = \frac{1}{2}\, \varrho\, w^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

3) Inelastic impacts of the second type, when the impinging particles lose all their relative velocity and are thus completely carried away by the meteor; $\varkappa = 1$. The resultant total pressure is in the direction of motion; we have

$$p_n = \varrho\, w^2 \cos^2 \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5');$$

$$p_s = \varrho\, w^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4').$$

The total heat absorbed by the nucleus is equal to the relative kinetic energy of the penetrated air mass. The total work of aërodynamic resistance per cm² and second is $\varrho\, w^3$, but the heating effect is only one-half of that[2]), or

$$E' = \frac{1}{2}\, \varrho\, w^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

1) In the direction of motion.

2) The other half of the aërodynamic work is spent in accelerating the air to the velocity of the meteor.

In the case of velocities of the order of those occurring in meteors, the mean penetration into matter of an impinging molecule is from five to eight times greater than at ordinary velocities of gas molecules[1]); thus the air molecule penetrates deep into the solid or liquid surface of the meteor, losing its energy mostly in collisions with the meteor atoms placed at a certain depth below the surface of the nucleus; with decreasing velocity the penetrating power of the air molecule decreases and the molecule (which actually must be dissociated into atoms, as the result of the collision) will be unable to regain the surface of the meteor; or at least, when the air molecules return to the surface, they will possess an average translational energy corresponding to the temperature of the meteor, whereas most of their kinetic energy will be absorbed by the meteor; in such a manner a kind of "energy trap" establishes itself already at velocities of 5 km/sec (for nitrogen), which is much smaller than the lowest possible meteor velocity. On the other hand, sometimes an atom of the meteor material may be "knocked out" when hit by the air molecule at the very surface of the meteor; this atom will carry away some fraction of the kinetic energy of the impinging molecule, which fraction (together with the lattice energy) is no more available for heating the meteor; on the other hand, the energy dissipated beneath the surface will not have much chance to escape. Considering all these circumstances, a rough calculation indicates that the major part of the relative kinetic energy of the impinging molecules is transformed into heat and absorbed by the nucleus. For direct impact into a solid or liquid surface of the meteor nucleus a value of $\varkappa_0 = 0.80$ may be a fair estimate. We may add that we get such a high value of $\varkappa$ even when the individual collisions between the air molecule and the molecules of the meteor substance are perfectly elastic.

Sparrow[4] assumes $\varkappa = 0.37$ to 0.27 (nitrogen), from a consideration of a certain type of inelastic collision between air molecules and meteor atoms [something like our case 2), applied to individual atomic collisions]; with his mechanism of individual collisions, and our "penetration trap", a value close to 1 would have resulted. Direct observations of positive ions support our

1) cf.[1], pp. 16, 23, 29.

conclusions as to the high value of $\varkappa$ at high velocities; thus Voorhis and Compton[6] find (impact into metallic surface) for A^+, $\varkappa = 0.75$ (vel. 10 to 25 km/sec), for Ne^+, $\varkappa = 0.65$ (vel 15 to 40 km/sec), for He^+, $\varkappa = 0.35$ (vel. 30 to 50 km/sec), and for same, $\varkappa = 0.55$ (vel. 80 km/sec).

Table II is the result of approximate calculations of the shielding effect of vapours. The "diffusion" of the kinetic energy by secondary collisions is taken into account.

Table II.

Relative Shielding Effect of a Protecting Layer of Vapours (or air).

Thickness of layer, in units of the half-energy range, d	0	1	2	3	4	$d > 4$
$\varkappa$	(1.00)	(0.77)	0.58	0.43	0.32	$\frac{1.28}{d}$

In this table, the values for $d > 2$ are supposed to represent absolute values of $\varkappa$; for $d = 1$, a value of $\varkappa = \varkappa_0$ instead of 1 must be adopted.

In Section 3. *i* we find that the shielding effect of meteor vapours can never cause $\varkappa$ to fall much below its "unshielded value", $\varkappa_0$, and that $\varkappa = 0.60$ appears to be a fair approximation for all possible cases during the process of vaporization. During the process of heating up to, and including, fusion, $\varkappa = \varkappa_0 = 0.8$ may be a better estimate. Nevertheless, in our first approximation $\bar{\varkappa} = 0.60$ is assumed throughout.

Thus, in the case of naked eye meteors case 3) of aërodynamic resistance appears as the closest to actual circumstances; we assume, for the sake of convention, a certain weighted average between cases 1) and 3):

$$p_s = \varrho w^2 \quad \ldots\ldots\ldots\ldots \quad (4)\ (4'),$$

and

$$p_n = (2 - \varkappa')\, \varrho w^2 \cos^2 \alpha \quad \ldots\ldots\ldots \quad (8);$$

Here $\varkappa'$ may be called the "dynamical coefficient of accomodation". The central pressure (for $\alpha = 0$) is

$$p_0 = (2 - \varkappa')\, \varrho w^2 \quad \ldots\ldots\ldots\ldots \quad (9).$$

For the sake of convention we take $\varkappa' = 0.60$, $2 - \varkappa' = 1.4$, although $\varkappa'$ may be greater than $\bar{\varkappa}$, the "thermal" coefficient of accomodation.

The work of aërodynamic resistance per unit cross section and second is ϱw^3, and the total heat absorbed by the nucleus per second is (R = radius of meteor)

$$E = \frac{1}{2} \bar{\varkappa} \pi R^2 \varrho w^3 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (10),$$

where $\bar{\varkappa}$ is a value averaged over the cross-section. If dq is an amount of heat absorbed per gram of mass of the nucleus, the deceleration is found from (4) and (10) as

$$dw = -\frac{2\, dq}{\varkappa\, w} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10^a).$$

e. Stability of liquid non-rotating drops.

The aërodynamic forces are unevenly distributed over the surface of the drop and tend to break it to pieces; surface tension (σ) keeps the drop together. For a given aërodynamic pressure, there may be assigned a certain limiting radius of stability, R_s; a drop smaller than R_s (which denotes the radius of the original undistorted sphere) may keep together, drops larger than R_s certainly break. The exact theory of stability is very complicated; but the transition from stability to instability is more or less abrupt, and R_s may be determined fairly well from an approximate theory. Such a theory[1]) gave:

$$R_s = \frac{4\,\sigma}{p_s} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11).$$

As a check of the order of magnitude, at least, we may apply formula (11) to the case of a non-adhering gravitating drop ($g = 981$) supported by a flat surface; we get $R_s = \sqrt{\frac{\sigma}{330\,\delta}}$; this gives for mercury $2R_s = 0.72$ cm, for water $2R_s = 0.94$ cm, or values near the right order of magnitude. We notice that the application of (11) to the case of a drop breaking under its own weight is not quite legitimate because of the different character of the distribution of pressure; also, in the gravitating drop the average pressure decreases with flattening; whereas the drop flattening under aërodynamic pressure will be subject to a more or less constant pressure, and will thus break more easily.

1) Still too complicated to be described here.

f. Shedding off drops.

The conditions under which a partly solid, partly liquid body loses detached drops are still more complicated than the preceding case. To guess the order of magnitude, we worked out approximately the model of a solid circular shield, subject to aërodynamic pressure on the front side, and carrying on the rear side a liquid fraction η of the whole mass (solid + liquid) the effective spherical radius being R (i. e. total mass $= \frac{4}{3}\pi\delta R^3$), assumed equal to the radius of the shield, the condition of stability is found as

$$R \leqslant \frac{\sigma\,(1+\frac{3}{2}\eta)}{\eta^2(1+\frac{3}{4}\eta)\,p_s} = \frac{R_s\,(1+\frac{3}{2}\eta)}{4\eta^2(1+\frac{3}{4}\eta)} \quad . \quad . \quad . \quad . \quad (12).$$

The formula is a very rough approximation, and must not be used for η near 1, when evidently the shield cannot exist and when formula (11) is to be preferred. In the process of fusion, for a given radius, η cannot exceed the limit set by (12); all the extra liquid is shed off and sprayed by drops which are small as compared with the nucleus.

g. Atmosphere.

The actual dynamical structure of the upper atmosphere, i. e. the form of the density function, is not of fundamental importance for the development of the present theory (cf. Introduction). On the other hand, the composition of the upper atmosphere must be known to some extent; at least, we must know what is the rôle of hydrogen in the upper atmosphere. The hydrogen atom at meteor velocities carries only a small amount of kinetic energy[1]), so that under 20 km/sec no visible impact radiation can be produced; at higher velocities, visible impact radiation can be produced, but with very low efficiency, which means greater masses of meteors of a given brightness[2]), and a greater relative importance of the continuous radiation of the nucleus. Thus the assumed relative amount of hydrogen, as compared with nitrogen and oxygen, may influence our theory to a considerable extent. The relative amount of helium has not so great an influence. On the other hand, the relative

1) cf.[1], p. 13, Table II, also p. 18, form. (12), and pp. 26—32.
2) cf.[1], p. 39.

amount of nitrogen and oxygen, elements of similar atomic weight, or their actual molecular structure (degree of dissociation and ionization), is of no such importance in the development of our theory, although in the applications of the theory such questions may arise.

Not so very long ago the prevailing view on the structure of the upper atmosphere, sponsored by Humphreys[7], Jeans[8], and others, ascribed to hydrogen a predominant rôle at heights over 80 kilometres where meteors appear. Sparrow in his meteor theory[4] accepts this view, but he finds that the amount of hydrogen postulated by Humphreys is too large. On the other hand, Chapman[9] denies any rôle to hydrogen; according to his views, a layer of maximum ozone concentration at 50 km height is followed by layers of increasing content of monatomic oxygen, O and O^+ (produced by the dissociating and ionizing action of the solar short wave radiation); the green (auroral) line of the night sky, λ 5577, is explained as a forbidden transition of monatomic oxygen $({}^1S \rightarrow {}^1D)$[10].

We feel inclined to accept Chapman's view; at least, the amount of hydrogen postulated by Humphreys is contradicted by a number of observational facts; we notice that Humphrey's calculations are based on very uncertain determinations of the hydrogen content near the earth's surface, and on a still less certain assumption of an undisturbed "conductive" equilibrium [1]) of the atmosphere from a height of 11 km on. Only in such a form of equilibrium will the lighter constituents such as hydrogen, or helium, show a smaller density gradient, and may outnumber the heavier elements in the upper atmosphere. Let us first consider the probability of such a form of equilibrium.

Ordinarily only two kinds of atmospheric equilibrium (from the standpoint of mixing) are considered: the conductive and the convective equilibrium. The upper atmosphere, at least in its lower strata, is known as more or less isothermal, even with a possible inversion of temperature; it has been justly pointed out by different authors that under such conditions convective currents cannot persist, any rising current being rapidly stopped through adiabatic expansion making the ascending current colder and heavier than the undisturbed strata and vice versa. Thus convective equilibrium must be certainly rejected.

1) Corresponding to pure diffusion.

But there is another form of mixing which may produce almost the same large-scale effects as convection currents: turbulence. Near the earth's surface turbulence is a powerful mixing agent[1]), and it may play an important rôle also in the upper atmosphere; the wave-shaped *cirrus* clouds may be regarded as an indication that turbulence is certainly active at heights up to 10 km.

Turbulence is irregular wave motion. From the standpoint of mixing, only the vertical component of the wave motion is of interest. We may roughly estimate the relative importance of turbulence and diffusion in the following way. Let λ be the mean free path, u' the average vertical component of molecular velocities, ΔH the average amplitude of the turbulent wave, U the mean (absolute) vertical velocity of turbulence. The efficiency, from the standpoint of a transport of mass (or heat) in vertical direction, is proportional to the vertical component of velocity and to the mean depth of direct interchange; thus the relative efficiency of the two processes is

$$\text{turbulence : diffusion} = \frac{U\Delta H}{u'\lambda}.$$

On the other hand, for an isothermal atmosphere the amplitude of turbulence depends upon the maximum vertical velocity $= 2\,U$ (approximately). With $2\,U = 5$ m/sec (which does not seem excessive, taking into account the considerable wind velocities revealed by meteor trains), we find $\frac{1}{2}\,\Delta H = 250$ metres. Further, from our subsequent approximate theory (as well as Sparrow's) it follows that average meteors appear (centre of trajectory) in a stratum of density about $\frac{1}{200\,000}$ to $\frac{1}{2\,000\,000}$ of atmospheric density at sea level, which gives λ from 2 to 20 cm. Taking $u' = 250$ m/sec, the above data give the ratio of efficiency in mixing:

$$\text{turbulence : diffusion} = \text{from } 250 \text{ to } 25.$$

Turbulence at meteor heights may thus be much more efficient than diffusion, in which case the atomic composition of the atmosphere at these heights will be identical with the compos-

1) Its effects may be easily observed on smoke from chimneys; the so-called diffusion of gas clouds in chemical warfare is almost entirely due to turbulence.

ition at sea level (except for water vapour); we should proceed to much greater (130—150 km) heights to find diffusion as the major factor. Only beginning from such great heights a differentiation of the different constituents according to molecular weight may set in; but not before heights of about 200 km are reached can hydrogen (if present at all) become relatively conspicuous. For the meteor theory in such a case the presence of hydrogen may be entirely neglected.

The absence of hydrogen lines in the spectrum of the aurora, and especially of meteors[11], supplies serious evidence against the existence of considerable amounts of hydrogen in the upper atmosphere.

Rough calculations based on the height of appearance of fireballs, such as the well observed Pultusk meteorite (Jan. 30, 1868) which increased by about 6 magnitudes between 260 and 180 km height[1]), give an increment of density of 10:1 for each 33 km decreasing height. Such a figure is incompatible with hydrogen or helium at any reasonable temperature (radiative equilibrium), but very well agrees with monatomic oxygen (or nitrogen).

Even assuming the "conductive" equilibrium to hold at moderate heights, it may be shown that hydrogen in the amounts postulated by Humphreys and Jeans appears extremely improbable. At a certain height hydrogen and oxygen may be found in the right proportion to form an explosive mixture, and if not outnumbered by the inert nitrogen the mixture will explode at the first fireball or meteor penetrating the layer. Assuming that a temperature of 800° C of the mixture is sufficient to support the explosion, we find that an amount of hydrogen not exceeding $^1/_{23}$ fraction of the amount assumed by Humphreys can exist; the explosive layer (partial pressures of $N_2 : O_2 : H_2 = 17{,}2 : 1{,}0 : 2{,}0$) will be found at $H = 82{,}0$ km in such a case. With Humphreys's data, the explosive layer lies between 60 and 70 km height, and the temperature of explosion exceeds 1000° C. We do not believe in the "conductive equilibrium" at these heights; the above figures are quoted merely to show that even in the case of such a sort of equilibrium the possible amount

1) At these heights the radiation of the meteor is probably pure impact radiation at the solid surface before appreciable heating sets in; the brightness in such a case is proportional to the density of the atmosphere. Cf. [1], p. 24.

of hydrogen in the upper atmosphere must be much smaller than that supposed by Humphreys and others.

Thus we seem to be justified in building up our first approximation theory on the assumption of an upper atmosphere composed chiefly of heavy constituents such as nitrogen and oxygen. We note that our former computations referring to meteor radiation[1] were based on the same assumption.

h. Air masses and densities.

The variation of gravity with height and the effect of the curvature of the earth (which sets in only at great angles of incidence) being neglected, the total air mass per unit cross section encountered by the meteor down to a certain level is given by

$$M = \frac{B \sec z_r}{g} \left(\frac{\text{gr}}{\text{cm}^2}\right) \quad . \; . \; . \; . \; . \; . \; . \quad (13);$$

$$\text{also } dB = -g\varrho dH \quad . \; . \; . \; . \; . \; . \; . \quad (14).$$

Here B is the barometric pressure $\left(\frac{\text{dyne}}{\text{cm}^2}\right)$ at that level, z_r the zenith angle of incidence, or the zenith distance of the meteor radiant, g the acceleration of gravity ($\bar{g} = 953$ at 89 km height), H the height.

Formulae (13) and (14) apply to an atmosphere of arbitrary structure. In the particular case of a "uniform" atmosphere, of constant logarithmic decrement of density (or pressure) with height, we have:

$$\varrho = \varrho_0 \, e^{-\frac{H}{H_0}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (15),$$

$$B = g \, \varrho_0 H_0 \, e^{-\frac{H}{H_0}} = g H_0 \varrho \quad . \; . \; . \; . \; . \; . \; . \; . \quad (16),$$

$$M = \varrho_0 H_0 \sec z_r \, e^{-\frac{H}{H_0}} = \varrho H_0 \sec z_r \quad . \; . \; . \quad (17).$$

Here ϱ_0 is the density of atmosphere at $H = 0$; H_0 is the "effective height" of the atmosphere; the density and pressure decrease in the ratio $1 : e$ for a difference of height equal to H_0; if all the mass of the atmosphere above the level H is given a constant density equal to ϱ, the thickness of the layer thus formed is H_0. We have also:

$$H_0 = 8.270 \times 10^7 \frac{T}{\mu g} \quad . \; . \; . \; . \; . \; . \; . \quad (18),$$

where μ is the average molecular weight. For $g = 981$ and dry air ($\mu = 28.82$) at $T = 273^0$ we find $H_0 = 7.985 \times 10^5$ cm.

The model of a uniform atmosphere, although far from actual conditions when the whole atmosphere is considered, is nevertheless a useful approximation when dealing with a particular layer, more or less limited in height, such as the region of visibility of naked-eye meteors; the model presents the advantage of permitting a very simple analytical treatment; the value of H_0 in such a case is a certain effective, or average quantity, chosen to satisfy equations (17) or (16) over a given limited range in height; H_0 may vary from level to level.

Taking into account that in meteor phenomena the air mass M, traversed by the meteor, is the chief argument (independent variable), a meteor theory built up on the model of a uniform atmosphere is believed to hold also[1]) in the case of a non-uniform atmosphere of the same average parameter H_0, when we assume that all the characteristics of the meteor flight in the non-uniform atmosphere at a given value of M are identical with the characteristics calculated for the same value of M in the uniform atmosphere. Thus, by representing the results of our calculations as a function of M, we make them practically independent of our assumptions regarding the actual structure of the atmosphere.

Section 2.

First Approximation for Slowly Rotating Meteors.

As stated in the Introduction, it appeared useful to work out an approximate theory for the purpose of making estimates of the physical conditions prevailing during the flight of the meteor in our atmosphere; a second approximation is made possible only by the preliminary results of this first approximation. From the standpoint of simplification, our first approximation resembles former theories, notably Sparrow's[4]; the purposes and consequences, however, are different; our first approximation pursues auxiliary purposes and cannot be regarded as a satisfactory theory itself.

The following simplifying assumptions are here made:

1) the temperature is constant throughout all the solid or liquid nucleus; in other words, a sufficiently high conductivity for heat is postulated;

1) Within the limits of uncertainty of any theory whatever.

2) vaporization starts suddenly at a certain temperature above the temperature of fusion; thus the gradual rise of vapour pressure is neglected;
3) after vaporization has started, all the heat absorbed by the nucleus is spent in vaporization; this follows also directly from assumptions 1), 2), and 4);
4) heat losses by radiation are neglected;
5) the time of visibility is assumed equal to the time of vaporization 1);
6) deceleration is neglected [12];
7) the integrity of the liquid drop is assumed.

Condition 2) is artificial, never being fulfilled; its introduction makes the obtained shape of the light curve illusory without however seriously affecting the calculated mean heights and the duration of visibility (or the length of the visible path); on the other hand, assumption 3), although artificial, well represents the actual conditions on the major part of the visible path of an observable slowly rotating meteor (as follows from computations made for the second approximation, which we hope to discuss in one of the next papers of the present series).

Conditions 5) and 6) are practically fulfilled in all cases of observable meteors; 5), because the chief source of visible radiation is the collision of the vapour atoms with the air molecules[1]; 6), because the heat of vaporization is small as compared with the kinetic energy[12].

Condition 4) is a good approximation in most cases, except for faint and slow telescopic meteors, when 6) also fails (because a large fraction of the kinetic energy is spent in radiation).

Condition 1) is well fulfilled in the case of a liquid drop and fails for the solid nucleus, which circumstance, however is of minor importance; for a low viscosity of the liquid, the actual process consists in liquefaction on the front side and the sweeping of the fused material to the rear side; a further rise of temperature does not start before all the nucleus is liquid.

The actual limitations of all these assumptions follow from the approximate theory itself (cf. Section 3).

The total heat intercepted from the beginning by unit cross section of the nucleus when reaching a certain atmospheric

1) cf. [1], pp. 5 and 22—26; also [4] and [12].

level is a fraction $\bar{\varkappa}$ of the relative kinetic energy of the traversed air mass, or

$$F = \frac{1}{2}\bar{\varkappa}Mw^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19).$$

The mass of the nucleus per unit of its cross section is

$$\frac{4}{3}\pi\delta R^3 : \pi R^2 = \frac{4}{3}\delta R.$$

Let the total amount of heat required to heat the unit mass of the meteor from a certain initial temperature to vaporization be $q = q_1 + q_2$, where q_1 is the heat up to, and including fusion. Liquefaction is complete when $F = \frac{4}{3}\delta R q_1$, or when the air mass is

$$M_l = \frac{\frac{8}{3}\delta q_1 R_0}{\bar{\varkappa}w^2} \text{ (liquefaction)} \quad . \quad . \quad . \quad (20);$$

here R_0 is the initial radius. Similarly, vaporization starts when

$$M_a = \frac{\frac{8}{3}\delta q R_0}{\bar{\varkappa}w^2} \text{ (point of appearance)} \quad . \quad . \quad (20').$$

After the beginning of vaporization, the amount of heat absorbed by unit cross section of the nucleus when passing an infinitesimal air mass dM is $dF = \frac{1}{2}\bar{\varkappa}w^2\, dM$. This heat is spent in vaporization, i. e. in diminishing the mass per unit cross section, or

$$dF = -h\, d\left(\frac{4}{3}\delta R\right).$$

The following linear equation between radius and air mass results [with the aid of (20)]:

$$M = \frac{\frac{8}{3}\delta R_0}{\bar{\varkappa}w^2}\left[q + h\left(1 - \frac{R}{R_0}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad (21).$$

The equation is valid during vaporization only.

For $R = 0$, or for the point of disappearance we get:

$$M_d = \frac{\frac{8}{3}\delta(q + h)R_0}{\bar{\varkappa}w^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22).$$

The ratio of air masses at disappearance and appearance is

$$\frac{M_d}{M_a} = \frac{q + h}{q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23).$$

Formula (23) actually determines the effective length of the observable trail.

Assuming a uniform atmosphere, we get the density of the atmosphere at a certain point of the meteor path from

$$\varrho = \frac{M \cos z_r}{H_0} \quad . \quad . \quad (17') \text{ (cf. Section 1.}h.\text{)};$$

for a particular point, M may be taken from formulae (20) — (22).

Let us call such points on the trajectories of two meteors for which the ratio $\frac{R}{R_0}$ is the same similar points. From (21′), (17′), and (15) we get the following equations which permit us to draw some general conclusions regarding physical conditions at similar points of the trajectories of different meteors:

$$\varrho = \frac{\frac{8}{3}\,\delta R_0 \cos z_r}{H_0\,\bar{\varkappa} w^2} \left[q + h\left(1 - \frac{R}{R_0}\right)\right] \quad . \quad . \quad . \quad . \quad (24),$$

$$H = H_{10} \log \left\{ \frac{H_0\,\bar{\varkappa} w^2 \varrho_0}{\frac{8}{3}\,\delta R_0 \cos z_r \left[q + h\left(1 - \frac{R}{R_0}\right)\right]} \right\} \quad . \quad . \quad (25),$$

where

$$H_{10} = \frac{H_0}{\log e} = 2.303\ H_0 \quad . \quad . \quad . \quad . \quad . \quad (26)$$

is the increment in height for an atmospheric density ratio of 10 : 1. For meteors of the same composition at similar points of their paths from (24) we infer that the density of atmosphere is proportional 1) to the initial radius, 2) to the cosine of the angle of incidence, and 3) inversely proportional to the square of velocity.

From (25) we find that the difference in height of similar points of the trajectories of two meteors is

$$H_1 - H_2 = H_{10}\left(2 \log \frac{w_1}{w_2} - \log \frac{R_1}{R_2} - \log \frac{\cos z_1}{\cos z_2}\right) \quad . \quad . \quad (27),$$

where w_1, w_2, R_1, R_2 are the initial velocities and radii of the two meteors, z_1, z_2 — the angles of incidence.

For meteors of different composition but of the same radius and velocity ($\bar{\varkappa}$ and H_0 assumed to be constant) at similar points of their paths we find that the density of the atmosphere is proportional to δq at appearance, and to $\delta\,(q + h)$ at the point of disappearance.

The difference in height is in this case

$$H' - H'' = -H_{10}\left[\log \frac{\delta'}{\delta''} + \log \frac{q' + h'\left(1 - \frac{R}{R_0}\right)}{q'' + h''\left(1 - \frac{R}{R_0}\right)}\right] \quad . \quad . \quad . \quad (28).$$

Formulae (27) and (28) hold not only under the restrictions of the present approximate theory, but partly also in the general case.

From (23), (17), and (15) we get the difference between height of appearance and disappearance:

$$H_a - H_d = H_{10} \log \frac{q + h}{q} \quad . \quad . \quad . \quad . \quad . \quad . \quad (29);$$

thus the range in height depends only upon the heat ratio of the material and upon the vertical density gradient of the atmosphere.

The length of the observable path is evidently

$$L = H_{10} \sec z_r \log \frac{q + h}{q} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30),$$

or it increases with the increasing angle of incidence; this latter statement is confirmed, qualitatively at least, by direct observations[1]). As to the absolute value of L, formula (30) depends greatly upon the validity of our assumption 1); it is not advisable to use the formula for stone meteors.

Meteor observations actually represent a selection by luminosity. It is convenient therefore to replace radius by luminosity in the above equations. Formulae (55) and (54) of Section 3. *h*, together with (30), give with a fair degree of approximation:

$$\operatorname{Log} R_0 = -\frac{2}{15} m_0 - \log w - \frac{1}{3} \log \cos z_r + \text{const.}$$

Substituting this in (27), we get, for two meteors of the same composition:

$$H_1 - H_2 = H_{10}\left[3 \log \frac{w_1}{w_2} + \frac{2}{15}(m_1 - m_2) - \frac{2}{3} \log \frac{\cos z_1}{\cos z_2}\right] \quad . \quad . \quad . \quad (27');$$

1) E. g. in the Leonid shower of 1931 as observed in Arizona, when the shower started with long trails (50—80°), the radiant being low; when the radiant was high, the trails became short (5—10°).

here m_1 and m_2 are the absolute magnitudes of the two meteors. Formula (27') proves to be of considerable value in the analysis of meteor heights. The error of the formula is the smallest for the point of disappearance. As a differential formula it remains more or less valid over a wide range of meteor sizes for which many of our assumptions may fail.

For the heat constants of iron, according to Section 1. *a*, we find (assuming an initial temperature of 280° abs., and an effective temperature of vaporization of 2800°):

$$q_1 = 1.18 \times 10^{10} (\mathrm{erg/gr}); \; q_2 = 7.5 \times 10^9; \; q = 1.93 \times 10^{10};$$
$$h = 6.0 \times 10^{10}; \; q + h = 7.93 \times 10^{10};$$

$$\frac{q+h}{q} = 4.11 \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(iron)}.$$

For stone, the data are less certain; with an initial temperature of 280° and an effective temperature of vaporization of 2300° we find, according to Section 1. *b*:

$$q_1 = 2.01 \times 10^{10}; \; q_2 = 6.5 \times 10^9; \; q = 2.66 \times 10^{10};$$
$$h = 5.6 \times 10^{10}; \; q + h = 7.86 \times 10^{10};$$

$$\frac{q+h}{q} = 2.95 \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(stone)}.$$

The parameter of the atmospheric density gradient, H_{10} (or H_0), can be obtained from observations only; an accurate value can be derived only with the aid of a more refined theory. However, for the purpose of preliminary estimates we must know H_{10}, approximately at least. Approximate values of this parameter may be guessed in different ways.

1) Assuming the upper atmosphere to be in radiative equilibrium with the surface of the earth, $T = \frac{288^0}{\sqrt[4]{2}}$, $g = 953$ cm/sec², from (18) we get, assuming $\mu = 28.82$,

$$H_{10} = 2.303 \; H_0 = 15.8 \text{ km}.$$

This may be a minimum, because the average molecular weight must be smaller than assumed (dissociation, ionization) and the temperature may be higher.

2) From the Pultusk meteor, in Section 1. *g* we estimated $H_{10} = 33$ km for the topmost atmosphere (above 180 km).

3) Unpublished results of the Arizona Expedition [1]), referring

1) Based upon the heights of about 3600 meteors.

to the height of disappearance as a function of luminosity and to the mean height as a function of velocity, in connection with formula (27′) gave preliminarily a mean value of

$$H_{10} = 24.6 \pm 1.1 \text{ km}.$$

In the numerical estimates of Section 3., the round value of $H_{10} = 20$ km (or $H_0 = 8.7$ km) is adopted. Taking into account the preliminary character of our estimates and of our first approximation, there can be no serious consequences from a possible error in the adopted value of H_{10}; in any case the error cannot be large.

Section 3.

Estimates of the Physical Conditions, on the Basis of the First Approximation.

We repeat that the first approximation refers to small, slowly rotating meteors of an upper limit of size and rotation to be determined below. Certain results of our estimates, however, are so definite that they may be applied also without the above limitations.

a. Difference in height of stone and iron meteors.

When dealing with differences in the composition of meteors, it appears to be sufficient to consider only two extreme types — the typical iron, and the typical stone. From the observed falls of meteorites we know that stones largely outnumber irons, a circumstance which may be due to better preservation during the flight through the atmosphere of the stone core protected from heat action by the relatively small thermal conductivity; on the other hand, iron meteorites prevail among finds, again an evident result of selection, because irons are more likely to attract attention. Among observed meteor spectra[11] stones and irons seem to appear more or less equal in number, a circumstance which may represent actual conditions. Large meteor showers, such as the Leonids, however, seem to consist almost exclusively of stone[11]. In any case in a meteor theory one should keep in mind both kinds of objects.

For some purposes, it is sufficient to make the detailed computations for iron, with differential estimates of the properties of stone relative to iron.

The ratio in the absolute length of trail, under similar conditions, is given by (30):

$$\frac{L \text{ stone}}{L \text{ iron}} = \frac{\log \dfrac{q'+h'}{q'}}{\log \dfrac{q+h}{q}} = \frac{\log 2.95}{\log 4.11} = 0.765.$$

Small slowly rotating stones are expected to yield shorter visible trajectories than small irons (at the same angle of incidence). According to Section 3. *e* below, this applies to telescopic meteors only. Larger stones must show much longer trails.

From form. (28), with the proper numerical data, we find that stones are expected to appear $0.22\ H_{10} = 4.4$ km higher, and disappear $0.365\ H_{10} = 7.3$ km higher than iron meteors of an equal radius; the middle of the visible path is expected to lie about 5.6 km higher for stones than for iron meteors. If, however, objects of equal mass are considered, the stone must have a radius 1.32 times the radius of the iron, which, according to formula (27), requires by $H_{10} \log 1.32 = 2.4$ km lower height. Finally, for meteors of the same mass, shape, velocity, and angle of incidence, stones are expected to appear on the average by $0.16\ H_{10} = 3.2\ (\pm)$ km higher than iron meteors. We do not yet know the relative luminous efficiency of iron and stone; therefore, when objects of equal luminosity are compared, the difference in height of stone and iron may be considerably changed.

b. Fissure and spraying of the liquid.

For slow rotation, when centrifugal force is insignificant, the condition of fissure of a drop is $R > R_s$, where R_s is given by equation (11). We shall try to investigate the variation of R_s over the meteor path.

From equations (11), (4) (4′), (17′), and (20) we find the condition for the breaking of a drop just at the moment of complete fusion to be independent of velocity and given by

$$R_0 > R_s = \sqrt{\frac{3}{2}\frac{\sigma\varkappa H_0}{\delta q_1}\sec z_r} = \sqrt{\frac{C}{q_1}\sec z_r} \ldots (31), \text{ where } C = \frac{3}{2}\frac{\sigma\varkappa H_0}{\delta}.$$

For the point of appearance the same formula holds when q is substituted for q_1.

We find the condition of first rupture during the process of vaporization by setting the variable radius R in (21′) equal to R_s. Let

$$\frac{R}{R_0} = y \quad \ldots \ldots \ldots \quad (32);$$

from (11), (4) (4′), (17′), and (21′) we get for the initial radius of a drop which breaks first when the radius has decreased to $R = y\,R_0$, the value

$$R_0 = \frac{R_s}{y}\sqrt{\frac{C\sec z_r}{y\,[q + h\,(1 - y)]}} \quad \ldots \ldots \quad (33).$$

We notice that y varies only within the restricted limits from 1 to 0. For this interval, the expression (33) reaches a minimum when

$$y = y_0 = \frac{q + h}{2h} \quad \ldots \ldots \ldots \quad (34)$$

if $q < h$, a condition fulfilled for most substances. This minimum value of R_0 is

$$R_0 = \sqrt{\frac{4h}{(q + h)}\,\frac{C\sec z_r}{(q + h)}} = \sqrt{\frac{2C\sec z_r}{y_0(q + h)}} \quad \ldots \quad (35).$$

Meteors of initial radius smaller than (35) never break from aërodynamic pressure. The formula does not apply to the case of detached drops originating from a larger nucleus, or to the fissure products of an originally larger drop.

In deriving (34) and (35) we tacitly assumed σ to be constant; for a surface tension slowly varying with temperature the results hold equally well. Table III contains the limiting radii of fissure at different characteristic points of the trajectory computed from the preceding formulae and the numerical constants adopted above.

Table III.

Limiting Conditions [1]) of Fissure of the Liquid Meteor.

Vertical incidence ($z_r = 0^0$); $\bar{\varkappa} = 0.6$; $H = 8.7 \times 10^5$ cm. *a*: first fissure at liquefaction; *b*: first fissure at point of appearance; *c*: first fissure at point of optimum fissure, $y = y_0$, or ultimate limit of fissure

	$\bar{\sigma}$	C	y_0	Limiting initial radius R_0, cm		
				a	b	c
Iron	900	9.02×10^7	0.661	0.087	0.068	0.059
Stone	380	8.75×10^7	0.702	0.066	0.058	0.056

When the radius is smaller than the value given under the heading "*c*" of Table III, the drop is expected to maintain its integrity all over its path. For other angles of incidence, or other values of H_0 the values for R_0 of Table III must be multiplied by $\sqrt{\frac{H_0 \sec z_r}{8{,}7.10^5}}$. The uncertainty in the basic physical data is considerably diminished for R_0 by the square root in formulae (31) and (35). Large drops which burst will show evidently a more violent increase in light (because the surface increases and vaporization becomes more intense), and a shorter path; larger meteors may exhibit a strong and more or less sudden outburst of light due to the spraying of the liquid into millions of drops; bursts and "spindles" observed in bright meteors may be apparently explained in this way.

However, for stone meteors viscosity may prevent fissure (Section 3. *e*); also, it appears that rotational centrifugal force may be a more powerful spraying agent than aërodynamic pressure; in such a case the outbursts of light must be explained in a different way (cf. Sections 3. *d* and 4).

In the case of fissure into smaller drops, each of radius $\sim R_s$, the surface exposed to aërodynamic heating increases in the ratio $\frac{R}{R_s} = \frac{R_0 y}{R_s}$. According to (11), $R_s \sim \frac{1}{p_s}$; according to (24), $p_s \sim \varrho \sim R_0$. Hence $\frac{R}{R_s} \sim R_0^2$; if for case *c*, Table III, $\frac{R}{R_s} = 1$, for iron of $R_0 = 0.236$ cm the surface should increase through fissure

1) Independent of velocity.

$4^2 = 16$ times, whereas the number of fragments is of the order of 4000. For still larger meteors, however, there will be no opportunity for such a sudden outburst; the nucleus will spray the liquid during the whole process of fusion; thus, according to (12), a maximum fraction of liquid mass of the order of $\eta \sim 0.1$ can be retained by the "shield", the rest being sprayed away, when $R = 25\ R_s$, or $R_0 \sim 0.5$ cm; the detached drops are pulverized into pieces of the order of R_s, small as compared with R_0, and therefore practically instantaneously vaporized. Therefore, for such large iron meteors, even when rotation is slow, visibility is synchronous with the fusion of the large nucleus. Evidently the formulae of the first approximation do not apply to the case of such large meteors. Assuming an effective angle of incidence $z_r = 45^0$, the apparent magnitude of meteors at the limit of fissure observed 45^0 from the zenith may be estimated according to Table VIII as follows:

Table IV.

Apparent Magnitude (m) at Limit of Fissure (for slowly rotating meteors).

w, km/sec	16	25	40	60	100
m, Iron	8.3	6.9	5.6	4.4	3.0
m, Stone	9.3	7.9	6.6	5.4	4.0

The first approximation may be assumed to hold, from the standpoint of fissure, still for meteors two magnitudes brighter than those of the table.

The figures of Table IV depend upon certain assumed values of the "heat factor" β, postulated to be the same for iron and stone; as only the order of magnitude of the heat factor can be estimated, our figures in Table IV may be systematically in error; however, these figures are sufficient to show that the effect of fissure and spraying may be important for naked-eye meteors, especially for those of low velocity. Earlier authors did not take into account fissure and spraying in developing theories of meteor phenomena in naked-eye meteors; thus earlier theories are similar to our "first approximation". If the neglect of fissure in Sparrow's theory [4] is not of serious consequences, because from Table IV we conclude that such a theory applies at least to the fainter naked-eye meteors, in Lindemann and

Dobson's theory [13] the neglect leads to a striking contradiction between their postulates and consequences. These authors, by assuming a very small value of $\varkappa$, arrive at very high densities of the atmosphere along an average meteor path; with such densities, violent fissure into smaller drops must occur (even the viscosity of stone cannot prevent fissure in such a case), so that a theory based on the assumption of a meteor unbroken all over its trajectory cannot have much resemblance to what actually would occur. We may add that Lindemann and Dobson's chief postulates, which led to the small value of $\varkappa$ and great atmosphere densities, namely, the assumption of an air cap in front of the meteor, and the assumption of the adiabatic formula for the temperature in such an air cap, do not hold: the first assumption for reasons put forward by Sparrow [4] and Maris [14], and explained in the following section; the second assumption because it violates the fundamental law of the conservation of energy [1]). Hence it follows that Lindemann and Dobson's theory, containing gross errors in the postulates, and leading to consequences contradicting their own postulates, must be rejected.

c. *Formation of an air cap.*

When the velocity of the meteor is large as compared with the velocity of sound in the undisturbed atmosphere, the air mass, or the mass per cm^2, accumulated in front of the meteor is proportional a) to the density of the surrounding atmosphere (independent of velocity, because the velocity of forced escape sidewise from the air cap is proportional to the velocity of the meteor [15,16]), and b) to the radius of the meteor (by reason of geometrical similarity). Thus, the air mass

$$a \sim \varrho R.$$

Neglecting fissure and spraying in the following, we assume evidently maximum values for R and ϱ (because the non-breaking meteor will penetrate into deeper portions of the atmosphere than the actual meteor that breaks); thus we get in such a

1) cf. Sparrow,[4]: "... The use of the adiabatic equation by Lindemann and Dobson is therefore equivalent to the assumption that a elocity of 60 km/sec is small compared to one of 0.5 km/sec". Also [1], p. 5 s.

manner for all possible initial conditions maximum values of the thickness a of the air cap.

A rough way to estimate a is the following. In front of the meteor the compression is so great that in spite of the rise of temperature the linear thickness of the air cap is small as compared with the radius (according to Epstein's data[16] it is of the order of $\frac{R}{4}$ for the "ideal" case and actually smaller on account of cooling through contact with the nucleus and through radiation). We assume simply an average surface density $a \frac{\text{gr}}{\text{cm}^2}$, and an effective velocity of sidewise escape kw. The mass of air caught per second by the cross section of the meteor is $\pi R^2 \varrho w$; this must be equal to the mass escaping sidewise, or to $2\pi R a k w$; hence we get

$$a = \frac{1}{2k} R\varrho.$$

In the "ideal" case, k is a large fraction; for 45^0 angle of impact, or for a quadratic profile of the projectile moving along a diagonal, Epstein's data[16] give $a = \frac{3}{2} b\varrho$, where b is one-half of the diagonal; within a close order of magnitude, for a sphere we may assume $b = R$, and

$$a = \frac{3}{2} R\varrho \quad \ldots\ldots\ldots \quad (36),$$

or $k = \frac{1}{3}$ for the "ideal" case.

With cooling in a ratio $\frac{T}{T_0}$, the velocity of escape sidewise will be smaller in the ratio $\sqrt{\frac{T}{T_0}}$ (neglecting viscosity), or a larger in the ratio $\sqrt{\frac{T_0}{T}}$. On the other hand, the fraction of heat absorbed by the nucleus is evidently close to

$$\bar{\varkappa} = 1 - \frac{T}{T_0},$$

$$\text{or } \frac{T}{T_0} = 1 - \bar{\varkappa} \quad \ldots\ldots\ldots \quad (37).$$

Hence we get for a cooling air cap

$$a = \frac{3R\varrho}{2\sqrt{1-\bar{\varkappa}}} \quad \ldots\ldots\ldots \quad (38).$$

According to this formula, a increases with increasing $\bar{\varkappa}$; according to Table II, the inverse relation takes place ($d \sim a$). A certain solution $0 < \bar{\varkappa} < 1$ must satisfy both conditions.

We are interested in the existence of an air cap only so far as it prevents the flow of heat toward the nucleus, i. e. so far as it makes $\bar{\varkappa}$ smaller; on the other hand, from (28) we infer that the largest value of $\bar{\varkappa}$ gives the largest a. Assuming $\bar{\varkappa} = 0.6$ as a maximum value which may be considered as differing sufficiently from 1, we get a maximum estimate of a [1]):

$$a_m = 2\, R \varrho \left(\frac{\text{gr}}{\text{cm}^2}\right) {}^{2)} \quad . \; . \; . \; . \; . \; . \; . \; (38').$$

Substituting $R = R_0\, y$, and ϱ from (17′) and (21′), we get for a_m an expression which contains the product

$$y\,[q + h\,(1 - y)],$$

already investigated in Subsection b. Along the meteor trail it reaches a maximum when $y = y_0$ (cf. form. (34)). At this point of the trajectory the mass per cm² of the air cap is the largest for the given meteor; for this maximum maximorum of a we get

$$a_m = C_1\, \frac{R_0^2 \cos z_r}{w^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; (39),$$

$$\text{where } C_1 = \frac{4}{3}\, \frac{\delta\,(q + h)^2}{H_0 \bar{\varkappa} h};$$

with our adopted data $C_1 = 2.09 \times 10^6$ for iron. Our incipient air cap corresponds to the case when a_m is equal to the air mass of the half-energy range, λ_0 ($d = a/\lambda_0 = 1$, cf. above). We must take into account the change of λ_0 with velocity [3]). If λ_0 refers to N. T. P., the transition value of a_m is $1.293\, \lambda_0 \times 10^{-3}$. Substituting this in (39), we calculate R_0 for different velocities and for an average $z_r = 45^0$. The results are [4]) (iron):

1) In other words, we are looking for the condition of an incipient air cap, such that a is of the order of the half-energy range, $d = 1$ in Table II, or $\frac{\varkappa}{\varkappa_0} = 0.77$; with $\varkappa_0 = 0.8$, $\bar{\varkappa} = 0.6$ appears to be close to the maximum possible value in such a case.

2) Round value of the coefficient used.

3) According to [1]; cf. Subsection h.

4) In [1], p. 23, formula (15), a factor of 10^6 is omitted. The calculations were made with the correct formula, but in Table VII, loc. cit., R_0 is given in mm instead of cm and the luminosities are overestimated. Fortunately, even this gross mistake has no effect on the conclusions made there.

w, km/sec	14.8	29.6	59.2	118.4
$10^5 \lambda_0$, cm	8.4	11.7	14.1	21.0
R_0, cm	0.40	0.95	2.14	5.07

Comparing these results with Table III, case a, we see that the air cap starts at values of R_0 much larger than the limit of fissure at fusion. Actually at the moment of complete fusion the original drop is broken into thousands and millions of drops, each of which has too small a radius for the formation of an air cap. Thus the most favourable conditions for the formation of an air cap must be sought during the process of fusion. We still overestimate the air cap if we assume that the meteor remains unbroken up to the moment of complete fusion; the maximum air mass in this case we find from (38') (with $R = R_0$), (20) and (17'):

$$a_m = C_2 \frac{R_0^2 \cos z_r}{w^2} \quad . \; . \; . \; . \; . \; (40), \text{ where}$$

$$C_2 = \frac{16}{3} \frac{\delta q_1}{H_0 \bar{\varkappa}}.$$

For iron we have

$$C_2 = 9.4 \times 10^5.$$

Table V gives the results of a computation according to (40) for $z_r = 45^0$; although computed for iron, the data may be regarded as representing the conditions for stone equally well, at least with respect to a close order of magnitude. The apparent magnitudes are guessed on the basis of some computations connected with the second approximation.

Table V.

Limiting Radius for the Formation of an Air Cap.

w, km/sec	14.8	29.6	59.2	118.4
Minimum R_0, cm	0.60	1.4	3.2	7.6
Appar. magnitude	0	−5	−11	−16

We notice further that the minimum radii found in this way correspond to an air cap of thickness less than the half-energy range (because shedding off drops and breaking makes the radius, and the air cap smaller); to obtain a noticeable effect in $\bar{\varkappa}$ we should have a layer at least four times thicker than that (cf. Table II) which means the doubling of R_0; the corresponding limiting magnitudes must be set in this case by 2 mag. brighter.

Thus we may repeat our former conclusion that "the formation of air caps is limited to fireballs of quite unusual brightness"[1]). When dealing with the statistical material of visual observations of meteors, we feel safe in neglecting the formation of air caps.

d. Rotation and oscillation of the nucleus.

In the preceding considerations we neglected the dynamical consequences of a possible rotation of the meteor. When the rotation is small, its influence on meteor phenomena is insignificant; rapid rotation, however, causes a spraying of the liquid during the process of fusion into small drops which are vaporized much faster than the integer meteor drop would have been; thus vaporization in this case is practically almost synchronous with fusion; the rotating meteor evaporates at a greater height and in a shorter time than the non-rotating meteor.

In Paper II we tried to evaluate the probable average velocity of rotation of meteors caused by collisions with other meteors; surprisingly high values are obtained; although the estimate of the average velocity of rotation is supposed to give only the order of magnitude, and although not all possible factors are taken into account, it appears highly probable that meteors actually rotate at very high speed, the mean equatorial velocity being of the order of 3000 cm/sec for $R = 0.1$ cm. If this is true, the theory of a slowly rotating meteor applies only to exceptions, whereas the rule is the theory of a meteor in rapid rotation[2]). Let us consider this case more closely.

Centrifugal force compels the fused material to move towards the "equator" of rotation, where it is shed off in drops the size of which depends upon the centrifugal acceleration, which, in notations of *II*, is equal to $\frac{U_e}{R}$. Let the effective

1) Cf. [1], p. 23.

2) For the larger stones the viscosity of the liquid may cause the two cases, of a rotating, and of a non-rotating meteor, to remain similar; cf. Subsections *e* and *f*.

radius of the drop shed off be r; a very close estimate of this quantity may be obtained from the equation

$$\frac{2}{3}\pi r^3 \delta \frac{U_e^2}{R} = 2\pi r \sigma,$$

in known notations. From this equation, for iron, and practically also for stone (with the adopted values of σ at fusion), we get

$$r = \frac{19.1\sqrt{R}}{U_e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41).$$

Substituting for U_e its expression from (9) of *II*, we get

$$r_0 = \frac{1}{60} R_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42).$$

The first drops shed are thus sixty times smaller in diameter, and sixty times sooner vaporized than the integer drop would have been. The proportionality of r and R obtained is curious.

As the process of spraying for a given nucleus goes on, the angular velocity of rotation decreases on account of loss of momentum through the liquid travelling from the "poles" toward the "equator" which it leaves with the highest possible speed. For constant angular velocity $U_e \sim R$; hence (41) yields $r \sim R^{-1/2}$, and $\frac{r}{R} \sim R^{-3/2}$. Because the angular velocity is not constant, but must decrease gradually, the increase of the ratio $\frac{r}{R}$ will proceed faster than estimated above, with a negative power of R larger than $\frac{3}{2}$; we may estimate it roughly at $\frac{r}{R} \sim R^{-5/2}$. Thus, for the validity of (42) as an initial condition, when the shrinking radius of the meteor reaches about one-fifth of its original value, $\frac{r}{R} = 1$, or no spraying or breaking can take place any more. The remnant core of the rotating meteor will move on without further fissure. The maximum diameter of individual drops (not counting fissure from aërodynamic pressure) is equal to this remnant core, or not more than about one-quarter of the initial diameter. It is clear that in the case of rapid rotation, the theory of meteor phenomena is very different from the theory, the basis of which has been exposed in the preceding sections.

The upper limit to what we called "slow rotation", in connection with the first approximation, may be defined now.

An increase, as the result of spraying, of the surface exposed to aërodynamic heating to double the original value may be considered as determining the margin between "slow" and "fast" rotation. This corresponds to $\frac{r}{R} = \frac{1}{2}$; equation (41) yields in this case:

$$U'_e = \frac{38.2}{\sqrt{R}} = 121 \sqrt{\frac{0.1}{R}} \frac{\text{cm}}{\text{sec}} \quad . \quad . \quad . \quad . \quad . \quad (43).$$

The margin equals $\frac{1}{30}^{th}$ of the probable velocity of rotation, estimated in *II*, (9). Thus, for an average naked-eye meteor, an equatorial velocity below 100 cm/sec, or less than about 100 revolutions per second, may be called "slow" from the standpoint of centrifugal force. From the reasoning put forward in *II*, such a low rotational speed appears extremely improbable, unless the space density of hyperbolic meteors is thirty times smaller than that assumed there.

Aërodynamic resistance can hardly stop or change the original rotation of a meteor considerably if it is fast enough. For a symmetrical body (sphere) the effect is practically nil, the relative loss of rotational momentum through friction during the flight in the atmosphere being comparable with the relative loss of linear velocity which is known to be small. For a non-symmetrical body aërodynamic resistance under certain conditions may stop slow rotation. In the interaction of aërodynamic pressure and rotation, two phases may be distinguished: the phase of acceleration (about one-half of the period of rotation), and the phase of retardation. For unchanged external conditions, and when the linear velocity of rotation is small as compared with the velocity of the projectile, the total momentum of acceleration is equal to the total retardation [1]), the sum of the momentum over one revolution being zero. Therefore, in an atmosphere of constant density, an initial rotation may continue indefinitely (at least during all the short time of visibility of the meteor), unless it is stopped during the first negative phase. Actually the meteor moves in an atmosphere of increasing

1) This holds exactly for the particular case of aërodynamic pressure considered here, when hydrodynamic stream lines are not formed, cf. Section 1. *d.* When an air cap is formed, conditions are very different, cf. below.

density, so that the positive and negative momenta also steadily increase. For the estimate of the order of magnitude we assume the period of revolution to be constant, and the duration of each of the two phases equal to one-half of the period; also, we neglect the difference of aërodynamic pressure due to the change of relative velocity during rotation (U_e assumed small as compared with w); actually the relative aërodynamic velocity is greater for the negative phase, and the duration of the negative phase is greater than for the positive phase; thus our assumptions mean underestimating the effect of retardation of rotation for a given density of the atmosphere.

Let the angular velocity of rotation be ω, the period thus $\frac{2\pi}{\omega}$. For a body of a given shape and dimensions the change of angular velocity during one phase is $\Delta\omega = \pm kw^2 \varrho$, the value of the constant k depending upon the shape of the nucleus; according to (15), with $\frac{dH}{dt} = -w \cos z_r$, or $H = -wt \cos z_r + \text{const.}$, we have $\varrho = e^{\frac{wt \cos z_r}{H_0}} \times \text{const.}$ Let us subdivide time into intervals of the half-period of rotation, according to the phases of interaction, and let the ordinal number of a given phase be n; we have

$$t = \frac{\pi n}{\omega}, \text{ and}$$

$$\Delta\omega = \pm kw^2 e^{bn},$$

$$\text{where } b = \frac{\pi w \cos z_r}{\omega H_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44).$$

The sum of all positive impulses up to n is equal to the sum of members of the form (44) for $n = n$; $n - 2$; $n - 4$; . . . . $n = -\infty$, and is equal to $+ kw^2 \frac{e^{bn}}{1 - e^{2b}}$; the sum of all the negative impulses up to $n + 1$ is evidently $- kw^2 \frac{e^{b(n+1)}}{1 - e^{-2b}}$; the angular velocity at the end of the $(n + 1)^{st}$ impulse (the last one being negative) is thus

$$\omega = \omega_0 - \frac{kw^2 e^{bn} (e^b - 1)}{(1 - e^{-2b})} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45).$$

When (45) yields $\omega < 0$, it means that rotation is stopped and converted into oscillations.

When b is small, which means that the atmospheric density changes little during one half-revolution, we have

$$\omega = \omega_0 - \frac{1}{2}\, k w^2\, e^{bn} \quad . \quad . \quad . \quad . \quad . \quad (45'), \text{ or}$$

$$\omega = \omega_0 - \frac{1}{2}\, k w^2\, \varrho \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45''),$$

where the arbitrary constant of the density formula is included in the factor k.

Thus, for given angular and linear velocity, and for a meteor of definite shape and dimensions, there exists a certain value of $\varrho = \varrho_s = \dfrac{\omega_0}{\frac{1}{2}\, k w^2}$ at which rotation is stopped. The larger ω_0 is, the larger is ϱ_s. Now, a meteor of given size is able to penetrate down only to a certain depth where the atmospheric density attains the largest possible value ϱ_m; when $\varrho_m < \varrho_s$, rotation cannot be stopped. Without giving details, it suffices to mention that a computation was made for a case of "typical asymmetry", — a triangular iron prism of 30^0 angle rotating around its axis and moving at right angles to it. The limiting value of the period of rotation was found to be of the order of $\dfrac{\sqrt{\sec z_r}}{75}$ seconds (based on the first approximation), independent of velocity and absolute dimensions. Thus, for a speed of rotation of about 100 revolutions per second or more the original rotation will persist all over the visible path of the meteor; curiously enough, this is near the margin of "slow" rotation (43). As in most actual cases we may expect a closer coincidence of the centre of mass with the resultant vector of the aërodynamic force than in the assumed case of the iron prism, even smaller speeds of rotation are likely to persist. But even in the case where rotation is stopped by aërodynamic resistance, it is replaced by oscillations, which maintain a period of the same order of magnitude as the period of the original rotation. From the standpoint of the equalization of the heating effect over the surface of the meteor such oscillations must be almost as efficient as rotation.

We notice that the period of rotation itself, or the mean angular velocity was assumed constant for the whole meteor trail in the preceding formulae; the assumption being admissible with respect to the order of magnitude, it cannot be retained for a more accurate treatment; the period of rotation (when near the critical value), and of the subsequent oscillation of an asymmetrical body, must change steadily along the path of the meteor.

A meteor of asymmetrical shape, if not rotating originally, when moving in a resisting medium will be forced to oscillate by aërodynamic pressure; the period and amplitude of the oscillation decrease with penetration into deeper portions of the atmosphere; the period of forced oscillation, for bodies of not too unusual a shape, may lie between $\frac{1}{10}$ to $\frac{1}{100}$ of a second for the visible portion of the path of the meteor, independent of velocity and size. It is obvious that a completely fused nucleus is not subject to all the above described effects.

All the preceding considerations are valid only in case no trace of an air cap is present, e. g., for meteors of radii considerably smaller than those of Table V; as, according to (40), the air mass of the cap changes as R_0^2, about $\frac{1}{3}$ to $\frac{1}{4}$ of the radii of Table V are sufficiently small to render the aërodynamic effect of the air cap negligible; for average meteor velocities this condition is still satisfied for practically all naked-eye objects. For the brighter meteors, however, the accumulation of air mass in front of the projectile, even when insufficient to protect the nucleus from heating, may cause systematical deviations of the aërodynamic pressure depending upon the curvature of the surface (whereas the pressure produced by independent individual molecular impacts depends only upon the area of the contour projected upon the plane perpendicular to the direction of motion); a concave surface experiences a systematically greater pressure than a convex one. Let us consider first the case of the ideal continuous aërodynamics and the extreme case of the two hemispherical cups of a Robinson anemometer. For such cups, the aërodynamic pressure upon the concave surface is from 2.5 to 2.9 times greater than upon the convex surface; the excess of pressure on the concave surface equals thus about 0.9 of the average pressure. If such a pair of Robinson cups is made to move freely in the air, a rotational impulse about 0.9 of the aërodynamic deceleration of the centre of gravity is acquired. The effective deceleration of a meteor near the end of its path may be estimated from the condition that a fraction $\varkappa$ of the relative kinetic energy lost is spent in vaporizing the meteor; the neglect of the condition of changing mass does not affect the order of magnitude of this effective quantity. Thus, from equation (10^a) we get

$$\Delta w = -\frac{2(q+h)}{\bar{\varkappa} w}.$$

For iron, the following values are obtained ($\bar{\varkappa} = 0.60$):

w, cm/sec	20×10^5	80×10^5
$-\Delta w$, cm/sec	1.32×10^5	3.30×10^4.

The equatorial velocity of rotation of ideal Robinson cups should thus attain about 0.45 Δw, which is a considerable speed.

For meteor projectiles of irregular shape, such a systematical effect must inevitably exist, although in a much smaller degree, perhaps from 1 to 10 per cent of the Robinson cup effect; for the phase of fusion, when a real asymmetry can exist, the impulse Δw must be taken equal to about $\frac{1}{5}$ of the above; hence rotational effects for fireballs result of the order of 200 to 2000 cm/sec ($w = 20.10^5$), or 60 to 600 cm/sec ($w = 80.10^5$).

For ordinary naked-eye meteors the air cap pressure effect is smaller, depending upon the number of elastically reflected atoms hitting a concave surface a second time; the reduction factor is thus smaller than $1 - \varkappa_0 = 0.20$ and may be estimated at $\frac{1}{20}$ th; thus, for such meteors which originally did not rotate, forced velocities of rotation acquired from the "Robinson cup effect" may result, amounting to from 10 to 100 cm/sec ($w = 20.10^5$), or 3 to 30 cm/sec ($w = 80.10^5$), thus "small" as compared with the expected order of magnitude of the initial rotation (3000 cm/sec). These estimates are important so far as they show that it is extremely improbable for a meteor to be devoid of rotation while moving in the terrestrial atmosphere.

e. *Heat transfer within a slowly rotating nucleus.*

Our theory of the first approximation is considerably simplified by neglecting differences of temperature within different parts of the nucleus. The question arises to what limit such a simplification is permissible.

We notice that from considerations put forward above meteors are supposed to rotate, or to oscillate, with a minimum speed of slow rotation corresponding to about 30 revolutions per second for ordinary naked-eye meteors; thus the period of one revolution is short as compared with the duration of the visibility of the meteor and therefore this slow rotation seems to be quite efficient in producing an equalization of the heating effect of different portions of the surface of the nucleus. If in the following estimates we neglect rotation as an equalizing factor, thus considering a non-rotating meteor, or a meteor moving along its axis of rotation, we evidently exaggerate differences of temperature existing in the nucleus. The least favourable for an equalization of temperature is the case of a solid nucleus where the only means of heat transfer is conductivity.

Let us first consider the phase just before fusion, when the nucleus is solid throughout. With the aid of the formulae of the approximate theory, we find for the difference of temperature between the front and the rear the maximum value

$$\Delta T < \frac{8}{9} \frac{\delta q' R_0^2 w \cos z_r}{k_t H_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad (46),$$

where $q' = q_1 - f$ is the heat up to fusion. With the constants adopted as before, for $z_r = 0^0$, $\Delta T < 200^0$, we find, for the moment of starting fusion:

$w =$	20×10^5	80×10^5	
iron, $R_0 <$	0.07	0.035	$(\Delta T = 200^0)$
apparent mag. in $45^0 >$	7	5	
stone, $R_0 <$	0.018	0.009	
appar. mag. $>$	11	9	

The adopted difference of temperature is not small; taking into account that the difference increases as R_0^2, we conclude that in the non-rotating solid nucleus equalization of temperature never takes place in naked-eye meteors. In the case of rotation equalizing the heating over the surface the values of R_0 must be increased four times [(or ΔT is 16 times smaller than in (46)]; thus the margin of equalization at starting fusion is: for iron, $R_0 = 0.28$ to 0.14 cm, for stone, $R_0 = 0.07$ to 0.035 cm, for the same velocities as above. It may be assumed that practical equalization of temperature in the solid nucleus sets in for iron meteors fainter than the 2^{nd}, and for stone meteors fainter than the 6^{th} apparent magnitude. This conclusion, derived for the moment of starting fusion, remains practically valid also for the earlier stages, although the conditions of equalization of temperature are somewhat more favourable then; also, oblique incidence of the meteor trajectory favours equalization ($\Delta T \sim \cos z_r$).

Fusion of the nucleus evidently starts at a spot exposed to the highest aërodynamic pressure, on the front side of the nucleus; the aërodynamic pressure compels the fluid to move and collect on the rear side of the projectile when rotation is slow (the fluid moves to the "equator" in the case of fast rotation); let us consider this effect more closely.

The pressure produced by the mostly inelastic impacts of the air molecules, in the absence of an air cap, is not perpendicular to the surface, but more or less in the direction of motion (cf. Section 1. *d*); therefore a tangential backward directed force acting upon the liquid layer exists, the force resembling more a force of viscosity or friction than true hydrodynamic pressure; such a tangentially dragging force, independent of the thickness of the layer, is much more efficient in propelling a viscous fluid than hydrodynamic normal pressure, the efficiency of which is proportional to the thickness of the liquid layer. To get an estimate of the existing conditions, the following theory may be regarded as sufficient. Instead of using variables in their complicated relationship, we introduce average, or effective quantities. Let ΔR denote the effective thickness of the liquid layer, flowing with a surface velocity u_s from the front hemisphere toward the rear of a nucleus of radius R, and let $\frac{\Delta R}{R}$ be small (only in such a case is the sweeping of the liquid toward the rear efficient, from the standpoint of the meteor theory). The total tangential force of aërodynamic resistance, acting upon the front hemisphere, is equal to $\frac{2}{3}\pi R^2 p_s$; this we set equal to the total viscosity friction in the liquid layer, $2\pi R^2 \nu \frac{u_s}{\Delta R}$; hence we get

$$\frac{1}{3} p_s = \frac{u_s \nu}{\Delta R} \quad \ldots\ldots\ldots \quad (a).$$

Further, for pure fusion (without vaporization, or rising temperature), equilibrium conditions require the amount fused per unit of time to be equal to the amount flowing from the front to the rear, or

$$\frac{E}{f} = \frac{u_s \delta}{2} \cdot 2\pi R \, \Delta R \quad \ldots \ldots \ldots \quad (b).$$

Substituting into (b) E from (10), and u_s from (a), we finally get the following expression for the thickness of the layer:

$$\Delta R = \sqrt{\frac{3}{2} \frac{\bar{\varkappa} \nu}{f \delta} R W} \quad \ldots \ldots \quad (47).$$

This expression does not contain absolute pressure, and, therefore, for a meteor of a given size and velocity, the equilibrium thickness of the liquid layer is a constant all over the portion of its path where fusion takes place (under the restrictions mentioned above).

As, with our assumptions, all the heat is spent in fusion, this heat is transported to the solid surface by conductivity; and whereas the bottom of the liquid is at a constant temperature of fusion (assumed 1800°), the top is hotter by an amount ΔT, so that

$$E = 2\pi R^2 k_t \cdot \frac{\Delta T}{\Delta R} \quad \ldots \ldots \ldots \quad (c).$$

With the aid of equations (10), (20), and (17), we get

$$\Delta T = \frac{2}{3} \frac{\delta q_1}{k_t H_0} \cdot R \, \Delta R . \, w \cos z_r \quad \ldots \ldots \quad (48);$$

here ΔR is to be taken from (47).

Equation (48) yields the maximum difference of temperature in the liquid layer, computed conventionally for the moment of complete fusion. We notice a considerable similarity of (48) and (46), as might have been expected. Further, as roughly speaking the luminosity I of a meteor is proportional to $R^3 W^3$, or $RW \sim I^{1/3}$, we notice that in (48) and (47) the actual argument is luminosity,

$$\Delta R \sim I^{1/6} \nu^{1/2}, \text{ and}$$
$$\Delta T \sim I^{1/2} \nu^{1/2}.$$

This is not exactly true, because the luminous efficiency changes probably with velocity and mass.

In equations (47) and (48) an important criterion presents itself, allowing us to distinguish between two fundamentally different kinds of meteor phenomena. If ΔR, and ΔT are large, it means that the liquid has not time enough to flow away, and that vaporization at the liquid surface sets in before the nucleus is completely molten. If ΔR and ΔT are small ($\Delta T < 500^0 \pm$ for stone, $< 1000^0$ for iron), the liquid is swept from the front side before it gets heated, and a further rise of temperature or perceptible vaporization does not start before all the nucleus is molten. The decision depends chiefly upon ν, the coefficient of viscosity, and k_t, the thermal conductivity (assumed to be the same for the liquid and the solid).

For stone, at the temperature of fusion, values of ν of about 100 have been observed (cf. Section 1. *b*), and, for the more elevated mean temperature of the liquid layer, a value between the extreme limits $10 > \nu > 0.1$ may be suggested (ibidem). In view of the cold bottom of the liquid layer, the upper limit $\nu = 10$ may be regarded as the more probable. With this value, and with the rest of the constants as assumed before, the following table is computed from (47) and (48):

Table VI.

Characteristics of Fusion, Stone, $\nu = 10$, Slow Rotation. m = apparent magnitude at $z = 45^0$; $z_r = 0^0$.

	$R = 1$ cm			$R = 0.1$ cm			$R = 0.02$ cm		
	ΔR	ΔT	m	ΔR	ΔT	m	ΔR	ΔT	m
$w = 16$ km/sec	0.035	(24 000^0)	−1	0.011	800^0	7	0.005	70^0	13
$w = 40$ „ „	0.054	(92 000)	−5	0.017	(3100)	5	(0.008)	270	10
$w = 90$ „ „	0.081	(320 000)	−9	0.026	(10 000)	2	(0.012)	870	7

For $R = 0.02$, ΔR comes out of the order of R itself, and thus has no analytical meaning; in such small stones, the large viscosity prevents any transport of matter; the slower meteors of such a radius ($m = 10$ and 13) are molten through before vaporization starts, whereas for $m = 7$, $\Delta T = 870^0$ suggests boiling (under low pressure) at the surface while the centre is still melting.

For $R = 1$ cm, the values of ΔT are enormous; this means that the layer never reaches the computed thickness ΔR, but starts boiling when its thickness is of the order of $\Delta R \times \frac{1000}{\Delta T}$: such a thin layer sticks to the surface by reason of viscosity. Thus, for $R = 1$ cm, the substance of the nucleus is practically vaporized, so to speak, on the spot, the thickness of the liquid layer being of the order of 0.001 cm only; most of the solid nucleus remains cold inside. This case evidently resembles the phenomena observed in large meteorites reaching the ground. $R = 0.1$ cm represents an intermediate case, although more similar to $R = 1$ cm, than to $R = 0.02$ cm.

The above results may be summarized as follows: if $\nu = 10$ (the most probable effective viscosity for stone), no sensible transport of the liquid under the influence of aërodynamic pressure (rotation assumed to be small) takes place; stone meteors brighter than the 7^{th} apparent magnitude are vaporized from the surface of a thin liquid layer, the nucleus remaining solid; those fainter than the 7^{th} magnitude maintain a more or less uniform temperature throughout, complete fusion preceding the beginning of perceptible vaporization.

If we assume the less probable inferior limit of viscosity $\nu = 0.1$ for stone, the values of both ΔR and ΔT decrease ten times; the characteristics of fusion are then as follows: for stone meteors brighter than the 2^{nd} apparent magnitude the conditions are the same as they were in the case of $\nu = 10$ for $m < 7$; for those fainter than the 4^{th} magnitude, an efficient transport of the liquid under the influence of aërodynamic friction establishes itself, which helps to equalize the temperature of front and rear (or of pole and equator for a rotating projectile); the liquid is collected in the portions less exposed to aërodynamic pressure (rear side, especially rear pole of rotation); all the solid nucleus is fused before a further rise of temperature and sensible vaporization start. For meteors fainter than the 7^{th} magnitude thermal conductivity takes the place of the immediate transport of the liquid.

As to iron, its low viscosity ($\nu = 0.01$) and high thermal conductivity altogether change the picture; the values of ΔR equal $\frac{1}{40}^{th}$, those of ΔT only $\frac{1}{1200}^{th}$ of the corresponding data of Table VI. For all naked-eye and telescopic iron meteors the frictional transport of the liquid is very efficient, ΔT is small during fusion, the nucleus melts completely into a practically isothermal drop before a further rise of temperature and perceptible vaporization start. For the larger projectiles, considerably exceeding the limit of fissure (Table III), a spraying of the liquid (cf. Section 1. *f* and 3. *b*), instead of an accumulation at the rear takes place. Only for iron fireballs brighter than magnitude — 12 the liquid layer cannot attain its equilibrium depth and starts boiling at the surface instead.

As to the liquid drop, convection currents produced by aërodynamic friction start there for the same reason which causes the transport of liquid during the process of fusion; only the transport of matter and heat in the drop is much more efficient than in the former case, because the resistance from viscosity is much smaller in the whole drop than in a thin liquid layer of same radius covering a solid nucleus. It is obvious that in all cases when complete fusion of the nucleus takes place before vaporization, the drop so formed is kept practically isothermal by the frictional convection currents (a backward current around the periphery of the "contour of resistance", balanced by a forward current around the central portion of the contour); in other words, only such liquid isolated drops can be formed which are maintained at a more or less uniform temperature either by mechanical convection, or by thermal conductivity (very small drops). Computations corroborating these conclusions have been made; they are not given here.

f. The process of fusion of a nucleus in fast rotation.

With the average speeds of rotation considered as probable in *II*, and in Section 3. *d*, the liquid formed during fusion is supposed to get immediately sprayed in the form of very small drops; as the centrifugal force in the case considered is much greater than aërodynamic friction, in all cases where the transport of the liquid by aërodynamic friction is found to be efficient the spraying by centrifugal force must be still more efficient; thus, according to the preceding subsection, all fast rotating iron meteors actually spray their substance during a more or less isothermal process of fusion; the vaporization takes place in the minute drops, the products of the spraying. The question arises whether in stone meteors the large viscosity is able to prevent such a spraying.

The liquid is supposed to move, under the action of centrifugal force, toward the equator of rotation, where it separates by drops as considered in Section 3. *d*. By analytical estimates similar to those made above and with the aid of formula (41), assuming that the drop is formed when the liquid layer has a thickness of the order r (same formula), we find the time of separation of the drop approximately equal to

$$\tau = \frac{8\pi r^2 R_0 \nu}{U_e^2 \delta} = \frac{9200\, \nu R_0^2}{U_e^3 \delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (49).$$

$\tau = 0^s.01$ may be regarded as sufficiently short an interval as compared with the life time of the meteor; assuming this as the maximum admissible for efficient spraying, and with $\nu = 10$ we find the condition

$$U_e > 130 \; R_0^{2/3} \text{ cm/sec} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50).$$

Equation (9) of *II* gives

$$U_e = 1140 \; R_0^{-1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

as a probable mean value. Condition (50) is fulfilled in this case for $R_0 < 6$ cm, thus over more than all the range to be considered.

Considering the rate of supply of the liquid from fusion to be in equilibrium with the amount flowing away toward the equator under the action of the centrifugal force, we find (notations as before) for the effective thickness of the liquid layer

$$\Delta R = \sqrt{\frac{8}{3} \frac{q_1 \nu}{f H_0 \delta} \frac{R^3 w \cos z_r}{U_e^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (52),$$

and $\frac{\Delta T}{\Delta R}$ the same as in (48). Assuming (51), with $\nu = 10$, for $\Delta T = 500^0$ as an upper admissible limit for fusion without vaporization, we find the following upper limits for the size of the stone:

Table VII.

Size of Fast Rotating Meteors with $\Delta T = 500^0$ in the liquid layer. Vertical incidence.

w	16×10^5	40×10^5	90×10^5
R_0	0.43	0.28	0.18
m	2	0	0
ΔR	0.0008	0.0003	0.0001

As in the present case $\Delta T \sim R_0^3 w^{3/2} \nu^{1/2} (\cos z_r)^{3/2}$, the limiting radii represent a relatively sharp margin; meteors rotating at the assumed speed and smaller than these limiting radii must spray their substance instantaneously during the process of fusion even if the viscosity is as large as $\nu = 10$; thus there seems to be no doubt that most naked-eye stone meteors except the brightest, if rotating fast, undergo the process of centrifugal spraying. For other values of ν, the limiting radius is $R_0 \sim \nu^{-1/6}$, thus varying slowly. For $\nu = 0.1$, the limiting magnitudes $(I \sim R_0^3 \sim \nu^{-1/2})$ must be set by 2—3 magnitudes brighter than those in Table VII. Stone meteors of greater size than the limiting one, even when rotating fast, are expected to behave like non-rotating meteors (cf. preceding subsection).

g. Radiation from the nucleus and deceleration.

The heat of vaporization being small as compared with the kinetic energy of an average meteor, the meteor is vaporized before it loses much of its initial velocity [cf. formula (10^a), also Section 3. *d*], deceleration thus being negligible for the naked-eye and the brighter telescopic meteors. This would hold also for the smallest objects but for radiation losses. Black body radiation of the nucleus, being comparatively insignificant for most observable objects, grows in relative importance with the decreasing size of the bodies; for very small particles the radiation of the nucleus may absorb all the kinetic energy, and the particle may be decelerated to zero velocity before it reaches a temperature sufficiently high for vaporization.

For a given surface temperature T, the relative importance of vaporization and radiation may be set equal to $\frac{h\xi}{Q}$. For an iron surface, the data of Table I lead to the following values of this ratio:

T	1800^0	2000^0	2200^0	2400^0	2600^0	2800^0	3000^0
$\frac{h\xi}{Q}$	0.062	0.31	1.09	3.0	6.9	13.6	24.6

Thus, for temperatures above 2200^0 vaporization takes the major part of the heat absorbed by an iron surface, whereas below 2200^0 the chief loss of heat is by radiation. For stone, the corresponding limiting temperature may be set at about 1600^0, as a guess.

Let us consider very small meteors for which radiation losses become important; neglecting the heat spent in heating the nucleus, the equilibrium condition between aërodynamic friction and black body radiation, together with the condition of deceleration [cf. (10^a)]

$$4\pi R^2 Q\,dt = \frac{4}{3}\pi\delta R^3 . \frac{1}{2} w\,dw,$$

lead to the following expression (iron):

$$T = 6.86\, w_0^{3/4} \varrho^{1/4} e^{-\frac{H_0 \sec z_r \varrho}{13.9\, R}},$$

w_0 being the initial velocity. This expression yields a maximum

$$T_{max} = 3.18\, w_0^{3/4} \left(\frac{13.9\, R \cos z_r}{H_0}\right)^{1/4} \quad . \quad . \quad . \quad . \quad (53).$$

Setting $T_{max} = 2200^0$, we find the following maximum sizes of iron particles which are stopped in their motion before sensible vaporization sets in:

	$w_0 = 16 \times 10^5$	$w_0 = 64 \times 10^5$
$\sec z_r = 1$	$R = 5 \times 10^{-4}$ cm	$R = 8 \times 10^{-6}$ cm
$\sec z_r = 10$	$R = 5 \times 10^{-3}$ cm	$R = 8 \times 10^{-5}$ cm.

The apparent magnitude of such meteors lies between the 25^{th} and the 40^{th}; thus only such "ultra-telescopic" meteors may reach the ground without losing much of their mass through vaporization.

For stone, and for an upper limit of temperature $T_m = 1600^0$, the limiting radii are found about the same as for iron.

We conclude that for all kinds of observable meteors vaporization exceeds radiation considerably and that the deceleration of the nucleus is comparatively small. On the other hand, fine meteoric dust may be stopped by the atmosphere without getting vaporized; under especially favourable circumstances (low relative velocity and oblique incidence) the dust particles may be as large as 0.01 cm in diameter; for average velocities, particles of the order of 10^{-5} cm will certainly be caught "uninjured".

h. Working tables of absolute magnitudes.

The absolute magnitude of a meteor, m_0, we define as its apparent magnitude seen from a distance of 100 km and free of atmospheric absorption. The zenithal magnitude, m_z (cf. [12]), does not differ much from m_0 for most "ordinary" meteors. With the existing system of stellar magnitudes we may write

$$m_0 = 24.6 - 2.5 \log I \quad \ldots \ldots \quad (54),$$

where I is the "visual" intensity of radiation in ergs per second, "visual" denoting the spectrum interval 4500 to 5700 Å. The average value of I over the whole visible path of a meteor is proportional to the total kinetic energy divided by the duration of visibility; thus we may write

$$I = \frac{2}{3} \beta \delta \frac{\pi R_0^3 w_0^3}{L} \quad \ldots \ldots \quad (55),$$

where L is the absolute length of the visible path, β — the visual efficiency of radiation (cf. A). The chief uncertainty of our estimates lies in the factor β. In A we tried to estimate the order of magnitude of β; the results were there given in Tables XII and XIII; the first table corresponds to the case of a sufficiently massive meteor for which the dissipation of the molecular kinetic energy takes place within the coma, or the cloud of meteor vapours; the second table corresponds to meteors which are too small for the formation of a coma, so that the dissipation of the molecular energy takes place in the free atmosphere. Whereas the absolute values of β in these tables cannot be regarded as established except for the order of magnitude, the difference between these tables is more certain. It is therefore important to know, at what minimum sizes of the meteor nucleus the formation of a coma begins. The treatment can only be rough. In spite of considerable simplifications, the theory is rather complicated and cannot be given here in all its details.

We consider first the case of a non-breaking spherical iron nucleus as in Section 2. At a certain distance behind the nucleus let d_t be the maximum transverse thickness of the coma, measured in units of the half-energy range of an average molecule; the latter, according to Paper A, Table X, changes as $w^{1/2}$, whence $d_t \sim w^{-1/2}$ for a given linear thickness and density of the coma. The fraction s_c of secondary collisions which happen within the coma is estimated as follows:

d_t	0	3	6	9	$\geqslant 12$
s_c	0.00	0.23	0.51	0.72	1.

If β_0 and β_c are the luminous efficiencies corresponding to no coma, and to a compact coma, we may set:

$$\beta = \beta_0 + s_c\,(\beta_c - \beta_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56).$$

Let $2\,x$ be the diameter of the coma at a distance y behind the nucleus, and γ the mean density over that diameter.

We have

$$d_t = \frac{2x\gamma}{bw^{1/2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57),$$

where $bw^{1/2}$ is the half-energy range in $\frac{\mathrm{gr}}{\mathrm{cm}^2}$; $b = 2{,}76 \,.\, 10^{-11}$.

The boundary of the coma is formed by the combined effect a) of the expansion, which may be regarded as an adiabatic outflow of gas into empty space (density of coma >> density of atmosphere), with a velocity $\frac{dx}{dt} = \frac{wu^{1/2}}{1+u}$, and b) of the backward motion relative to the nucleus, due to the inertia of the admixed air molecules, $\frac{dy}{dt} = \frac{wu}{1+u}$. Here u denotes the effective proportion of admixed air (cf. A, p. 34);

$$u = \frac{2h}{\varkappa w^2}\,\frac{x^2}{R^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58),$$

in adopted notations.

The paraboloid outline of the effective boundary of the coma results from the above equations as

$$2\sqrt{\frac{\varkappa w^2}{2h}}\,\frac{y}{R} = \frac{x^2}{R^2} - 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (59).$$

Further we find from the above equations and from the consideration of the total amount of vaporized material flowing backward through the cross-section πx^2 with the velocity $\frac{dy}{dt}$ (except for u very small):

$$\gamma = \varrho\,\frac{(1+u+u^2)}{u^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60).$$

From (57), (60), and (58) d_t for given ϱ and u can be determined. An effective value of $u = \bar{u}$ is defined by the condition that one-half of the energy of secondary collisions be liberated for $u \leqslant \bar{u}$. In the notations of Paper A, p. 34, this means

$$\int_0^{\bar{u}} \beta_2\,q_2\,d\left(\frac{u}{1+u}\right) = \frac{1}{2}\int_0^{\infty} \beta_2\,q_2\,d\left(\frac{u}{1+u}\right).$$

From this, $\bar{u} = 0.20$ is found. Thus, for the effective value of d_t we find generally

$$d_t = \frac{27.6 \sqrt{\frac{\bar{\varkappa}}{2h}}}{b} w^{1/2} \varrho R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61).$$

This formula applies to an arbitrary spherical nucleus of the radius R and velocity w, in an atmosphere of density ϱ; no restrictions are imposed except for the adopted dependence of the mean free path upon velocity.

For a particular meteor d_t varies as ϱR; we have discussed the same product in Section 3. b, c; along the visible path it reaches a maximum at $\frac{R}{R_0} = \frac{q+h}{2h}$; the mean value of $R\varrho$ may be assumed as equal to its value at $R = R_0$, $\varrho = \varrho_1$ as defined by equation (24); it is even an overestimate when we consider the breaking of the meteor into drops. With the constants assumed for iron we get, for the particular model of a non-breaking nucleus, with $\bar{\varkappa} = 0.6$ and $H_0 = 8{,}68 \cdot 10^5$:

$$d_{t\,eff.} = 1{,}72 \cdot 10^{12} R_0^2 w_0^{-3/2} \cos z_r \quad . \quad . \quad . \quad . \quad (62).$$

If the nucleus breaks into pieces of radius r, d_t must be set smaller in the proportion $\frac{r}{R_0} \frac{\varrho}{\varrho_1}$, where ϱ is the density of the atmosphere at the moment of separation of the fragments. In this way, from $d_{t\,eff.}$ the fraction s_c for formula (56) could be estimated. For an integer iron nucleus we find ($z_r = 45^0$):

	$w_0 = 16.10^5$	$w_0 = 40.10^5$
s_c	R_0 cm	R_0 cm
1.00	$\geqslant 0.12$	$\geqslant 0.24$
0.50	0.085	0.17
0.25	0.06	0.12
0.08	0.033	0.067

Now, from the preceding discussion of the conditions of integrity we find the maximum size of an iron drop $r = 0.059$ cm even without rotation (Section 3. b), and ϱ (near fusion) $= 0.6\,\varrho_1$. This makes $s_c = 0$ practically in all cases. $w_0 = 16.10^5$ presents an exception, but in this case (cf. A) $\beta_c = \beta_0$ practically, and the actual value of s_c is unimportant. Thus, for iron meteors except very bright fireballs, in any case for $R_0 < 1$ cm, $\beta = \beta_0$: the amount of radiation per atom is the same as for a single atom moving in the atmosphere; for the same reason, "temperature radiation" (*A*, pp. 36—38) must be negligible in iron meteors.

For stone, non-rotating and of high viscosity, $\nu = 10$, the nucleus actually remains integer and the following values may be considered as valid:

	$w_0 = 16.10^5$	40.10^5	100.10^5
s_c	R_0 cm	R_0 cm	R_0 cm
1.0	$\geqslant 0.16$	$\geqslant 0.31$	$\geqslant 0.62$
0.5	0.11	0.22	0.44
0.25	0.08	0.16	0.32
0.08	0.044	0.087	0.17

For fast rotation, the nucleus must actually break into drops, and $s_c = 0$, if the radius is smaller than the limiting value given below:

	$w_0 = 16.10^5$	40.10^5	100.10^5
$\nu = 10$, R cm $\leqslant$	0.43	0.28	0.17
$\nu = 0.1$, R cm $\leqslant$	0.93	0.60	0.37

Having estimated s_c according to these principles, the table of absolute magnitudes given below was calculated, from formulae (54), (55), (56), and Tables XII and XIII of Paper A; these tables were assumed provisionally to apply equally to stone and to iron. An empirical mean value of $L_0 = 18.10^5$ cm was adopted. If L is the actual effective length of the visible path, a correction of $+2.5 \log \frac{L}{L_0}$ must be added to the tabulated values. Larger, but yet unknown systematic corrections, depending upon composition and velocity, must be added to these values of m_0. In the case of an integer nucleus (the larger stone meteors), to m_0 as defined by (54) an additional correction for temperature radiation, in agreement with Paper A, has been added as follows:

R_0, cm	0.25	0.30	0.4	0.5	0.7	1.0
Δm_0	0.0	—0.1	—0.2	—0.3	—0.5	—0.9.

Table VIII.

Provisional Absolute Magnitudes.
a = iron meteors; b_1 = stone, $\nu = 10$, slow rotation; b_2 = stone, $\nu = 10$, fast rotation; b_3 = stone, $\nu = 0.1$, fast rotation.

w_0 km/sec and Case	R_0, cm											
	1.0	0.7	0.5	0.4	0.3	0.2	0.15	0.10	0.07	0.05	0.03	0.01
	m_0											
16 a	−1.7	−0.5	0.6	1.3	2.2	3.5	4.5	5.8	7.0	8.1	9.7	13.3
„ b_1	−2.0	−0.4	0.9	1.7	2.7	4.1	5.1	6.6	7.8	9.0	10.6	14.2
„ b_2	−2.0	−0.4	0.9*	2.2*	3.1	4.4	5.4	6.7	7.9	„	„	„
„ b_3	−2.0*	+0.4*	1.5	2.2	3.1	4.4	5.4	6.7	„	„	„	„
25 a	−3.0	−1.8	−0.7	0.0	0.9	2.2	3.2	4.5	5.7	6.7	8.4	12.0
„ b_1	−4.1	−2.5	−1.2	−0.4	0.6	2.0*	3.5*	5.1	6.5	7.6	9.3	12.9
„ b_2	−4.1	−2.5	−1.2	−0.4*	1.8*	3.1	4.1	5.4	6.6	„	„	„
„ b_3	−4.1	−2.5*	+0.2*	+0.9	1.8	3.1	„	„	„	„	„	„
40 a	−4.4	−3.2	−2.1	−1.4	−0.5	0.8	1.8	3.1	4.3	5.4	7.0	10.6
„ b_1	−6.3	−4.7	−3.4	−2.6	−1.6*	0.8*	2.3	3.8	5.1	6.3	7.9	11.5
„ b_2	−6.3	−4.7	−3.4	−2.6	−1.6*	1.7*	2.7	4.0	5.2	„	„	„
„ b_3	−6.3	−4.7*	−1.2*	−0.5	+0.4	1.7	„	„	„	„	„	„
60 a	−5.6	−4.4	−3.3	−2.6	−1.7	−0.4	0.6	1.9	3.1	4.2	5.8	9.4
„ b_1	−7.9	−6.3	−5.0	−4.2*	−2.0*	+0.1*	1.3	2.7	3.9	5.1	6.7	10.3
„ b_2	−7.9	−6.3	−5.0	−4.2*	−2.0*	+0.5*	1.5	2.8	4.0	„	„	„
„ b_3	−7.9	−6.3	−5.0*	−1.7*	−0.8	+0.5	„	„	„	„	„	„
100 a	−7.0	−5.8	−4.7	−4.0	−3.1	−1.8	−0.8	0.5	1.7	2.8	4.4	8.0
„ b_1	−9.9	−8.3	−5.9*	−4.4*	−3.0*	−1.2*	−0.1	1.3	2.5	3.7	5.3	8.9
„ b_2	−9.9	−8.3	−5.9*	−4.4*	−3.0*	−1.2*	+0.1	1.4	2.6	„	„	„
„ b_3	−9.9	−8.3	−5.9*	−4.4*	−2.2*	−0.9	+0.1	„	„	„	„	„

In the above table, for stone meteors the transition from β_0 to β_c causes a more or less sudden increase of the luminosity with the radius; the figures which correspond to this sudden change are marked with asterisks. A more or less deep depression in the frequency curve of meteor luminosities may be expected at these transition magnitudes with an excess of frequency at higher luminosities; such an effect may eventually be subjected to an observational test.

i. The shielding effect of meteor vapours.

In Section 1. *d* general mention was made of the shielding effect of the vapours which decrease the flow of heat toward the nucleus; now we can consider this effect in more detail. The shielding effect can be important only when the vapour pressure exceeds the normal component of the aërodynamic pressure, i. e., when

$$P > p_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a).$$

It is easy to show that when $P < p_n$ the vapour can only form a layer of a thickness which is small as compared with the half-energy range, of the order of

$$d < \frac{u_0}{w}{}^{1)},$$

or about $d < 0.01$, which is negligible (cf. Table II).

Let us consider a spherical nucleus for which at a certain angle of incidence, a_0, $P = p_n$. The vapours have free escape only for $a > a_0$, and the total amount vaporized per second is $2\pi(1 + \cos a_0) R^2 \xi$. With the aid of formulae (1) (with $T = 3000^0$), (8) and (10) we get, independently of the density of the atmosphere and of the radius:

$$\frac{P}{p_n} = \frac{2{,}1 . 10^4\, w}{(1 + \cos a_0)\, h \cos^2 a} \cdot \frac{\bar{\varkappa}}{(2 - \varkappa')} \quad . \quad . \quad . \quad . \quad (63);$$

here $\bar{\varkappa}$ is an average value for the whole nucleus, whereas $\varkappa'$ is a constant, independent of the thermal shielding. We assume $2 - \varkappa' = 1.4$ (Section 1. *d*).

According to equations (9) and (8), $p_n = p_0 \cos^2 a$. This leads to another form for (63):

$$\frac{P}{p_0} = \frac{2{,}1 . 10^4\, w}{(1 + \cos a_0)\, h} \cdot \frac{\bar{\varkappa}}{(2 - \varkappa')} \quad . \quad . \quad . \quad . \quad . \quad (63').$$

When $P = p_0$, which means also $a_0 = 0$, the front (maximum) aërodynamic pressure equals the vapour pressure; this is evidently the limiting case for which a complete vapour shell

1) This follows when we consider that the free length of path of the particle, of the order of $d = 1$, is travelled with a speed $w + u_0$ relative to the atmosphere, whereas the speed of the particle relative to the nucleus is u_0; thus, when losing $\frac{1}{\sqrt{2}}(w + u_0)$, the particle should reach a distance from the nucleus of only $d < \frac{u_0}{w + u_0}$, in units of the half-energy range; actually it has to lose much less, namely $\Delta w = u_0$ before it gets stopped relative to the nucleus.

starts forming around the nucleus. When $P > p_0$, a vapour "microatmosphere" shields the nucleus from the direct impacts of the air molecules. When $P < p_0$, a "bald" cap extending from $a = 0$ to $a = a_0$ is formed subject to direct impacts of the air molecules; a_0 is evidently defined in this case by the equation

$$\cos^2 a_0 = \frac{P}{p_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64).$$

Let the "unshielded" value of the accomodation coefficient be $\varkappa_0$. Evidently

$$\bar{\varkappa} < \varkappa_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65);$$

$\varkappa_0 = 0.8$ may be estimated.

For the critical case $\frac{P}{p_0} = 1$, $a_0 = 0$, we get from (63′), with $h = 6.10^{10}$:

$$\bar{\varkappa}_c = \frac{8.10^6}{w} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66^a).$$

Of course, we do not yet know under what conditions the critical case sets in; but the above equation, together with (65) furnish us with an inferior limit for the velocity below which a closed vapour shell cannot be formed. For the closed vapour shell we may safely put $\bar{\varkappa}_c < 0.50$ (cf. Table II), whence

$$w > 160 \text{ km/sec.}$$

Most meteor velocities fall without doubt below this limit; thus, already from such general considerations it is obvious that the only case of practical importance is $P < p_0$.

Nevertheless, to have things clarified as much as possible, we start by assuming the opposite case of $P > p_0$, the least favourable for $\bar{\varkappa}$. In formula (63′) $a_0 = 0$ must be set, and we get

$$\bar{\varkappa}_c = \frac{8.10^6}{w} \cdot \frac{P}{p_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66).$$

An increasing P, according to this formula, requires an increasing $\bar{\varkappa}$; on the other hand, with P increasing the shielding effect of the vapours must increase, and $\bar{\varkappa}$ must diminish; hence, for a certain value of w, definite solutions for $\bar{\varkappa}_c$ and $\frac{P}{p_0}$ must exist. The maximum shielding effect may be estimated as follows. If D is the density of vapours at the surface of the nucleus, the density at a distance x is less than $\frac{DR^2}{x^2}$ (because velocity of flow and temperature increase with the distance); assuming the latter value means committing an overestimate. Similarly, the pressure may be estimated at $< \frac{PR^2}{x^2}$. On the front side of the meteor, the expansion of the vapours may be considered to proceed until equilibrium with the aërodynamic pressure is reached, or $p_0 = \frac{2PR^2}{x^2}$, the factor 2 allowing for partly oblique incidence; hence

$$x_{max} (\widetilde{=}) R \sqrt{\frac{2P}{p_0}}.$$

The effective shielding vapour mass per cm² of cross section may be set equal to

$$a = 2 \cdot \frac{1}{2} \int_R^{x\,max} \frac{DR^2}{x^2} dx = \left(1 - \sqrt{\frac{p_0}{2P}}\right) DR \left(\frac{\text{gr}}{\text{cm}^2}\right) \quad . \quad . \quad (67).$$

Here, of the two mutually cancelling factors, 2 and $\frac{1}{2}$, the first allows for oblique incidence, the second for the neglect of the acceleration of the expanding and steadily heated meteor vapours.

The "kinetic depth", d, we obtain from (67) dividing it by $2{,}76.10^{-11}\, w^{1/2}$ (cf. preceding subsection). Setting the amount vaporized equal to $\frac{E}{h}$ (cf. form. (10)), and substituting ϱ_1 for the point of appearance (form. (24) with $R = R_0$) as representing average conditions (cf. preceding subsection), we get (with $D = 1{,}92.10^{-5}\xi$, cf. Table I, and different constants as before):

$$d_c = 0.67\, R^2 w^{1/2} \cos z_r \left(1 - \sqrt{\frac{p_0}{2P}}\right) \quad . \quad . \quad . \quad . \quad (68),$$

referring to iron.

Further, Table II for $\varkappa < 0.5$, or $d > 2$, is well represented by

$$\varkappa = \frac{1.28}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69),$$

which is actually a formula for the conductivity for heat.

Formulae (66), (68), and (69) lead to the following quadratic equation with respect to $\varkappa_c$ (or to $\frac{1}{\varkappa_c}$):

$$\frac{1.28}{\varkappa_c} = 0.67\, R^2\, w^{1/2} \cos z_r \left(1 - \sqrt{\frac{4.10^6}{w\varkappa_c}}\right).$$

Setting $y = \frac{1}{\varkappa_c}$, $n = \frac{1.92}{R^2\, w^{1/2} \cos z_r}$, $b = \frac{4.10^6}{w}$, we get:

$$y = \frac{(2n+b) \pm \sqrt{b^2 + 4nb}}{2n^2} \quad . \quad . \quad (70) \quad (y > 2).$$

It is obvious that for a larger R the shielding effect must be greater; thus, to find a minimum value for $\varkappa_c$ we may set $R \sim \infty$, or n small as compared with b; actually, at meteor conditions, $w \sim 4.10^6$, $n = 0.1\, b$ even for R as small as 0.1 cm. In this case (70) yields the following two solutions, valid for practically all meteors with $R > 0.1$ cm:

$$y_1 = \frac{b}{n^2}, \quad \varkappa_1 = \sim \frac{1}{10^6 \cdot R^4};$$

$$\text{and } y_2 = \frac{1}{b}, \bar{\varkappa}_2 = \frac{4.10^6}{w} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71).$$

The first solution has no physical meaning, as it may give for example, for $R \sim 10$ cm, $\varkappa_1 \sim 10^{-10}$, $d \sim 10^{10}$ (equation (69)), or $a \sim 10^{10}.2{,}76.10^{-11}\, w^{1/2} = 500$ gr/cm², which is more than the mass of the nucleus per cm² [1]). The second solution is in dimensional agreement with (66^a) and is the only real one. Thus, with increasing radius, for a completely shielded nucleus the value of $\bar{\varkappa}$ tends to a certain minimum value, inversely proportional to the velocity and depending upon velocity alone, the limit being practically reached already for $R \geqslant 0.1$ cm. From the above (form. 66^a), (71) must be valid only beginning from about $w \geqslant 16.10^6$, when (71) gives $\varkappa_c \leqslant 0.25$, and 66^a gives $\bar{\varkappa}_c \leqslant 0.50$; there is a contradiction, in a ratio 2:1, for the two formulae which agree only for $w = \infty$, $\bar{\varkappa}_c = 0$. In any case, the order of magnitude of $\bar{\varkappa}$ is little affected even at unusually high velocities.

Now we may proceed to the more important case of $P < p_0$. The front side of our spherical nucleus may be divided into

1) In the present subsection a is the mass per cm² formed by the meteor vapours only, without counting the air mass of the air cap.

two zones: the zone of unshielded impact, for $\alpha < \alpha_0$, with a high value of $\varkappa = \varkappa_0$, and with a relative cross section of $\sin^2 \alpha_0$; and the shielded area, with a low value of $\varkappa = \varkappa_1$, with the relative cross-section equal to $\cos^2 \alpha_0$. The shielded area starts with $\frac{p}{p_n} = 1$, and hence $x_{max} = R\sqrt{2}$ (cf. above) may be a fair estimate for this area. By analogy with equation (68) we set as above (the divisor $\cos\ \alpha_0$ allows for oblique incidence):

$$\frac{1{,}28}{\varkappa_1} = \frac{0.67}{\cos \alpha_0} R^2 w^{1/2} \cos z_r \left(1 - \sqrt{\frac{1}{2}}\right) \quad . \quad . \quad . \quad (72),$$

valid for $\varkappa_1 < 0.5$. Further, obviously

$$\bar{\varkappa} = \varkappa_0 \sin^2 \alpha_0 + \varkappa_1 \cos^2 \alpha_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (73).$$

Substituting in this $\bar{\varkappa}$ from (63′), and applying (64), we get with the proper constants

$$\varkappa_1 = \frac{4{,}0.10^6 (1 + \cos \alpha_0)}{w} - \varkappa_0 \operatorname{tg}^2 \alpha_0 \quad . \quad . \quad . \quad (74).$$

On the other hand (71) gives:

$$\varkappa_1 = \frac{6.56 \cos \alpha_0}{R^2 w^{1/2} \cos z_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72').$$

These two equations determine $\varkappa_1$ and α_0. The solution may be simplified. From (72′) it appears that $\varkappa_1$ approaches zero for large R, and is negligible even at as small values as $R = 0.2$ cm. Thus, for large radii we set $\varkappa_1 = 0$. We then get from (74):

$$\sin^2 \frac{\alpha_0}{2} = \frac{b}{2(\varkappa_0 + b)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (75), \text{ or}$$

$$\cos \alpha_0 = \frac{\varkappa_0}{\varkappa_0 + b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75'), \text{ and}$$

$$\bar{\varkappa} = \frac{b(2\varkappa_0 + b)}{(\varkappa_0 + b)^2} \varkappa_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76),$$

$$\text{where } b = \frac{4.10^6}{w} \text{ as before.}$$

On the other hand, for very small radii the shielding effect is nil and we must have $\bar{\varkappa} = \varkappa_1 = \varkappa_0$.

We notice that (75) and (76) are valid for all velocities, and that $\alpha_0 \neq 0$ except for $w = \infty$, in which case (76) becomes identical with (66^a) but for the factor $\varkappa_0$. It appears that the case $P < p_0$ is the only possible real case, whatever the velocity, and that a complete vapour shell can never be formed.

Assuming $\varkappa_0 = 0.80$, from (72′) and (74) the following values of $\bar{\varkappa}$ and α_0 are found for different values of the radius and velocity.

Table IX.

Values of $\bar{\varkappa}$ the Effective Fraction of Heat Absorbed by the Nucleus During Vaporization.

Iron sphere; $z_r = 0^0$; $H_0 = 8{,}68 . 10^5$ cm.

$w = 16.10^5$			$w = 25.10^5$			$w = 50.10^5$		
R cm	$\bar{\varkappa}$	α_0	R cm	$\bar{\varkappa}$	α_0	R cm	$\bar{\varkappa}$	α_0
(∞)	0.75	76^0	(∞)	0.71	70^0	(∞)	0.60	60^0
0.095	0.76	„	0.10	0.72	„	0.11	0.63	„
0.048	0.78	...	0.052	0.76	...	0.068	0.68	...
$\leqslant$0.037	0.80	...	$\leqslant$0.040	0.80	...	0.052	0.74	...
						$\leqslant$0.043	0.80	...

$w = 100.10^5$			$w = 160.10^5$			$w = 250.10^5$		
R cm	$\bar{\varkappa}$	α_0	R cm	$\bar{\varkappa}$	α_0	R cm	$\bar{\varkappa}$	α_0
(∞)	0.44	48^0	(∞)	0.38	40^0	(∞)	0.24	34^0
0.10	0.49	„	0.092	0.45	„	0.083	0.32	„
0.063	0.58	...	0.057	0.55	...	0.051	0.46	...
0.052	0.66	...	0.046	0.63	...	0.041	0.58	...
0.045	0.73	...	0.040	0.72	...	0.036	0.69	...
$\leqslant$0.040	0.80	...	$\leqslant$0.036	0.80	...	$\leqslant$0.032	0.80	...

The same table applies to other values of z_r and H_0 after multiplying R in the table by the factor $\sqrt{\frac{H_0}{8{,}68.10^5 \cos z_r}}$. The upper limit of R for which the table is valid is set by the formation of an air cap (cf. Table V).

For stone, the average molecular weight of the vapours is but slightly smaller than for iron, and so also is the temperature of vaporization (cf. Section 1. *a*, *b*); the ratio $\frac{\xi}{P}$ is therefore practically the same as for iron. More important is the difference in the heat of vaporization; in an integer viscous stone nucleus with its low conductivity for heat, vaporization in the thin surface layer takes place simultaneously with fusion, and the whole heat, $q+h=7{,}7.10^{10}$, for stone must be taken, instead of h alone; for a given velocity and radius the vapour pressure is smaller, and $\bar{\varkappa}$ larger; Table IX may be adapted to this case by increasing the tabular w by 30 per cent, which, however, does not introduce essential changes in $\bar{\varkappa}$. On the other hand, the per-volume heat of vaporization is smaller for stone than for iron, so that, for a given radius, the stone is vaporized at greater heights, with smaller ϱ and D, and with a greater resulting $\bar{\varkappa}$; to account for this, R in Table IX must be increased by 50 per cent.

Table IX refers to the average conditions during the process of the vaporization of a nucleus. During the process of pure fusion, and before it, no perceptible shielding can take place, and $\bar{\varkappa}=\varkappa_0=0.80$ must be assumed for this stage. Due to differences of temperature, the vapour pressure on the front side may be somewhat greater than the pressure on the rear side, which means an increased shielding effect; the extreme case is no vaporization from the rear side; the values of w in Table IX must be halved in this case; however, even a slow rotation will strongly counteract this effect, so that in an average case the effect must be much smaller.

Similarly, the shape of the nucleus must influence to some extent the value of $\bar{\varkappa}$. Deviations from spherical shape mean an increase of the relative surface for a given mass, thus a smaller density of the atmosphere, and of the vapours also; this would require a value of the effective radius in Table IX which exceeds the tabular one by a factor of the order of $\sqrt{\frac{S}{3v}\sqrt[3]{\frac{3v}{4\pi}}}$; here v denotes volume, S — surface.

For a very flat prism with dimensions 1:4:4, the factor is still as small as 1,25. Taking into account that table IX is not very sensitive to changes of radius, or velocity, and that the different effects are partly in opposite directions, we conclude that Table IX apparently represents the variation of $\bar{\varkappa}$ in most imaginable cases well, with an uncertainty that perhaps is less than 10 per cent. The value of $\bar{\varkappa}=0.6$ chosen above for the first approximation seems well to represent average conditions during vaporization. For a constant absolute magnitude (iron, Table VIII) Table IX yields:

$w =$	16.10^5	25.10^5	50.10^5	100.10^5
$m_0 = 3, \bar{\varkappa} =$	0.75	0.71	0.66	0.70
$m_0 = 0, \bar{\varkappa} =$	0.75	0.71	0.60	0.47.

Thus, for an average naked-eye meteor ($m_0 = 3$), the constancy of $\bar{\varkappa}$ for different velocities appears to be a good approximation.

Section 4.

Synopsis.

The estimates of the different physical conditions which are made in this paper permit us to obtain a clearer view of the actual problems set by the meteor theory. A meteor theory must be able to account for the chief observational facts: the display of visible radiation over a certain limited range in height. As for meteors radiation and vaporization are almost inseparable, the major problem consists in describing the rate of the vaporization of the meteor substance along its trajectory in the terrestrial atmosphere. Vaporization may take place in two different ways: 1) directly from the surface of the nucleus; 2) from small drops shed off during the process of fusion. In the second case the drops are almost instantaneously vaporized after their separation from the nucleus and fusion of the nucleus coincides in time with the visible display of the meteor; smaller amounts of heat, thus greater heights of the visible trajectory, correspond to the second case as compared with the first. The two fundamentally different processes of vaporization may have their shades and gradations.

From the variety of cases which may present themselves only a few typical ones seem to be important. These cases are listed below. For the sake of convenience, absolute magnitudes according to Table VIII are partly quoted as defining the limits of the validity of the different cases[1]). The limits can be given only approximately, of course.

1) As noticed before, the marginal cases being actually defined by two variable parameters, R_0 and w_0, are characterized by little variation in $m = f(R_0, w_0)$; the absolute magnitudes quoted here are thus a substitute for

a) Case of an isothermal sphere getting vaporized from the surface. This case is the closest to our first approximation; the difference consists in taking into account the variation of vapour pressure with temperature, the heat losses from black-body radiation, and eventually fissure of the liquid nucleus. Slowly rotating small iron meteors, of $m_0 > 0$, $R_0 < 0.2$ cm, belong to this class; those of $R_0 > 0.06$ cm will burst after liquefaction into smaller more or less equal drops which at $R_0 \sim 0.2$ cm already are small enough to be vaporized 10–20 times faster than the integer nucleus would have been; a real burst at the end of fusion may thus result, preceded, however, by intense spraying of the liquid during fusion; thus, $R_0 = 0.2$ cm represents a transition toward case b). Further, slowly rotating stones ($\nu = 10$), $m_0 > 6$ (fainter than $m_0 = 6$) belong to case a), fissure never occurring for them.

In case a), vaporization requires a supply of heat per gram of the nucleus: q preceding vaporization, h during vaporization.

a′) Case of a non-isothermal sphere with complete fusion preceding vaporization. Only iron meteors of slow rotation $m_0 < 0$, $R_0 < 0.1$ cm may belong to this group (a narrow group with high velocities and intermediate radii, not represented in Table VIII).

The supply of heat required by the nucleus for vaporization is exactly the same as in case a), and from the standpoint of the display of light the case must be closely similar to a).

b) Case of an isothermal sphere with complete spraying during fusion. To this case belong: iron meteors of slow rotation, $m_0 > 0$, $R_0 > 0.2$ cm; iron meteors of fast rotation, $m_0 > 0$, all radii; stone meteors ($\nu = 10$) rotating fast, $m_0 > 6$.

The supply of heat per gram of the nucleus is: $q_1 - f$ preceding spraying, f during spraying. The formulae of the first approximation apply also to this case, with f and $q_1 - f$ substituted for h and q.

c) Case of a non-isothermal sphere with complete spraying during the fusion. To this class belong: iron meteors of slow

the more complex characteristic (R_0, w_0); the values of m_0 need not be, and may not be accurate, because of the lack of knowledge with respect to β; at the same time, these conventional values of m_0 define in a unique manner (R_0, w_0), according to Table VIII. Cf. Section 3. c, f, h.

rotation, $m_0 < 0$, $R_0 > 0.2$ cm; iron meteors of fast rotation, $m_0 < 0$, all radii; stone meteors ($\nu = 10$) rotating fast, $-1 < m_0 < 6$.

Fusion starts at the surface when the inner portions of the nucleus have not yet reached the temperature of fusion. The heat supply per gram of nucleus is: less than $q_1 - f$ preceding spraying; more than f during spraying (q_1 in the limiting case). The formulae of the first approximation cannot be well applied to this case.

d) Case of a non-isothermal sphere with the liquid vaporized "on the spot". Stone meteors ($\nu = 10$) belong to this case, brighter than $m_0 = 6$ for slow rotation, and brighter than $m_0 = -1$ for fast rotation. Also, slowly rotating iron fireballs of $m_0 < -12$, and still larger fast rotating ones may be counted with this case, but for such bright objects complications from an air cap arise, which make them a special class (cf. next).

The display of light in case d) starts early and ends late. The heat supply per gram during vaporization is: more than h, attaining $q + h$ in the limiting case, thus larger than in all the preceding cases. This case permits of simple mathematical treatment (cf.[5]), very different from the first approximation.

e) Same as case d), but with an air cap. Very bright fireballs, and meteorites belong to this class.

In the second approximation, cases a) and a′) may be joined together; the same refers to b) and c). Thus numerical computations are required for three cases, of which d) may be treated analytically, the other two cases by mechanical quadratures.

According to the order of the relative length of the trail (or range in height) the different cases may be classified as follows (order of increasing length): b); c); a); a′); d); e).

The same is the order of the decreasing height of the point of disappearance, for constant radius and velocity. With respect to the length of the trail, however, for large size meteors c) may follow a′).

Bursts may be explained partly by the sudden spraying of the liquid, apt to happen in transition cases d) — c), and a) — b), or a′) — c); minor bursts may be due to the spraying of fusible inclusions (Troïlite); occluded gases will help spraying and favour bursts.

Conclusion.

Our first approximation of the physical theory of meteor phenomena developed above can be applied to the analysis of certain kinds of observational data, such as the average heights of meteors and their relation to velocity, luminosity, and the density gradient of the atmosphere. The first approximation gave us also a means of describing qualitatively and quantitatively the different physical conditions of meteor phenomena; the knowledge of these conditions is required to form a basis for the second approximation, a more detailed theory of meteors, a theory which must explain the variation of luminosity along the trail, as well as a number of other important details without which a complete understanding of meteor phenomena cannot be attained. The second approximation is developed by a purely numerical method, because a satisfactory analytical treatment seems to be impossible, except in a few particular cases, in view of the complexity of the conditions. It may be added that most of the computations required for the second approximation are completed. We hope to give soon the results, together with a comparison of the theory with suitable observational data.

Tartu, February 13, 1937.

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