I. On the Empirical Mass-Luminosity Relation[1].

By Jacob Gabovitš.

1. Introduction. Empirical mass-luminosity relations have been established on several occasions; the question of the statistical error in such relations mostly remains obscure. Thus, in a recent paper, Huffer (*Ap. J.* **80,** 269, 1934) published a list of known mass ratios of visual binary stars; from these data and the orbital elements he computes the masses of sixty stars and gives an empirical mass-luminosity relation using the visual absolute magnitudes instead of the bolometric magnitudes. The dispersion of the masses from the mean curve obtained from Huffer's material is considerable, but it is due more probably to observational errors than to a real cosmic spread. The aim of the present note is to confirm this supposition, as well as to establish a reliable empirical mass-bolometric magnitude relation; only the best observed binaries with good trigonometric parallaxes were chosen for this purpose.

2. Selection of Material. The masses of the two components of a visual binary star (μ_1 and μ_2) are given by the formulae:

$$\left.\begin{aligned} \mu_2 &= \frac{k\,a^3}{P^2\,\pi^3} \\ \mu_1 &= \mu_2\left(\frac{1}{k}-1\right) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1),$$

where a denotes the semi-major axis in seconds of arc, π — the parallax, P — the period in years, and $k=\frac{\mu_2}{\mu_1+\mu_2}$. As the greatest uncertainty in the determination of mass appears to originate from the error in parallax, we decided to use only stars with a probable error in the parallax not exceeding 8 per cent

[1] Seminar in Astrophysics, 1936/37, conducted by E. Öpik.

of the parallax itself. Further, we rejected stars with unknown spectra, since the bolometric correction in such cases is quite uncertain. In complex systems all visual components which are spectroscopic binaries were also rejected. The following stars were further excluded: 70 Ophiuchi A (invisible companion), 85 Pegasi (unreliable mass ratio), 61 Cygni (uncertain orbital elements), and o^2 Eridani B and Sirius B (white dwarfs). All

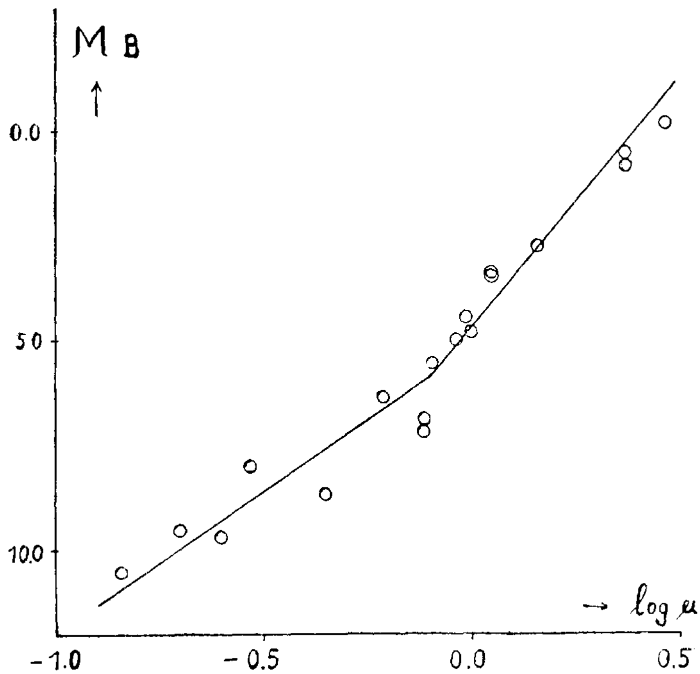

Fig. 1. The mass-luminosity relation. Abscissae — logarithm of mass; ordinates — bolometric absolute magnitude.

unsuitable stars having thus been eliminated, a small but well determined material of masses and bolometric magnitudes is left over.

3. Observational Data. Table I gives the observational data for the stars considered. The consecutive columns give: (1) the name of the star; (2) the spectrum; (3) the apparent visual magnitude (from *T. P.* **25.** 6); (4) the period of revolution in years; (5) the semi-major axis of the orbit in seconds of arc; the orbital elements are taken from Finsen's catalogue,

Table I.

The Masses and Bolometric Absolute Magnitudes of Visual Binary Stars.

1 Star	2 Sp.	3 vis. mag.	4 P	5 a	6 k	7 $\pi \pm \Delta\pi$	8 μ $\odot = 1$	9 M_v	10 C	11 ΔM_B	12 M_B	13 $\log \mu$
η Cas A	F9	3.7	479^y	11″.9	0.24	0″.182 ± 0″.005	0.93	5.0	0.41	0.02	5.0	−0.03
„ B	M0	7.4					0.29	8.6	1.38	0.62	8.0	−0.53
o^2 Eri C	M5	11.0	248	6.89	.31	.202 3	0.20	11.3	2.25	1.75	9.5	−0.70
Capella A	G1	0.74	0.285	0.0536	.44	.071 4	2.97	0.00	0.75	0.13	−0.13	0.47
„ a	F5	1.2					2.33	0.5	0.39	0.02	0.5	0.37
Sirius A	A0	−1.58	49.9	7.62	.32	.373 2	2.33	1.28	(10000°)	0.48	0.80	0.37
Procyon A	F3	0.48	40.2	4.26	.26	.291 4	1.44	2.80	0.28	0.03	2.77	0.16
γ Vir A	F0	3.65	182	3.74	.50	.089 7	1.12	3.40	0.22	0.04	3.36	0.05
„ B	F0	3.68					1.12	3.43	0.22	0.04	3.39	0.05
ξ Boo A	G5	4.80	151	4.87	.49	.147 6	0.81	5.64	0.63	0.08	5.56	−0.09
„ B	K5	6.8					0.78	7.6	1.20	0.46	7.1	−0.11
44 ι Boo A	G1	5.28	205	3.58	.54	.079 5	1.01	4.77	0.50	0.03	4.74	0.00
β 416 A	K5	5.99	42.2	1.83	.43	.147 6	0.61	6.82	1.20	0.46	6.36	−0.21
26 Dra A	G1	5.34	80.6	1.51	.47	.066 5	0.97	4.44	0.47	0.03	4.41	−0.01
μ Her B	M3	10.2	43.0	1.29	.50	.109 6	0.45	9.9	1.85	1.20	8.7	−0.35
70 Oph B	K6	6.0	87.7	4.495	.50	.196 4	0.78	7.5	1.29	0.55	6.9	−0.11
Kr 60 A	M3	9.2	44.5	2.36	0.37	0″.258 ± 0″.004	0.25	10.8	1.85	1.20	9.6	−0.60
„ B	M4	10.7					0.14	12.0	1.99	1.4	10.6	−0.84

Union Observatory Circular No. 91 (except for 26 Draconis; cf. Huffer, *loc. cit.*); (6) the mass ratio from Huffer's list; (7) the trigonometric parallax and its probable error, from Schlesinger's new Parallax Catalogue; (8) the mass, computed from formula (1); (9) the visual absolute magnitude, computed from the data in columns (3) and (7); in the case of the M type stars the TiO correction, as given in *T. P.* **28.**5, Table II, is added; (10) the colour index, taken from *T. P.* **27.**1 (F and G stars), and *T. P.* **28.**5 (K and M stars); as to Sirius A (H. D. spectrum AO), it seems to be advisable in this case to use the ionization temperature instead of the colour temperature; we assume for Sirius A: $T = 10\,000^0$ Abs.; (11) the bolometric correction (for derivation cf. *T. P.* **28.**3, formulae 7 and 9); (12) the bolometric absolute magnitude; (13) the logarithm of mass.

4. The Mass-Luminosity Relation. Fig. 1 shows the correlation between *log*mass and bolometric absolute magnitude, obtained from the data in Table I. Although the material is scanty, the correlation is of a small dispersion and thus well defined. Actually the mass-luminosity law, as given in our graph, may be fairly well represented by two linear relations between $\log \mu$ and M_B. For the A — *d*K stars we get:

$$\log \mu = 0.40 - 0.085\ M_B \quad \ldots\ldots \quad (2),$$

and for the M dwarfs:

$$\log \mu = 0.77 - 0.15\ M_B \quad \ldots\ldots \quad (3).$$

Introducing the luminosity L ($\odot = 1$) into the expressions (2) and (3), we obtain:

$$L = 0.90\, \mu^{4.7} \quad \ldots\ldots \quad (2\text{ a})$$

and

$$L = 0.56\, \mu^{2.7} \quad \ldots\ldots \quad (3\text{ a}).$$

Thus, the luminosities of the A — K stars vary nearly as the fifth power of the masses, whereas the luminosities of the M-type dwarfs are approximately proportional to the cube of the masses.

The observed dispersion of *log*mass from the mean curve [actually we take the dispersions from the straight lines (2) and (3)] is calculated from $\Delta_0 = \pm \sqrt{\frac{\Sigma \Delta^2}{n - k}}$, where n is the number of stars, and k the effective number of groups, or normal points; we assume $k=3$, because three points determine our pair of intersecting straight lines. We find:

$$\Delta_0 = \pm 0.07.$$

The observational error dispersion (Δ_e) of $\log \mu$ is given by

$$0{,}674\,\Delta_e = \sqrt{\Delta_k^2 + \Delta_a^2 + \left(\Delta_m \cdot \frac{d \log \mu}{d M_B}\right)^2 + (3\,\Delta_\pi)^2},$$

where Δ_k is the p. e. of $\log k$, Δ_a — the p. e. of $\log \frac{a^3}{P^2}$, Δ_m — the photometric error of M_B, and Δ_π — the p. e. of $\log \pi$. As the error of $\log \mu$ is mostly due to the parallax error, we shall consider the "minimum" value of Δ_e, putting $\Delta_k = \Delta_a = \Delta_m = 0$, and taking into account Δ_π only. From column 7 of Table I we get $\Delta_\pi = \pm 0.04$, whence

$$\Delta_e = \pm 0.18.$$

Thus the observed dispersion of $\log \mu$ amounts to only two-fifths of the "minimum" error dispersion, and the true cosmic spread of the masses from the mean curve must be practically zero. We conclude that, as revealed by our selected first-class data, the stars (chiefly of the main sequence) probably follow a strict mass-luminosity law.