VIII. On the Upper Limit of Stellar Masses.

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A widely adopted opinion, first expressed by Eddington, is that stars of large mass are unlikely to persist for a very long interval of time; because of the large ratio of radiation pressure to gravitation ($1 - \beta$ approaching 1), these stars, under the action of external or internal disturbing factors, may perhaps more easily divide into smaller masses than small stars with a small value of $1 - \beta$.

Alternative speculations, intended to demonstrate the existence of an upper limit to stellar masses, and based upon theoretical considerations of stellar structure, have been put forward by H. Vogt and W. Anderson.

Disregarding the uncertainty of such theoretical speculations, an uncertainty which still appears unavoidable when dealing with stellar interiors, we notice that the theoretical reasons are never physically prohibitive for the existence of large masses; the persistence of such masses, never being regarded as impossible, becomes from the theoretical standpoint a question of mere probability. The considerable attention paid to the question of an upper limit to stellar masses seems to have been stimulated by a general belief, or impression, that observations indicate the existence of such an upper limit. Below we try to demonstrate that the belief is not well founded.

In the first place, the absence of very large stellar masses from our observational records may be a mere statistical phenomenon, referring to the probability of origin, and not to the probability of persistence. A universal law makes large masses less numerous than small masses; the law holds, in broad outline, equally for meteors, planets, and stars; the trivial explanation of the law is that from a limited amount of material there can be made more small bodies than large ones. If V is the total volume (may be proportional to the total number of objects, or to the volume of space) of a sample, $p(m)$ the probability for a mass to exceed m, the expected observable number of masses greater than m is $Vp(m)$. When $p(m)$ is a rapidly decreasing function (such as a Gaussian at large values of the argument), the "observational", or catalogue limit of m is with a mostly low accidental error practically defined by the equation

$$Vp(m) = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1);$$

further, the limit of m defined in such a manner varies but slowly with V, and an apparent "upper limit" of m may result when the different volumes V in different samples remain of the same order of magnitude.

Thus, if our knowledge of the masses of celestial bodies were limited to those directly striking the earth (V = volume swept by the earth in historical time), 100 or 1000 tons might appear to us as an upper limit of mass; with respect to meteoric bodies we know that there is no physical reason for the existence of such an upper limit of mass and that actually no upper limit exists. The apparent upper limit of stellar masses ($\sim 70\odot$ at least) may be of a similar character. To test this, with respect to the order of magnitude we are allowed to assume Kapteyn's Gaussian "Luminosity-Curve" for the frequency of stellar luminosities (which generally represents the high-luminosity branch of the distribution satisfactorily), and to assume luminosity proportional to the cube of the mass,

$$L = m^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

which again is an order-of-magnitude simplification. We get, using a well known approximation for the Gaussian integral at large values of the argument:

$$Vp(m) = \frac{0.045 \quad V e^{-x^2}}{\sqrt{\pi}\, x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

where $x = \dfrac{\log \ m/m\odot + 0.36}{0.473}$, in agreement with Kapteyn's figures (Mt. Wilson Contr. **188**); V is the volume of space in cubic parsecs, of constant stardensity equal to the stardensity in the neighbourhood of the sun.

Below are computed upper limits to stellar masses, according to formulae (1) and (3).

Table 1.

Statistical Apparent Upper Limits of Stellar Masses for a Gaussian Distribution

V, cubic parsecs	10^3	10^6	10^9	10^{12}	10^{15}
Total stellar population	45	$4{,}5.10^4$	$4{,}5.10^7$	$4{,}5.10^{10}$	$4{,}5.10^{13}$
$m/m_\odot$ limit	4.7	11.5	32	74	162

The case $V = 10^{12}$ corresponds approximately to the stellar population of the whole galaxy; the statistical upper limit, 70—80 solar masses, corresponds approximately to what is believed to have been observed actually[1]. It is interesting to follow the gradual increase of the "limit" of mass as the volume of selection increases; for $V = 10^3$ ($\pi > 0''.16$), among the nearest stars a normal giant or an AO star, of mass 4—5$\odot$ may be found; for $V = 10^6$ ($\pi > 0''.016$), B stars, of mass 10—12$\odot$ occur; $V = 10^9$ corresponds more or less to the actual selection of our observational data with respect to stars of high luminosity and mass in our galaxy; *V* Puppis, *Y* Cygni, with masses ranging up to 20$\odot$, indicate that the tabular limit, 32$\odot$, is not much beside the point. The brightest star in the Large Magellanic Cloud, *S* Doradus, according to Shapley has a luminosity $\sim 4.10^5\odot$, which with the conventional cubic relation gives a mass of about 70—80$\odot$, which corresponds to $V = 10^{12}$ in our table (the true mass of *S* Doradus is probably greater, as for the Trumpler stars). If there is anything peculiar in the frequency of the more massive stars, it appears that they are observed more readily and in greater numbers than would follow from our assumed statistical law (Trumpler's O stars, loc. cit.); thus, if there is any deviation, it is in the sense that the existence of massive stars may be more secure than the existence of the smaller ones. In any case, the actual data do not imply a physical upper limit to stellar masses; the frequency of large masses seems to be governed practically by the same law as the frequency of small masses (with an excess of large masses); the law is apparently a statistical one connected with

[1] Larger masses as found by Trumpler for O stars, 100—300$\odot$ (P. A. S. P. **47**, 249, 1935), do not interfere yet with our formal way of reasoning, in which luminosity is the cube of mass, and the frequency of luminosities is actually considered. The luminosity of Trumpler's stars is only $\sim 2.10^4\odot$, corresponding to a "luminosity-mass" of $\sim 30\odot$ for our formulae.

the mode of origin of the stars, not with their physical stability. From Table 1, $V = 10^{15}$ corresponding to a population of about 1000 galaxies, we infer that the maximum stellar "cube-law" mass we may some day discover in a spiral nebula may be of the order of 160☉, corresponding approximately to absolute magnitude — 12. These data, of course, are extrapolations.

From the above it appears that our present state of observational data does not allow us to decide whether there exists an actual upper limit to stellar masses, or whether the observed limit is only the result of a "statistical perspective". But even if such a real limit could ever be observationally established, the interpretation is not necessarily bound to considerations of physical instability. On the contrary, below we try to show that a limit of this order of magnitude, produced by energetic causes, is very likely to exist, even when stellar masses of different size are mechanically equally stable.

According to the modern physical theory of stellar interiors, as first outlined by Eddington, there exists a certain mass-luminosity relation for gaseous stars, such that the luminosity is more or less independent of the degree of compression of the stellar material; for supergiant stars any kind of atomic transmutations (Atkinson) as a source of stellar energy seems to be inadequate even with the short time scale; these sources are soon exhausted and the star (as a whole, or its nucleus only) starts to contract; unless a new source of energy comes into action (annihilation of matter), the star must live at the expense of its gravitational energy — which, after all, represents an ideal mechanism for converting mass into radiant energy (cf. W. Anderson, T. P. **29**$_1$, 1936); a star of sufficiently great mass, > 1.6☉ for matter not containing hydrogen (cf. Chandrasekhar, Zts. f. Astroph. **5**, 321, 1932), cannot become a "degenerate" white dwarf; it is thus compelled to radiate with non-decreasing intensity (with the exhaustion of hydrogen, even an increase of the luminosity may follow). Thus, in one or another way, the supergiant rapidly burns down its mass. With (2) as the conventional law of luminosity (☉ = 1), the equation of variable mass (year as unit of time) becomes:

$$\frac{1}{m}\frac{dm}{dt} = -\frac{2}{3}\cdot 10^{-13}\, m^2, \text{ whence}$$

$$\frac{1}{m^2} = \frac{1}{m_0^2} + \frac{4}{3} \cdot 10^{-13} t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4);$$

here m_0 is the initial mass, m the mass after the time interval t. Assuming $m_0 = \infty$, (4) yields a maximum value for the masses of stars of an age t. Now, it is probable, although not certain, that most stars are more or less of the same age, $t = 3.10^9$ y (the "short" time scale); substituting this into (4), we get as an upper limit of stellar masses observable at present

$$m = 50\odot.$$

Such is the present mass of a star which started 3.10^9 years ago with an infinite mass.

This, again, is remarkably close to the "observed", as well as to the "statistically predicted" upper limit of the "cube-law" mass. Without doubt, there may exist massive stars of a younger age, and consequently of a greater present mass. For $t = 7{,}5.10^8$ y, $m = 100\odot$ is the maximum. Further, the luminosity for large masses varies less rapidly than with m^3, so that a longer life, or a larger upper limit of the true mass results. On the other hand, $m_0 = \infty$ cannot represent a real case; for a finite initial mass, the final mass is obviously smaller.

It is not advisable to lead our discussion further, because our considerations are based on simplifications and schematizations. We feel contented to point out, however, that by using more detailed laws instead of those expressed by formulae (2) and (3) (cf. Eddington, Internal Constitution of Stars, Cambridge 1926, p. 308 ff.), we do not arrive at essential changes in our conclusions.

Summary. There is no observational evidence for the existence of a definite upper limit to stellar masses; the observed frequency of large masses corresponds more or less to what would occur in a Gaussian distribution of mass logarithms such as suggested by Kapteyn for stellar luminosities; on the other hand, a true upper limit to stellar masses may exist as the result of the radiation of mass during the life time of the stars, if the life time amounts to a large fraction of the supposed age of the universe (3.10^9 y on the short time scale).

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