STELLAR STRUCTURE, SOURCE OF ENERGY, AND EVOLUTION

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Abstract.

A critical review of the problem of stellar structure and evolution is given, special attention having been paid to the question of the sources of stellar energy. The most probable picture, obtained from a discussion of alternative hypotheses, is summarized in the present abstract.

Atomic synthesis at present has become an experimental reality; however, details of the process in stars have not yet been established with certainty. The observed synthesis $C^{12} + H^1$, with the observed amount of carbon in the sun, is able to supply the radiation of the sun for 500 million years, at $T_c \sim 2.10^7$. If the central temperature is allowed to rise above 10^8, various observed nuclear reactions (involving capture of α-particles and the formation of neutrons) suffice to convert all the hydrogen of the sun into heavier elements, providing energy for 10^{10} years. Thus, atomic synthesis at a certain stage of stellar evolution must play, quantitatively, all the role ascribed to it by its advocates. At the most probable central temperature of the sun, $1{,}2.10^7$, corresponding to adiabatic equilibrium and 38 per cent hydrogen, the well-studied nuclear reaction $Li^7 + H^1 \rightarrow 2\,He^4$, with the amount of lithium as in the solar atmosphere, alone provides for one-fifth of the energy of the sun; the small amount of lithium itself, however, cannot present a persistent source of energy, but must be restored by a process of synthesis from hydrogen; thus, the incorporation of one proton in the above mentioned reaction must be preceded by the incorporation of seven more protons required for the synthesis of lithium, and the whole cycle, terminated by the Li + H reaction, must yield eight times more energy, or by 60 per cent exceed the actual needs of the sun. The excess of energy generation thus found lies

within the uncertainty of the estimate; it appears highly probable that a cycle of nuclear reactions, starting from hydrogen, and including the Li + H reaction, represents the main source of energy for the sun, and for the main sequence stars.

The starting reaction of the atomic synthesis is most probably the not yet observed direct synthesis of the deuteron from protons (with the expulsion of a positron). If this is a reaction of protons in their ground state, the observed rate of energy generation in the sun, together with the requirement of equilibrium with the rate of the known Li + H reaction, sets for the probability of proton capture by a proton (after penetration has taken place) as low a limit as $q = 1{,}3.10^{-19}$; in such a case there is no hope of detecting the reaction in the laboratory. The energy generation, ε, should have to vary then with about the sixth power of the temperature, $s = 6$. The correlation of radius and mass for stars of the main sequence leads, however, to $s = 19$ (from 15 to 25). Formal agreement in s can be obtained upon the assumption that one of the reacting protons must be excited to a nuclear level of about 10 000 volts; this yields $q = 10^{-14}$, thus small but more hopeful for laboratory experiments. It might be worth while to try the synthesis of deuteron from protons "activated" by X rays from 10 000 to 50 000 volts, and bombarded by H canal rays of suitable velocity.

A special giant source of energy which works at low temperatures and densities appears to be impossible: whatever assumptions as to the law of energy generation are made, if these assumptions are in harmony with general physical principles, they invariably lead to the conclusion that the central temperatures of the giants must be at least as high, or higher than the central temperatures of the main sequence stars; hence the giants must be differently built (with a greater concentration of mass towards the centre) as compared with the main sequence.

Annihilation of matter as a source of stellar energy probably cannot come into play before the atomic synthesis is exhausted (at least in a central core); in a superdense core the existence of such a process cannot be denied in principle, but its effect is practically the same as for energy generation by gravitational contraction; without losing much of the general character of our discussion, we may altogether disregard annihilation of matter as a possible subatomic process, especially as such a conception does not seem very welcome to the physicist; the gravitational energy of a contracting superdense core renders possible the radiation into space of a large fraction of the stellar mass, and is an adequate substitute for the doubtful process of annihilation of matter.

Stellar structure may exhibit different types, with different degrees of concentration of mass towards the centre: without appealing to unknown physical properties of matter created *ad hoc*, we have shown that in the course of stellar evolution governed by atomic synthesis and gravitation the stars *must* arrive at widely different types of structure, characterized by widely different ratios $T_c : \left(\frac{\beta M}{R}\right)$. The difference in the structure of giants and dwarfs is explained in this way, as well as minor differences between the dwarfs themselves.

Convection currents arise at the centre of a star (Kramers opacity) as the result of a law of energy generation $\varepsilon \sim \varrho T^s$, when $s > 6.5 - 3\, n_0$, where n_0 is the adiabatic polytropic index for the temperature-density relation. In the case of atomic synthesis being an important source of energy, convection currents always start at the centre (except when the source of energy exhibits progressive exhaustion towards the centre). Convection in stellar interiors is highly efficient for the transport of heat; therefore, wherever convection starts, adiabatic, instead of radiative, equilibrium sets in: in the sun, the convective transport of heat is provided for by as small relative deviations as 10^{-8} of temperature and pressure from their adiabatic values. Only for a very low density of matter, especially near the boundary of a star, convection becomes inadequate as a means of heat transport. The role of rotational currents is only a subordinate one. According to the extent of the central convectional region, different types of structure of main sequence stars may arise. On account of convection, negative density gradients inward are impossible (except under certain rather peculiar conditions).

The net flux of energy, L_r, cannot be computed directly from the flux of radiation, Q_r, nor can the distribution of the true sources of energy inside a star be derived from Q_r in the presence of convection; in case of a sufficient degree of concentration of the energy sources, their distribution may be quite arbitrary; Q_r in such a case is prescribed by the adiabatic structure, whereas convection undertakes the transport of the excess $L_r - Q_r$; in other words, when adiabatic equilibrium takes place in the central region at least, no functional connection can be established between the type of structure ($n_{eff.}$) and the law of energy generation (s). As, however, heat that has passed outwards of a spherical shell of the radius r cannot be transported back against the temperature gradient, the total luminosity of a star, L, must be closely equal to the maximum flux of radiation, Q_{max}, which corresponds to a certain radius $r = r_0$ depending upon the actual structure. Although L may theoretically exceed Q_{max}, because convection can transport almost an unlimited extra amount of heat, in the normal course of the evolution (contraction) of a star it settles down automatically at an equilibrium radius R, with $L = Q_{max}$; there is no imaginable way of getting the star into the "overcompressed" state required for the generation of the extra amount of energy. Therefore, a normal mass-luminosity relation must hold for adiabatic structures, as it holds in the case of pure radiative equilibrium.

The different probable types of structure of actual stars, in the order of increasing ratio $T_c : \left(\frac{\beta M}{R}\right)$, are listed below.

The complete adiabatic model. This is of homogeneous composition throughout; the "dead zone" of convection at $r = r_0$, separating the central from the marginal convective systems, is overcome by the rotational currents; as mixing is complete, the uniformity of composition is not disturbed by the progress of atomic synthesis. The position of the sun in the radius-mass correlation (Fig. 3), as well as the agreement of the heat output of the sun with the value calculated for this model from the Li + H reaction, render it highly probable that the sun is built according to the present model. The evolutional course of the adiabatic model, with the gradual exhaustion of hydrogen, is characterized by a steady increase in luminosity and a slight secular increase of the radius (if $s > 7.5$), which after reaching a certain maximum ($1.2\ R_{\odot}$ for $s = 19$ for the sun) begins decreasing again; after the exhaustion of hydrogen, and of all the minor sources of subatomic energy, the collapse begins; smaller masses become white dwarfs, larger masses which cannot become degenerate end their lives as Wolf-Rayet stars of high bolometric, but low visual or photographic luminosity, on account of their high effective temperature. The average temperature of the earth, so far as it depends upon the radiation of the adiabatic sun, should at present increase at the rate of 1^0 C each 150 million years; the increase proceeds at an accelerated rate, rendering life on earth impossible after 4000 million years; the exhaustion of hydrogen in the sun takes place after 10^{10} years, when the surface of the earth is heated to over $+600^0$ C; after that, the collapse of the sun into the white dwarf stage lowers the temperature of the earth to about -150^0 C. As to the past, the geologic history of the earth does not contradict such a conception; disregarding irregular fluctuations such as the ice ages (which are of too short a duration to influence the mean geologic temperature), a gradual increase of the terrestrial temperature at about the theoretical rate seems to be indicated; the greater frequency and extent of the glaciations in the Palaeozoicum and Praecambrium, as compared with the later eras (which contain only one ice age, the recent Diluvial), is one of the most important arguments in favour of the earth getting gradually warmer. From this standpoint, the investigation of glaciation must be considered one of the most important problems of archaean geology.

The composite adiabatic-radiative model of homogeneous composition. This originates when the extent of the central convective region does not reach to r_0; the model is only of genetic interest, because in actual stars the atomic synthesis changes the composition of the core, and transforms the model into the next following.

The composite adiabatic-radiative model of non-homogeneous composition. The exhaustion of hydrogen in the convective core increases the mean molecular weight there, a circumstance which renders impossible any further efficient interchange of matter between core and envelope. The outer layers may be in complete, or in partial radiative equilibrium; in the latter case a zone of radiative equilibrium separates the inner from the marginal convective regions. With progressing exhaustion of hydrogen, the ratio $T_c:\left(\frac{\beta M}{R}\right)$, and the radius increase, producing at an advanced stage of exhaustion of the core a kind of "semi-giant" structure. Procyon is probably in such a state. During the evolution, the range in luminosity is smaller, the time-scale shorter than for the complete adiabatic model. With the complete exhaustion of the subatomic energy sources in the core, the composite model enters the giant stage of evolution, defined by the following model.

The giant model. While the exhausted central core contracts at a rate determined by the supply of gravitational energy, the outer shell containing hydrogen cannot follow indefinitely; the inner portions of the shell settle down automatically at certain moderate effective values of temperature and density, so that the release of subatomic energy does not exceed the amount which the shell is able to transport to the surface; the peculiar distribution of the energy sources creates the typical giant structure, with a contracting superdense core and an extended shell (simple, or composite) of expanding tendency. The ratio $T_c:\left(\frac{\beta M}{R}\right)$ may increase almost indefinitely. The core gradually sucks in the exhausted material of the envelope, and the evolution also ends in a Wolf-Rayet star, rendered almost invisible on account of its high surface temperature.

Condensation of meteoric material at an early (nebular) stage of a star's life may create an initial core of small hydrogen content, favouring evolution towards a composite, eventually a giant model. An initial small exhausted core of such a kind need not disturb the complete adiabatic equilibrium in the outer regions; it is not impossible for the sun to possess such a core.

Statistical equilibrium of evolution. The observed relative frequency of main sequence stars and of giants of different luminosities in the average Galactic space (surroundings of the sun, $D < 125$ parsec) and in the Globular Clusters may be accounted for by the following hypotheses: all stars are created as main sequence objects, of which a fraction a remains completely adiabatic, whereas the fraction $1 - a$ is originally composite and develops into giants; the duration of the main sequence stage of a composite model equals 0.3 of the life of a complete adiabatic model of equal mass, and the life of a giant is at least 50 times longer than that; all stars in the Globular Clusters are of the same age of 3.10^9 years, whereas the stars in the average Galactic space have been continually born at a uniform rate during the last 3.10^9 years; the initial hydrogen content equals 40 per cent. The theoretical upper limit of luminosity of a main sequence star 3.10^9 years old, i. e. of an adiabatic model on the edge of collapse, is found equal to — 2.3 (bolo. mag.), as compared with the observed limit, — 2.0, in Globular Clusters; the corresponding mass is $1.9\odot$. More massive main sequence stars in the Galaxy must be either younger than 3.10^9 years, or contain initially more than 40 per cent hydrogen. The fraction of adiabatic models born is $a = 0.5$ for masses below $4.3\odot$, and 0.94 for larger masses; the latter result is somewhat puzzling. The interpretation of the statistical picture cannot yet be considered as definitive.

The Mount Wilson n and s subdivisions of A and B spectra are tentatively identified as the adiabatic, and the composite (semi-giant) models, respectively.

Appreciable nuclear dissociation in a collapsing core does not seem to be possible, the increase of pressure counterbalancing that of temperature; this conclusion seems to be quite definite, as possible departures from the dissociation formula at very high temperatures do not much alter the results of our computations. The formation of a pure neutron core, under conditions of thermodynamic equilibrium, or by diffusion, is impossible.

White dwarfs may be explained as collapsed main sequence stars with a small initial hydrogen content (for Sirius B, $\leqq 26$ per cent); a different initial hydrogen content in the components of a binary may be easily explained (cf. above, condensation of meteoric material); if, nevertheless, objections to that effect should be made, we should first want to have a plausible explanation of the origin of binary stars.

The ice ages, due most probably to temporary minima of solar radiation, may be explained by a temporary expansion of the sun; an ice age of one million years' duration can be produced by a total expansion of less than 1 per cent of the solar radius; during the expansion, a fraction of the heat generated is spent on mechanical work, and is withdrawn from the amount radiated into space. The withdrawal is possible only on the expense of the convectional transport of heat in the outer portions of the star, at $r > r_0$.

The spread of luminosities around a mean mass-luminosity relation, due to the hydrogen content changing with age, is not as large as has been anticipated; considering the relative speed of the exhaustion of hydrogen (proportional to the luminosity), and the equalizing effect of electron scattering, the maximum spread $\pm$ 0.46 mag is expected at $M = 2.5\odot$, decreasing for smaller as well as for larger masses.

Introduction.

Below it is attempted to give, on the basis of our present state of knowledge, a picture of stellar structure and evolution. Although guided by a few basic ideas, we have tried, whenever possible, to consider alternative hypotheses, and the conclusions we arrive at were forced upon us by the combined theoretical, experimental, and observational evidence.

We are not going to construct at present a new mathematical theory of stellar interiors, based on fixed premises chosen a priori; the amount of theoretical work already done on this subject is large enough to offer a choice, and to obtain an answer, at least a qualitative one, to the question: what happens to the stars, when the initial physical conditions are given. Stellar structure is a physical, not a mathematical problem. What matters are the premises, not the exact mathematical deductions from given premises; we want to know the actual physical conditions determining stellar structure and evolution; a "correct" mathematical theory may then easily follow *. We believe that a mere qualitative picture, taking into account all the complexity of the conditions in stellar interiors, is still a better approximation to the truth than an exact mathematical theory based on simplifications which do not take into account certain most important factors of stellar structure and evolution.

For our purposes we have to compare the present state of our physical knowledge, experimental as well as theoretical, with individual and statistical observational data referring to the stars. Nuclear physics at present has progressed far enough to lead to certain definite conclusions regarding stellar structure. Thus, it may be considered an experimentally established fact that nuclear transmutations may take

* Cf. Eddington[1], pp. 101—103, where a brilliant discussion of the role of physical premises as compared with mathematical deduction is given.

place at a considerable rate in the interior of stars with a central temperature as low as in Eddington's model, namely, at twenty million degrees, and even less. On the other hand, we cannot affirm that the energy liberated by these atomic transmutations is in all cases the only, or even the chief source, of stellar energy. With respect to the source of stellar energy, as well as in different other respects, we must allow for a vast realm of the unknown; nevertheless, it seems to be possible, by the aid of general physical principles, to set up limitations to the unknown, and to arrive at definite conclusions with respect to stellar structure and evolution.

When dealing with actual stars as they are observed at present, the question of how long they have already existed as individual stars is of primary importance. A maximum age, and in many cases a probable age of the order of 3.10^9 years may be estimated: this is the age of the universe on the short time scale. The arguments in favour of the short time scale are at present so numerous and weighty that the alternative of a much longer time scale need hardly be considered (cf. [2,3]).

Section 1.

Gravitational Energy.

a. Radiation of mass.

Gravitation as a source of stellar energy is doubtlessly inadequate for stars built on something similar to the standard model of Eddington; for white dwarfs, however, gravitation appears to be perfectly sufficient; similarly, gravitation may play an important, perhaps a predominant, role in nuclei of planetary nebulae, and in massive stars with superdense cores. Ultimately, gravitation may turn out to be the most powerful source of energy, a substitute for the annihilation of matter as postulated by Eddington, Jeans and others; in a manner imagined by W. Anderson [4] the star, fed by gravitation only, may radiate away a considerable fraction of its mass (not more than about one-half of it, however, and perhaps much less when the compressibility is limited); the number of individual atoms may remain unchanged in such a case, and the mass of each atom decrease, being converted into radiation which afterwards leaves the star; there is in fact no escape from Anderson's conception, unless the principle of the conservation of energy (mass being a form of energy) is to be abandoned in the case of stars radiating at the expense of their gravitational energy.

b. Limit of degeneracy.

According to a theorem established by Chandrasekhar [5], when the stellar mass exceeds $6.6\,\mu^{-2}\odot$ (μ = molecular weight) "the perfect gas equation of state does not break down, however high the density may become"; there may be limitations to this statement at very high densities; nevertheless, except for such unpredictable limitations, in the absence of other sources of energy, a giant star may live actually on its gravitational energy for intervals of time which are large as compared with the short time scale. The limit of mass for which such an unlimited contraction with practically undimmed luminosity appears to be possible, is for $\mu = 1$ ($\sim$ 30 per cent hydrogen), about $6.6\odot$. However, as pointed out below, the contraction and formation of a central core can start only with the exhaustion of hydrogen, thus $\mu \sim 2.1$, and the limiting mass may be estimated at $\sim 1.6\odot$ in agreement with Chandrasekhar [5]. This, however, can be valid only until a disintegration of the atomic nuclei into protons and neutrons starts at very high temperatures, which means a smaller μ again, and a higher limit of mass; however, such a nuclear dissociation cannot come into question for the existing stellar masses (cf. Section 4).

c. Nuclear dissociation and source of energy.

It is interesting to note that such a star, contracting indefinitely, cannot make use of the energy of atomic synthesis in the manner imagined by Sterne [6]; in a state of thermodynamic equilibrium as considered by him (temperatures $> 10^{9}$), energy is liberated when the material is cooling; the temperature of the contracting gaseous star rises, however, and atomic transmutations must now absorb energy instead; the star must provide energy for radiation *plus* atomic synthesis (which in this case consists in the formation of the lighter elements out of the heavier ones), and gravitational energy must pay for both, unless there is a third source of energy (which is very unlikely). The consideration of variable density, disregarded by Sterne, changes the picture altogether (Section 4); at a certain degree of compression the dissociated material begins to recombine again (Figure 1), and heat is liberated at an increasing temperature; but appreciable dissociation does not happen in such a case, for $M < 50\odot$; thus the mechanism of energy generation imagined by Sterne (and Milne, cf. [23]) does not work.

Section 2.

Annihilation of Matter.

a. The time scale.

The long time scale, which for some while seemed to dominate the minds of astronomers (including that of the writer, cf. [10]), required for the stars a powerful source of energy which, in the terminology of Eddington and Jeans, is called the annihilation of matter. We keep this label to designate processes in which a large fraction of the atomic mass, much larger than the packing fractions of the elements, 100 per cent in the limiting case, is converted into radiation or into kinetic energy, without making use of the potential energy of gravitation. The actual process need not be specified; the original idea of electrons and protons annihilating each other has lost much of its probability after the discovery of the neutron and the positron, but nevertheless it cannot be denied altogether.

With the adoption of the short time scale the hypothesis of the annihilation of matter is forced upon us only when we assume that the more luminous stars are as old, or almost as old, as the universe itself, whereas most stars get along excellently with atomic synthesis; further, if superdense cores are postulated, it becomes, from our present standpoint, practically impossible to discern between annihilation in our restricted sense, and between the gravitational conversion of mass into radiation.

b. Giants without superdense cores.

The necessity of postulating annihilation shows itself only when we consider giant stars to be built more or less according to a "standard" model without a superdense core; we propose to consider the question first from this standpoint. Thus, let us assume giants and dwarfs to be built more or less according to a homologous model (which in the general case need not be a polytropic one), so that the central densities and temperatures follow more or less the order indicated by Eddington's standard model; thus, dwarfs are hotter and denser than giants, and

$$T_c \sim \frac{\beta \mu M}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)^*.$$

The process of annihilation may be imagined to belong to one of the following classes: (α) it may be a reaction involving one single corpuscle (an atom with bound electrons, or a nucleus); (β) it may be a process of collision of two (or more) corpuscles, involving a potential barrier and thus stimulated by increasing temperature and density; (γ) it may be a process of collision of two (or more) corpuscles without a potential barrier, possibly with a limitation of the angular momentum; this leads to indifference with respect to temperature, or even to a slight decrease of the rate with increasing temperature, whereas density still favours the process. The origin of cosmic rays may be considered on this occasion, because these have been referred to as possibly connected with annihilation of matter, and as in analogy with hypothetical processes which may occur in stars [7]. It is proposed to consider the different types of processes separately.

Process (α). Radioactive processes belong to this class; the observed processes of radioactivity, however, involve much smaller energies than atomic synthesis, and cannot come into question for our purposes. Spontaneous annihilation of an H atom involves sufficient energy; such a process, however, must occur like radioactivity unaffected by temperature and density, at least when the temperature is below 10^9; it should occur everywhere with an intensity proportional to the concentration of the active material; the secular stability of stars,

* Cf. [1], p. 135.

depending upon the automatic adjustment of the energy generation to the loss prescribed by mass, radius, the law of opacity, and composition (cf. [8, 9, 10]), would be impossible in such a case, unless the stars should obtain the exact amounts of the active material required to maintain their radiation. Also, if cosmic rays could be explained in such a manner, the absence of a process of this sort in the earth's crust would speak against the possibility of it. Thus, if process (α) exists, it cannot play a conspicuous role in the energy generation of stars, and cannot help us out of our difficulties; this process may be left out of consideration in the following discussion. For annihilation of matter as a source of stellar energy, only collision processes need be considered.

Process (β). Collisions of two particles separated by a potential barrier are qualitatively similar to Atkinson's atomic synthesis [11, 12, 13, 6]; the speed of reaction is proportional to density and to an exponential function of the temperature, the latter varying extremely rapidly; although the requirements of secular stability are automatically fulfilled in this case, a single process of annihilation involving particles which are relatively abundant in the universe cannot be accepted, because in such a case dwarfs should produce much more energy than giants. An exhaustion of the energy source in dwarfs through annihilation cannot be postulated, because giants are the ones which must arrive at exhaustion first.

An escape may be found in a hypothesis put forward by Atkinson in connection with his theory of atomic synthesis: let a certain atomic nucleus A be indispensable for the reaction of annihilation, and let A, more or less stable at the low temperature of a giant, disappear by atomic synthesis at the higher temperatures of the dwarfs; obviously A can belong only to the lighter atomic nuclei; if the dwarfs are well mixed by convection currents, A may disappear altogether from the star, and the powerful source of energy will stop working in the dwarfs. If, however, the mixing is incomplete (which probably is often the case), at a certain distance from the centre there will be found a region of intense annihilation; the smaller energy production of the dwarf may in this case be ascribed to the smaller mass involved.

The greatest difficulty is to conceive how a dwarf, with such an enormous store of energy released at low temperatures, can ever get into its present condensed state; and still more so — why diffuse stars of small mass should be entirely absent. A star contracting from infinity must soon reach the state when the central temperature is sufficiently high for annihilation to balance radiation (prescribed by the mass-luminosity law); no further contraction can follow before the exhaustion of the energy source, which for dwarfs cannot come into consideration at all. Thus, on the basis of a condensation theory of the origin of the stars, with our postulated source of energy, the central temperatures of the dwarfs should be lower than those of the giants, which is exactly opposite to the actual state of affairs (on the basis of the standard model, of course).

The hypothesis that A cannot be changed or produced by atomic synthesis in stellar interiors (such elements might be those of high atomic weight), and that A, present in the giants, is absent from dwarfs, must be rejected: it is inconceivable how a certain element, which must be present in the original diffuse matter of which stars are imagined to have been built, should get only into giants (showing an inferior limit of mass), and be entirely absent from stars of small masses.

Cosmic radiation cannot originate in the diffuse interstellar matter from process (β). This process must be considered improbable, and the respective hypothesis useless, as unable to explain the existence of actual stars if their central temperatures are defined by (1).

Process (γ). To obtain the greatest contrast with case (β), we assume a constant upper limit of the angular momentum in the nuclear collision, $vr \leq$ const.; thus the target area is $v^{-2} \sim T^{-1}$; further, the probability of the reaction to take place we assume as $\sim v^{-1} \sim T^{-\frac{1}{2}}$ (which, according to Bethe[14], must hold for the capture of neutrons by nuclei); the number of collisions is proportional to the density, ϱ, and to the velocity, $\sim T^{\frac{1}{2}}$; finally we get for the probability of the reaction to happen per unit of time

$$W \sim \varrho\, T^{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

With (1), setting $\beta = 1$ for not too large masses, we get

$$W \sim \varrho^{\frac{2}{3}} M^{-\frac{2}{3}}.$$

When comparing giants and dwarfs we find that ϱ decreases as M increases; thus smaller values of W result for giants as compared with dwarfs. The difficulty is substantially the same as in process (β), and the situation becomes worse when we consider triple or multiple collisions where the effect of density is enhanced. Our approximation ($\beta = 1$) has practically no influence on the conclusion.

For the explanation of cosmic radiation the situation is slightly more favourable when we forget about the stars: if T is taken small enough, a sufficient probability of reaction may result. The absence of annihilation on the earth may be explained by the shielding effect of bound electrons which make nuclear collisions of small velocity impossible; in interstellar space there is considerable ionization; in fact, ionized hydrogen alone should be regarded as responsible for cosmic radiation from interstellar space in such a case, because complete ionization of other elements appears to be impossible there. For the interstellar space of our Galaxy $T = 3^0$ K may be estimated as a minimum (approximately the temperature of black body equilibrium; gaseous substances should attain a much higher temperature, on account of line absorption), and $\varrho \sim 10^{-24}$. As compared with a supergiant of $T = 3.10^6$ K, $\varrho \sim 10^{-6}$, we find that the star should exhibit a 10^{12} times greater activity than interstellar space with respect to the annihilation of matter. It is known that the total amount of cosmic radiation is about the same order of magnitude as the integrated light from the stars, and that the total mass of diffuse matter is also comparable with the total mass of the stars in the Galaxy. A ratio of activity of 10^{12}, however, means that a process important in stars must have zero intensity in interstellar space, and a process important in interstellar space must be enhanced in a star to approach the intensity of 10 000 Super-Nova explosions. The assumption of a partially high density of interstellar matter, such as exists on the surface of solid bodies (meteors), cannot save the situation, because the number of collisions still would depend upon the density of the surrounding gaseous medium. The conclusion seems to be definite that cosmic radiation cannot be traced to processes of annihilation (or atomic synthesis) happening in the diffuse matter of interstellar space. From the above consideration of the three possible types of the annihilation of matter we conclude that if the stars are built more or less according to Eddington's model, annihilation of matter cannot be an important source of energy. The hypothesis does not serve its purpose: it does not help us to escape from the assumption of central condensations in giant stars. Being also in disharmony with our physical knowledge, this hypothesis may safely be rejected altogether, at least for temperatures below 10^{10} K. The only process of radiation of stellar mass which we need take into account takes place through gravitation as described in Section 1.

Section 3.

Atomic Synthesis.

a. Rate of the reaction.

The theory of atomic synthesis as a source of stellar energy has been put on a sound physical basis by Atkinson[12, 13]; although the actual chain of processes involved in the synthesis of heavier elements out of hydrogen cannot yet be indicated with certainty, some of the reactions such as $Li^7 + H^1 \rightarrow 2\,He^4 + 17$ Mev are well established experimentally. The reaction takes place at a sufficient rate at comparatively low stellar temperatures, in collisions of a high multiple of kT, and thanks to the circumstance that penetration according to wave mechanics (Gamow) is possible when the relative energy of the collision is smaller than the potential barrier. A comprehensive review of the theory is given by B. Strömgren in B. VII (Ergänzungsband) of the Handbuch der Astrophysik. A somewhat more precise theory of transmutation than Atkinson's is given by Wilson[15], but his conclusion that at $T = 4.10^7$ K the rate of reaction is negligible is untenable — the reaction is perceptible even at much lower temperatures; Steensholt[16] made a numerical solution of a stellar model in hydrostatic equilibrium on the basis of Wilson's theory and finds "that it is quite possible to build up stars generating energy by proton capture in the way imagined by Wilson, assuming internal temperatures of the order of 10^7—10^8 K. This is in direct contradiction to the view of Wilson... He failed, however, to inquire closely into what is to be understood by a reasonably large rate of reaction... this fact to some extent strengthens the position of Atkinson's views." For the reaction of two elements of nuclear charges $Z_1\, e$ and $Z_2\, e$, and of density $\varrho_1 = N_1\, m_1$ and $\varrho_2 = N_2\, m_2$, respectively, the rate of decay (reciprocal of the life time) of one of the elements is given by

$$\frac{1}{\varrho_1}\,\frac{d\,\varrho_1}{dt} \sim -\frac{q\,\sigma\,\varrho_2}{m_2\,Z_2}\,F\left(\frac{Z_1^2\,Z_2^2\,m_1}{T}\right)\ldots\ (3),$$

where F is a certain exponential function (cf.[6], p. 770 f.); the formula is the result of the combination of Gamow's probability of penetration with Maxwell's law of the distribution of molecular velocities; thanks to the extremely strong dependence of the speed of transmutation upon temperature, the important range of temperature for a given reaction is rather limited; for this limited range $F = T^s$ may be assumed with sufficient approximation, thus

$$\frac{1}{\varrho_1}\,\frac{d\,\varrho_1}{dt} \sim -q\,T^s\ \ldots\ (3'),$$

with s of the order of 10 to 20. $\sigma = 10^{-25}$ cm^2 is the assumed (constant) cross-section of the target[17]; $q =$ const. $\sim$ 0.01 to 0.1, for intense reactions observed in the laboratory, is the probability of capture when penetration has taken place; for uncommon reactions q may be much smaller. Atkinson apparently disregarded the importance of the factor q, assuming it to be always of the same order of magnitude which is not the case.

b. Overstability.

The only serious objection to atomic synthesis as an energy source has been the danger of pulsational instability, or "overstability", which seemed to follow when $s > 3$ (cf.[1], p. 201 f.); as the stars do not usually pulsate, Eddington and Atkinson tried to escape from this danger by supposing that the energy is produced in two steps, so that only the first step depends upon the temperature, whereas the second step during which the major part of the energy is released is independent of temperature and covers a time interval which is large as compared with the period of the pulsation of the star. Now, however, it appears that the danger of overstability has been exaggerated, on account of imperfect analysis; Cowling[18] has shown that if there is no convection, the star is vibrationally stable save when γ, the ratio of specific heats, is nearly equal to $^4/_3$; if there is convection, the lower limit of γ for which stability begins increases from the minimum value $^4/_3$ with the increase of the exponent s in (3′); for example, for $s = 20$, $\gamma \geqslant 1.44$ is the condition for stability.

The effective value of γ for the sun may be estimated according to Eddington[1], p. 191, in the following way. The heat content, inasmuch as it depends upon the temperature, consists of the following components: the kinetic energy of the particles which are all monatomic, with $\gamma = {}^5/_3$; the imprisoned radiation, with $\gamma = {}^4/_3$; the energy of ionization and excitation, with γ near 1. Let the effective ratio of specific heat for the ionized material alone, without radiation, be Γ. Eddington gives formulae for γ as a function of Γ and β (cf.[1], p. 191, Table 28). The average value of Γ for the whole interval of temperature from 0 to T may be computed by assigning to the separate values (kinetic and ionization) of γ weights proportional to the corresponding heat contents.

The kinetic energy may be taken according to[1], p. 289; the ionization depends upon composition. A considerable hydrogen content of the stars seems to be at present highly probable[19, 20], as this removes the discrepancy between the astronomical and the physical values of the opacity. We assume, therefore, as a combination of the data of B. Strömgren[21] and Russell[22], the following schematical mean composition for the sun:

Table 1.

Mean composition, and Energy of Ionization for the Sun.

Element	H	He	O	Fe	Other metals	All
Proportion by weight	0.37	0.05	0.29	0.06	0.23	1.00
Energy of ionization, volts	13.5	78	2020	17000 *	8000 *	—
„ „ „ , 10^{-12} erg : gr	13.1	18.9	121	293	241	114

* Without the two inner K-electrons.

For the components of the heat content and the mean value of γ (not a very defintite conception) we have the following data:

Kind of energy	Kinetic	Radiant	Ionization	Γ	$\bar{\gamma}$
γ	1,667	1,333	($\geqq$)1,000	—	—
Amount, erg: gr. a) $(1-\beta) = 0.003$	$14,1.10^{14}$	$0,09.10^{14}$	$1,14.10^{14}$	1,615	1,613
„ „ b) $(1-\beta) = 0.05$	$13,5.10^{14}$	$1,5.10^{14}$	$6,0.10^{14}$	1,462	1,448

Case a) corresponds to the composition according to Table 1, with $T_c = 1,91.10^7$ K, $\bar{\mu} = 0.98$ [21]; $\bar{\gamma} = 1,613$ indicates, according to Cowling, vibrational stability even with $s \geqq 20$. Case b) is computed for Eddington's "standard" data, $\bar{\mu} = 2.11$, $T_c = 3,95.10^7$ K, on the assumption of 100 per cent of completely ionized iron, including the two K electrons (35 000 volts altogether); $\bar{\gamma} = 1.448$ is already near Cowling's limit of instability for $s = 20$. When we allow a star of given composition to contract, the content of kinetic energy changes as R^{-1}, the radiant energy per unit mass $\left(= \frac{R^{-4}}{\varrho}\right)$ changes in the same proportion (thus $1-\beta = \text{const.}$), whereas ionization remains practically constant; as a consequence $\bar{\gamma}$ increases (approaching the limiting value for $\Gamma = {}^5/_3$), and the star gets farther away from overstability, especially because the exponent s decreases rapidly with increasing temperature[6]. Hence we conclude that the stars can very well be vibrationally stable with Atkinson's mechanism of energy generation — as stable as they are pictured by observation. Pulsations maintained by energy generation may be expected only for special values of the central temperature and density, especially when the ionization is in a stage of transition so as to be oversensitive to moderate changes of temperature; Eddington ([1], p. 203 f.) has shown that a peculiar behaviour of the coefficent of opacity at such transition phases of ionization may be itself a cause of vibrational instability; both causes of pulsation may perhaps be expected to cooperate in the Cepheids.

c. Giant and dwarf energy generation.

To account for the puzzle of giants which produce more energy at a low temperature than dwarfs at a high one Atkinson[12] assumes two main processes of energy generation by transmutations; one working intensely at low temperatures (hypothetically identified as He → Li → Be^8 → He) and stopped at high temperatures (rapid transmutation of Be^8 into heavier nuclei), when a second, or several, new sources come automatically into action.

The situation is exactly the same as discussed in Section 2.*b*, "process (β)", and the conclusion is identical*:

from the genetic standpoint it is inconceivable how the dwarfs (all dwarfs!), in the process of contraction, could ever reach the second, hot-temperature stage; the contraction must stop as soon as the first, "giant" source of energy comes into action, a source which, on the short time scale, should be inexhaustible for dwarfs; for giants and dwarfs the same group of processes of atomic synthesis must exist, which come into action step by step as the temperature increases; the start is made at approximately the same T_c (in giants higher than in dwarfs), and in giants exhaustion begins earlier and the temperature has to rise in order to open up the next source of energy. Giants cannot get enough energy at all unless their central temperatures are higher than those of the dwarfs, this can never be attained for a homologous structure of giants and dwarfs. Thus, giants must possess superdense cores, probably formed by the collapse of the central portions after the exhaustion of the original source of energy (exhaustion of hydrogen in the central region, cf. below). We thus arrive at the conception of Milne's superdense cores, only in giants of course, but on a different basis of reasoning than Milne's[23], who thought that subatomic energy can be efficiently released only at temperatures exceeding 10^{10} degrees; we know that this is not correct. As to Russell's "giant stuff", it may be most probably identified with gravitation.

* At stellar temperatures Be^8 is apparently never formed directly from Li, and Be^8 seems to be stable (cf. below); thus regeneration of He^4 follows without delay, and Atkinson's mechanism, by which dwarfs had to escape the regeneration of He, does not work.

d. The lithium-hydrogen reaction.

The possible chain of processes of atomic synthesis leading to the liberation of subatomic energy has been discussed by Atkinson[12, 13]; however, there is little certainty in the details, although the general picture is more or less clear; experimental data could only settle the question.

A transmutation of considerable energy generation, well investigated in the laboratory, is the hydrogen-lithium reaction. We may inquire into the probable importance of this reaction in the energy budget of the sun. In the reaction $Li^7 + H^1 = 2\,He^4$, the rate of energy generation is proportional to the amount of hydrogen present multiplied by its rate of decay; the rate of decay is given by equation (3) when ϱ_2 is the density of lithium. According to Russell[22], the abundance of lithium amounts to $3{,}8 . 10^{-8}$ of mass in the solar atmosphere, which is 1 000 times less than in the earth's crust. If the scarcity of lithium in the sun is caused by atomic transmutations, the relative amount of this metal in the solar atmosphere cannot be less than in the interior of the sun; assuming Russell's value, we possibly overestimate the rate of the generation of energy. On the other hand, by assuming a minimum value $q = 0.01$ for the probability of nuclear capture[24], we underestimate the rate of the reaction. Further, although we consider the capture of only one proton in our reaction, it is clear that the supply of lithium must be continually replenished, unless lithium were allowed to disappear within a rather short interval of time ($\sim 10^5$ years, for the actual sun); whatever the chain of the synthesis of lithium, if the synthesis starts from hydrogen it means seven protons more captured; the total energy released for each capture $Li^7 + H^1 = 2\,He^4$ is thus equivalent to the mass defect $8\,H^1 - 2\,He^4$, which corresponds to $6{,}4 . 10^{18}$ erg per gram of hydrogen. Similarly, for the rate of decay of hydrogen we have to take eight times the value which directly follows from (3); for the latter we used Sterne's table (cf.[6], p. 774) of the values

$$\varrho_2 \left[\frac{1}{\varrho_1} \frac{d\varrho_1}{dt} \right]^{-1}$$

for the hydrogen-lithium reaction.

According to B. Strömgren[21], we assume 37 per cent of hydrogen and $\bar{\mu}=0.98$ (corresponding to $\alpha=2.5$ for the mass-luminosity relation). The temperature and density, for the first approximation, we take according to the polytrope $n=3$ (Emden's tables; also cf.[1], pp. 83, 85 and 136). The computations are as follows:

Fraction of internal mass $\frac{Mr}{M}$	T 10^6 deg.	δ density gr/cm^3	$\log\left(\frac{1}{\varrho_1}\frac{d\varrho_1}{dt}\right)$ sec^{-1}	Life of hydrogen, years	Energy, erg per gr of solar mass and second
0.0000	19.1	76	−13.38	8.10^5	250
0.0025	19.0	74	−13.42	8.10^5	1300
0.0192	18.3	67	−13.62	$1,3.10^6$	1600
0.0595	17.4	58	−13.97	3.10^6	1000
0.125	16.3	47	−14.37	8.10^6	520
0.212	15.0	37	−14.88	$2,4.10^7$	170
0.312	13.7	28	−15.44	9.10^7	42
0.418	12.4	21	−16.10	4.10^8	6
0.498	11.1	15	−16.83	2.10^9	1
1.000	0	0	...	....	
Sum ...	...	...	....	....	4889

For $\frac{1}{\varrho_1}\frac{d\varrho_1}{dt}$ is assumed the value $8\,q\,\varrho\cdot\frac{1}{P}$, where P is the life time as tabulated by Sterne (graphical interpolation used); here $8\,q\,\varrho = 8.0{,}01.3{,}8.10^{-8}\,\delta = 3.10^{-9}\,\delta$ is taken (the factor eight allows for the eight protons finally bound after the Li + H reaction is accomplished). The contribution to the energy per gram of solar mass is $0{,}37.6{,}4.10^{18}\,.\,\Delta\left(\frac{M_r}{M}\right)\cdot\left[\frac{1}{\varrho_1}\frac{d\varrho_1}{dt}\right]$, for a given shell containing the fraction $\Delta\left(\frac{M_r}{M}\right)$ of the total mass.

The computations give a total of 4 900 erg per gram of solar mass and second as the energy developed by the hydrogen-helium synthesis with the H + Li reaction as the final phase for a polytropic model $n=3$. This is 2500 times larger than the actual radiation of the sun; a reduction of the internal abundance of lithium in such a ratio would lead to agreement, but it appears from the following that so large a reduction is not necessary. From the table we infer that 50 per cent of the energy is developed by the central fraction 0.03 of the whole mass, at an effective temperature of energy generation $T_\varepsilon = 17{,}4.10^6$ K $= 0{,}94\ T_c$. Such a concentration of the energy sources corresponds closely to the mathematical point-source case; practically this leads to convective adiabatic equilibrium, with $n = \frac{1}{\gamma - 1}$ (cf. below), for which T_c is lower than for the "standard" model $n = 3$. Assuming $\gamma = 1.615$, $n = 1.63$ (cf. Subsection *b*), the temperature (according to Table 2 below) amounts to 0,651 of the above. A second correction in the same direction is required to account for the different energy output (at $n = 1.63$, $\alpha = 2.2$, cf. Table 2, interpolation of $\frac{1}{\alpha}$); as compared with the case $\alpha = 2.5$, the polytrope $n = 1.63$ requires a slightly larger hydrogen content, corresponding to a further reduction of T_c by 2.0 per cent. Thus, for T_ε, the effective "working" temperature of the sun, we get at $n = 1.63$ only 0.638 of the first adopted value, or $T_\varepsilon = 17{,}9 \,.\, 10^6 \,.\, 0{,}638 = 11{,}4.10^6$ ($T_c = 12{,}1.10^6$; 38,5 per cent hydrogen instead of 37), which, according to Sterne's table, leads to a decrease in the rate of the reaction in the ratio 200 : 1 (the exponent s in (3′) becomes 13,3). Further, the central density for $n = 1.63$ is 7.6 times less than for $n = 3$. On the other hand, a 4 per cent increase follows from the increased hydrogen content. The corrected rate of energy generation in the sun from the Li + H reaction becomes now $\frac{4889.1{,}04}{200.7{,}6} = 3{,}2$ erg/gr. sec., which is very close to the actual rate (1,9 erg/gr. sec.).

Thus, the helium synthesis through lithium is able to account for the energy generation of the sun; the difference between the computed and observed figures is smaller than the uncertainty involved in the computation. This does not preclude the possibility of further synthesis; e. g., if oxygen is the last step, sixteen protons instead of eight are incorporated, and the Li synthesis step will share only about one-half of the total energy output; in such a case, however, the central temperature must be higher (central condensation), and the internal abundance of lithium much smaller than assumed (cf. Section 7).

The $Li + H \rightarrow 2\,He$ reaction was considered by Atkinson[12] as a low temperature reaction, characteristic of the "giant" source of energy: he, of course, postulated Be^8 as the first product, but Be^8 seems to be produced as a resonance effect only at high relative energies of the collision, of $4{,}5.10^5$ and 9.10^5 volts, energies which cannot come into question at $T \sim 2.10^7$ K. Atkinson's "giant stuff" actually works in dwarfs; as we have seen above from general considerations, the existence of a special "giant stuff" is improbable.

e. Probability of the direct deuteron synthesis.

In the chain leading to the synthesis of Li, some of the reactions must be extremely rare; such should be the reaction $H^1 + H^1 = H^2 + \beta_+$, which Atkinson[13] considered as the most probable starting reaction of the synthesis; to keep pace with the formation and decay of lithium, this reaction should occur at an equal absolute rate with the $Li^7 + H^1$ reaction; considering the comparative easiness of the reaction ($Z_1 = Z_2 = 1$, cf. (3)), the speed of which is comparable to the lithium reaction at a temperature nine times higher, a value of $q \leqslant 1{,}3.10^{-19}$ must be postulated: such a small upper limit to the probability of the capture of a proton by another is indicated by the fact that the sun actually does not explode from this reaction. In such a case, there is no hope of discovering the reaction in the laboratory: for canal rays the fraction of energy dissipated in central collisions being of the order of 10^{-4} of all collisions, 1 in 10^{23} collisions of protons with hydrogen atoms is expected to yield a deuteron nucleus; a whole gram of hydrogen canal rays yields less than six atoms of H^2.

f. Equilibrium of abundance for intermediate steps.

As to the intermediate members of the chain of atomic synthesis, no such limitations of the value of q can be derived for them from solar observations; the equilibrium abundances of the different intermediate elements will automatically settle themselves in ratios inversely proportional to the speeds of the corresponding reactions (Atkinson's theory of abundance); and there is apparently no certain way of testing the abundances even when the speeds of reaction are known from experiment, because the observed abundances refer to the solar atmosphere, not to the interior; although, as follows from the case of lithium, perhaps in the sun (if not in all stars) mixing may be complete, and the observed composition is indicative of the conditions in the interior. Only the first element in the chain (H^1), and the last effective one (He^4, or O^{16}, perhaps both), will not possess equilibrium abundance: the first will gradually disappear, the last will steadily accumulate. Some of the light nuclei which are known to react easily in the laboratory (such are H^2, Li^6, Li^7, also the neutron), must be extremely rare in the interior of the sun. The neutron must be practically absent (cf. below).

g. The starting reaction.

The starting point of the synthesis is the most important one; the approximate constancy of the central temperatures of the main sequence stars must be the direct consequence of the laws governing this first reaction, without regard to the following steps (the rate of the subsequent reactions is completely determined by the rate of the first reaction, cf. above, equilibrium of abundance). The dependence of the average energy generation upon temperature is also entirely determined by the first reaction. However, for short period fluctuations of the temperature (pulsations), during which the absolute abundance of the elements has not time enough to change considerably, all the links of the chain of synthesis contribute to the dependence of energy generation upon temperature. Unfortunately, we do not know anything definite with respect to this first step. Atkinson [13, 12] has made certain hypotheses which may remain unproved for a long while. There are practically three possibilities to be considered.

(α) The reaction ${}_1H^1 + {}_1H^1 = {}_1H^2 + \beta_+ + (0.41 \pm 0.05)$ Mev, already mentioned above. The probability of capture must be as small as $q \leqslant 1{,}3.10^{-19}$, to account for the rate of the generation of solar energy, and there is no hope of ever detecting the reaction in the laboratory; we are here confronted with a dilemma: if the reaction were detectable experimentally, the sun would blow up from the immense energy generation; or, rather, the sun could exist only as a diffuse star of spectrum

M of about nine times its present radius (Atkinson [13] finds that the probability of the reaction "should be much too high at main sequence temperatures"); if, however, the reaction takes place in the sun, there would be an upper limit to the probability of the reaction, which makes it undetectable experimentally. The well-known mass defects of H^1 and H^2 cannot allow of much change in the energy developed by the reaction; and even if this were zero (which is impossible), the liberated positron would soon combine with an electron, thus releasing about one million volts of energy; thus the energy of the reaction alone would be sufficient to make the sun explode if q is not as small as assumed above, not counting the much larger energy released in the subsequent steps of the synthesis after H^2 is formed.

Another possibility is the absence of hydrogen in the interior of the sun; with q of the order of from 0.01 to 10^{-6} for our reaction, hydrogen must be absent from 95 to 90 per cent of the internal mass; in this case, of course, not much energy from atomic synthesis could be obtained; however, such a possibility must be ruled out by the hydrogen content known to be large, from 0.30 to 0.37 of the whole mass [19, 20, 21].

(β) The reaction ${}_1H^1 + \beta_- = {}_0n^1 - E$, or the formation of neutrons in the collision of a proton with an electron at the expense of a yet not very accurately known energy E. This is an endothermic reaction and, as such, it can occur only when the relative energy of the collision exceeds E; on the other hand, apparently there exists no potential barrier, and the rate of the reaction is proportional to the Maxwell frequency of kinetic energies exceeding E, multiplied by an unknown probability q. Negative results of some experiments (cf. [13], p. 79 f.) are not conclusive, at most they show that q is small. The value of E, which depends upon the mass excess of the neutron, is not well determined. From recent data we may try to estimate it anew. Some of the atomic weights of the lightest atoms determined by different methods are as follows*:

* The weights refer to the neutral atoms; to get weights of the nuclei, such as they occur in atomic transmutations, 0.00054 must be subtracted for each bound electron; failure to observe this may lead to misunderstandings (such as in [4], p. 65).

	H^1	H^2	H^3	He^3
Oliphant, Kempton and Rutherford [25]	1.00807 ± .00007	2.0142 ± .0002	3.0161 ± .0003	3.0172 ± .0003
Bethe [26]	1.0081 ± .0001	2.0142 ± .0002	3.0161 ± .0003	3.0170 ± .0005
Bainbridge and Jordan * [27] . .	1.00815 ± .00002	2.01478 ± .00003	. . . . . .	. . . . . .
Weighted mean adopted	1.00814 ± .00002	2.01476 ± .00003	3.0161 ± .0002	3.0171 ± .0003

	He^4	Li^6	Li^7
Oliphant, Kempton and Rutherford [25]	4.0034 ± .0004	6.0163 ± .0006	7.0170 ± .0007
Bethe [26]	4.0034 ± .0002	6.0161 ± .0005	7.0169 ± .0005
Bainbridge and Jordan * [27] . .	4.00395 ± .00007	. . . . . .	7.01822 ± .00014
Weighted mean adopted	4.00390 ± .00007	6.0162 ± .0004	7.01811 ± .00014

Similarly, for the other nuclei:

Nucleus	Be^9	B^{10}	B^{11}	C^{12}
Atomic weight . .	9.0150 ± .0002	10.0161 ± .0001	11.0127 ± .0001	12.0040 ± .0001

Nucleus	C^{13}	N^{14}	N^{15}	O^{16}
Atomic weight . .	13.0078 ± .0002	14.0076 ± .0002	15.0050 ± .0003	16.0000

Nucleus	O^{17}	F^{19}	Ne^{20}	Ne^{21}	Ne^{22}
Atomic weight . .	17.0040 ± .0010	19.0000 ± .001	19.9992 ± .0002	21.0001 ± .0003	21.9987 ± .0004

The data of Bainbridge and Jordan are of much greater weight than the rest, as indicated by their probable errors. The weighted mean values may be used with more confidence in the computations of mass defects. The mass of the proton is thus 1.00760 ± 0.00002, and of the deuteron $_1H^2$ (nucleus of H^2) 2.01422 ± 0.00003. The mass of the neutron cannot be determined with present methods directly; it has been computed several times from energy considerations. A recent determination by Livingston and Hoffmann [28], partly based on their own experimental data ($_3Li^6 + _0n^1 \rightarrow _2He^4 + _1H^3 + 4.67 \pm 0.05$ Mev), assigns to the neutron 1.00884, with $H^1 = 1.00798$. In such calculations, however, it is advisable to use directly determined atomic weights for the charged particles, in order to reduce the error of the indirect method as much as possible.

* From measures of doublets.

We take the well studied reaction, which also served as a basis for Livingston and Hoffman,

$$_1H^1 + {}_0n^1 \rightarrow {}_1H^2 + (2.22 \pm 0.06) \text{ Mev.}$$

With the mean atomic weights cited above this gives for the mass of the neutron $1.00899 \pm .00007$, and for E in the hypothetical reaction

$$E = (0.79 \pm 0.07) \text{ Mev.}$$

This is only slightly less than the value considered by Atkinson[13], and his conclusion that the neutron formation cannot proceed at 2.10^7 K remains valid; our computations point to a temperature about $1{,}2.10^8$ K, with $s \sim 60$—70 in formula (3′). Thus, neutrons may be generated in such a way in overdense cores only, but not in the interior of stars if these are built more or less according to a polytrope $n \leqslant 3$.

(γ) Reactions starting from a heavier nucleus which must be assumed to have existed in sufficient amount "from the beginning", or to be continually regenerated by disintegration of nuclei of higher order (cf. Atkinson[12]). Considering the fact that lithium has not completely disappeared from the sun, and that from calculations made above it is very likely that lithium reactions form part of the main process of energy generation in the sun (if built more or less polytropically), the nucleus starting the chain of reactions must be lighter than Li^7. The only possible one is evidently He^4 (pointed out by Atkinson already), as may be inferred from its abundance, and its tendency towards regeneration revealed in many nuclear reactions. $Li^7 + H^1 \rightarrow 2\,He^4$ would in this case represent the regenerative process for helium. However, for He^4 to be a starting point at $T \sim 10^7$ without the interference of lighter nuclei except protons, it should be able to form sufficiently stable atoms by proton capture; now, He^5 is unstable, having a life of $\sim 6.10^{-20}$ sec. (cf. [29]), too short to form a step in the continued synthesis; also, the energy (2,93 Mev, computed from $_2He^5 \rightarrow {}_2He^4 + {}_0n^1 + 0.93$ Mev, cf. [29])* is far too high for He^5 to be formed at stellar temperatures (10^7). Li^5 does not seem more promising[13] (there may still be a loophole on the

* The mass of He^5 is thus 5.01389, according to our standard masses of $_2He^4$ and $_0n^1$.

assumption of the existence of the unknown Li^5). In such a case, the synthesis cannot start from He^4 without a steady supply of deuterons [13], and we have to go back to cases (α) or (β) (neutrons, of course, easily lead to the formation of deuterons). Another possibility is $He^4 + He^4 \rightarrow Be^8$ as the starting reaction, which however requires $T \sim 3{,}5.10^7$ (cf. Section 7), thus no longer a polytropic ($n < 3$) structure for main sequence stars.

From the preceding discussion we conclude that, if at least the main sequence stars are built more or less according to Eddington's model, i. e. without superdense cores, the only starting reaction for atomic synthesis can be the formation of a deuteron from two protons, with the expulsion of a positron. The law of energy generation (3′) in dwarf stars must in this case be represented by $s = 6.4$ (at $T_\varepsilon = 1{,}12.10^7$), thus by a much slower dependence upon temperature than hitherto supposed.

If, however, the main sequence stars possess central condensations and higher central temperatures, the process of neutron formation may come into question; in this case the direct deuteron synthesis from protons would be prohibited.

Atkinson supposed that the deuteron synthesis might be characteristic of giants only, being for some reason prohibited at the higher temperatures of the main sequence (cf. [13], p. 81); such prohibition may happen only in a process of thermodynamic equilibrium, when the reaction becomes reversible; now, for the reaction ${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + \beta_+ + E$, the atomic weights assumed above give for the energy of reaction

$$E = 0.41 \pm 0.047 \text{ Mev.}$$

Reversibility of the reaction at $T \sim 2.10^7$ may be expected only when E is small, i. e., about 0.02 — 0.03 Mev. The smallness of the probable error above practically excludes such a possibility. Besides, the reverse reaction would require a supply of free positrons, which is rather unlikely to exist at $T \sim 2.10^7$, because the positrons would be absorbed by free electrons and converted into radiation. In the present case, too, the "giant stuff" and "dwarf stuff" hypothesis is not supported by our physical knowledge.

Section 4.

Abundance of Elements and Mixing.

a. Equilibrium of atomic synthesis.

There have been a few attempts to explain the relative abundance of the elements theoretically. Atkinson[12] considers the abundance to be the result of the equilibrium of atomic synthesis reactions and, without doubt, for the intermediate members of the chain of reactions such an equilibrium must happen (cf. above), except for nuclei of a large initial abundance (oxygen, carbon?) for which the time scale may be too short for equilibrium to be established. For the starting (hydrogen?), and the final (helium, carbon, or oxygen?) members the age also comes into play, of course. At $T \sim 2.10^7$, however, only the relative abundance of the lighter nuclei can be explained in such a manner. It is true that neutrons, which are probably transiently formed in the process of the synthesis, may react with heavy nuclei; but the increase of the atomic weight by neutron capture alone is rather limited, and there cannot be seen any possibility for the formation of heavy nuclei from the lighter ones without the capture of positively charged particles; such a capture, however, is practically impossible at values of $Z > 8$, when $T \sim 2.10^7$.

b. Dissociative equilibrium.

Sterne[6] considers the abundance of elements as the result of thermodynamic (dissociative) equilibrium at high temperatures, of the order of 3.10^9 K. The relative abundance is determined by temperature, density, and by the energies of formation of different nuclei (corresponding to the packing fractions, or mass defects). Increasing temperature favours the abundance of nuclei with smaller mass defects, thus of the lighter nuclei, and vice versa. At $T = 2.10^9$, or lower, the equilibrium condition is practically all iron (or similar nuclei); at $T \geqslant 4.10^9$, it is all hydrogen (according to our present views, all neutron); oxygen must remain scarce ($\sim 10^{-16}$) under any circumstances. It is clear that matter in stellar atmospheres is not in thermodynamic equilibrium, because here we observe an abundance of hydrogen and oxygen, instead of their complete absence, and instead of 100 per cent of Fe and related elements. The explanation is that the rate of approach towards thermodynamic equilibrium is too slow at low temperatures (the rate being exactly the speed of Atkinson's transmutations). If we assume high temperatures inside the stars, and dissociative equilibrium there, we must have at $T_c \sim 2.10^9$ an iron core extending outwards until $T_c \sim 4.10^8$ is reached, when the rate of reaction becomes too slow; the formation of such an iron core, from material which originally contained plenty of hydrogen (as stellar atmospheres and nebulae do), could have been attained only after the lapse of a sufficiently long interval of time on an atomic synthesis basis, to allow all the mass excess of hydrogen to be radiated into space; a further rise of the central temperature ($T_c > 4.10^9$), which can be attained only through continued contraction, inverts the process: an amount of energy equal to the amount formerly spent on radiation would have to be regenerated by gravitation (during an actual collapse) and absorbed in the process of the dissociation of iron back again into hydrogen; there would be now a hydrogen (actually neutron) core, surrounded by a pure iron shell where the temperature would be $\sim 2.10^9$ with intermediate composition in between. It is inconceivable how a single lighter nucleus from the central region could ever reach the boundary of a star without being incorporated in the heavier elements of the intermediate shell, as the rate of reaction for Li + H is defined by a life of the order of 10^{-6} sec, and even for Fe + H it is about 1 hour, at $T \sim 2.10^9$ (cf.[6], p. 774, and our formula (3)), and for $\varrho \sim 10$ (Sterne's assumption; the density in the collapsed core must be much higher and thus the reaction much faster). Thus Sterne's dissociative equilibrium would lead to pure iron (or something similar) at the surface, as in the case of $T_c \sim 2.10^9$, contrarily to what is observed. *

Sterne's considerations need to be corrected so far as the neutron instead of hydrogen, being the particle of highest

* There is no danger of an atomic explosion of the collapsing nucleus, if the history of the stars is as described here: the gravitational "pit" into which the star has contracted and radiated itself, is deeper than the subatomic energy of the dissociated mixture, by an amount equal to the total energy radiated into space; thus the star cannot "jump out of the pit" (no matter whether it contracted as a whole, or only in its central core).

internal energy, is the dominating nucleus at very high temperatures. The energy of binding, $E = 0.79$ Mev (cf. above), being relatively low, protons with electrons are rapidly transformed into neutrons in dissociative equilibrium at temperatures of the order of 7.10^8 K; at such temperatures, however, protons must have been already completely absorbed by atomic synthesis (reactions observed in the laboratory suffice to accomplish this, cf. Section 7), thus there is no chance for such an "early" formation of free neutrons to happen.

There is another interesting point to be considered in connection with the dissociative equilibrium of the elements. During the transition phase, when the state of dissociative equilibrium changes rapidly with the temperature, the ratio of specific heats, $\bar{\gamma}$, is chiefly determined by the enormous amount of energy involved in the atomic reactions (at 4.10^9 K, the translatory energy of the particles is $\sim 5.10^5$ volts, whereas the subatomic energy is $\sim 8.10^6$ volts), for which $\gamma \sim 1.0$. Therefore $\bar{\gamma} < \frac{4}{3}$, and an unstable state is reached: the contraction is rapidly converted into a real collapse (cf.[1], p. 142), which is stopped only after dissociation has been completed; the value of $\gamma > \frac{4}{3}$ is expected to remain after that still close to $\frac{4}{3}$ on account of the closeness to dissociative equilibrium; in that case the star must be vibrationally unstable[18], and we may safely suppose that non-pulsating stars cannot have collapsed cores of the dissociative type ($T > 2.10^9$). Curiously enough, the argument of vibrational instability, which Sterne[6] thought to speak against Atkinson's mechanism of energy generation, turns out to be harmless to the transmutation theory of stellar energy, presenting instead an argument against Sterne's dissociative energy generation (disregarding other difficulties, cf. Section 1), as well as against the existence of collapsed cores in stars of the main sequence which are known to be, as a rule, vibrationally stable. In any case, the dissociative equilibrium hardly determines the internal composition of these stars — simply because the time scale is too short for dwarfs to have radiated away all their supply of subatomic energy stored in free hydrogen, a radiation which must have been accomplished before the first "iron" stage ($T \leqslant 2.10^9$) is reached.

So far as to Sterne's qualitative picture, where the effect of pressure (or density) upon dissociation is disregarded, as a first approximation. Surprising conclusions, however, are reached when the influence of density is considered.* We limit ourselves to the two-phase dissociation Fe → He; from Sterne's calculations for a more complex case it appears that the most important dissociation He → H follows soon the dissociation Fe → He, so that the start of Fe → He is actually the start of rapid complete dissociation and collapse if such can happen. From Sterne's figures we estimate that a degree of dissociation $x = 0.01$ is reached at $T = 2{,}2.10^9$, $\varrho = 10$ gr/cm³; we use the Saha formula for order-of-magnitude extrapolation starting from this point (the approximation is much better than might appear at first glance; the dissociation does not happen suddenly according to ${}_{26}Fe^{56} \rightarrow 14_2 He^4 + 2\beta_-$, but gradually, as ${}_{26}Fe^{56} \rightarrow {}_2He^4 + {}_{24}Cr^{52}$, with the absorption of about 3.5 Mev; thus it is not only a two-phase reaction, but the number of reacting particles is the same as in the Saha case of ionization); the effective "ionization potential" results as $I = 3.0$ Mev (in good agreement with the packing fraction of one incorporated α-particle, cf. above), and the dissociation formula for $x = 0.01 =$ const. becomes:

$$\log \varrho/_{10} = -1{,}5.10^{10}\left(\frac{1}{T} - \frac{1}{2{,}2.10^9}\right) + \frac{3}{2}\log\frac{T}{2{,}2.10^9} \quad \ldots (a).$$

Now let us consider a contracting superdense nucleus of mass M, central density ϱ_c, molecular weight $\bar{\mu} = 2.11$ (exhausted, no free hydrogen, all $\sim$ iron), and $\beta =$ ratio of gas pressure to total pressure; the source of energy is gravitation, and the nucleus is so dense in comparison with the rest of the star that it behaves like an independent polytropic model $n = 3$ (cf. below). The central temperature (cf.[1]) is then given by

$$T_c = 4{,}2.10^7\,\beta\left(\frac{M}{M_\odot}\right)^{\frac{2}{3}}\left(\frac{\varrho_c}{76}\right)^{\frac{1}{3}} \quad \ldots\ldots (b),$$

where β is determined from Eddington's quartic equation (cf.[1], p. 137, Table 14).

* At $T = 2{,}2.10^9$, the black-body radiation has a material density of ~ 200 gr/cm³; the energy stored in radiation equals the subatomic energy of transmutations at a density of matter $\sim 3.10^4$ gr/cm³. Sterne's calculations of the equilibrium of transmutations, for a total density of only 10 gr/cm³ (!) neglecting radiation, are an example of mathematical abstraction which disregards physical realities (cf.[6], p. 715).

By solving the pair of equations (a)+(b), with $T=T_c$, $\varrho=\varrho_c$ in (a), the unknowns, T_c and ϱ_c, corresponding to the degree of dissociation $x=0.01$ are found. Now, equation (b) is equivalent to the linear equation

$$\log \varrho_c = 3 \log T_c + \text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{b}'),$$

the constant depending solely upon the mass, whereas (a) approaches asymptotically the straight line

$$\log \varrho = \tfrac{3}{2} \log T + \text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{a}').$$

This pair of equations, (a) and (b), as shown by the accompanying figure, yields either two solutions (b_3 on the figure),

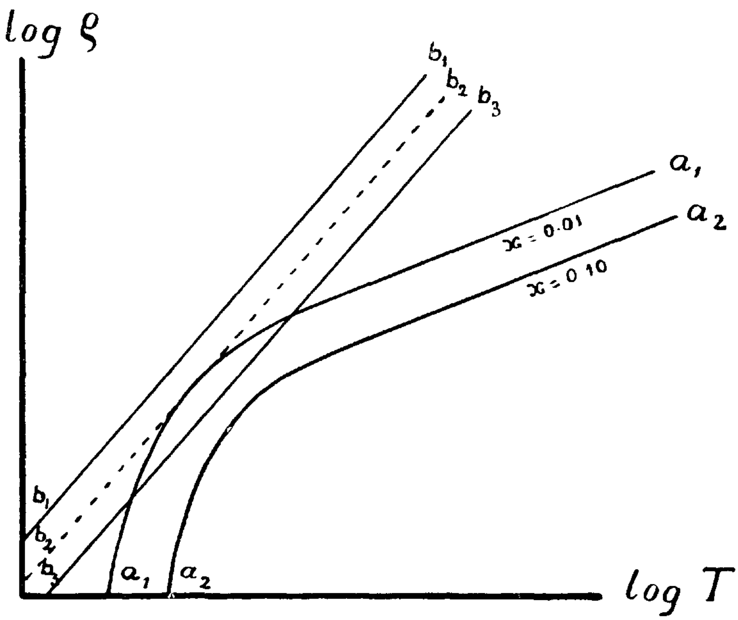

Fig. 1. Conditions of nuclear dissociation. b_1, b_2, b_3 = equation of state for different stars ($\log \varrho_c = 3 \log T_c + \text{const.}$); a_1, a_2 = equation of state for x = const.

or none (b_1). The case is similar to the conditions of degeneracy investigated by Chandrasekhar[5]. In our case, the condition of maximum dissociation attaining $x = 0.01$ (b_2 on the figure) is given by

$$4{,}2.10^{7} \left(\frac{M}{M_\odot}\right)^{\frac{2}{3}} \beta = 1{,}25.10^{8}.$$

This gives $M = 19_{\odot}$; for collapse, $x \geqslant 0.01$ is required; thus only masses much larger than $19_{\odot}$ arrive, in the process of contraction, at a sufficient degree of dissociation to cause a collapse. For other masses, the maximum attainable degree of dissociation ($x_{max.}$) is*:

Mass $M/M_{\odot}$ . . .	50	19	10	8	5	3	2	1.5
x_{max}	0.014	0.01	0.008	0.007	0.005	0.004	0.003	0.002
Energy of dissoci. erg: gr	8.10^{15}	6.10^{15}	5.10^{15}	4.10^{15}	3.10^{15}	$2{,}4.10^{15}$	$1{,}8.10^{15}$	$1{,}2.10^{15}$
ϱ_c, gr/cm³	3.10^{8}	6.10^{8}	9.10^{8}	$1{,}2.10^{9}$	2.10^{9}	4.10^{9}	6.10^{9}	$1{,}2.10^{10}$
T_c	$2{,}4.10^{10}$	$2{,}4.10^{10}$	$2{,}4.10^{10}$	$2{,}5.10^{10}$	$2{,}5.10^{10}$	$2{,}6.10^{10}$	$2{,}6.10^{10}$	$2{,}7.10^{10}$

We see that the maximum energy absorbed by the dissociation is only a small fraction of the energy lost during the preceding contraction $\left[= 1{,}2.10^{15}\left(\frac{M}{M_{\odot}}\right)^{\frac{2}{3}}\left(\frac{\varrho_c}{76}\right)^{\frac{1}{3}} \text{erg/gr}\right]$ and that the "collapse" thus amounts to only a negligible decrease in the radius: no real collapse thus takes place from nuclear dissociation for existing stellar masses, and no appreciable amount of dissociated material, to feed further radiation, can be formed. Possible degeneration (for $M < 1.6$) is an additional factor to prevent dissociation.

In the process of dissociation described, the small, but perceptible equilibrium content of the neutron must collect by diffusion at the centre; unfortunately, no pure neutron core can be formed in such a manner as imagined by Anderson[4], because the same law of thermodynamic equilibrium, which led to the formation of the small percentage of neutron, will provide for the constancy of the percentage and convert the excess of neutron collected at the centre into the heavier nuclei. There is no escape from the conclusion that nuclear dissociation as well as pure neutron cores cannot play an appreciable role in the energy balance, stability, and structure of actual stars. (Dissociation by pressure, of course, is pos-

* All our conclusions, derived from Saha's formula $\log \frac{x^2}{1-x}\varrho = -\frac{A}{T} + p \log T + C$ with $p = \frac{3}{2}$, remain essentially correct also for the case of possible deviations from the formula at very high temperatures, if these deviations are such that $\bar{p} \leqslant 3$.

sible, as it does not involve absorption of energy; the formation of electron-positron pairs from radiation is more likely to occur in the collapsing core; involving an energy of only 10^6 volts, and depending upon the density of radiation, instead of the material density, thus solely upon the temperature, it may be an important process in the core, especially as it increases the opacity, and lowers the molecular weight, thus reducing the luminosity).

c. *Initial distribution of abundance.*

It is the question whether we have any right at all to derive the observed abundance of elements from conditions prevailing at present in stellar interiors. A more or less similar distribution of the elements is revealed by the earth, the meteorites, and the stellar atmospheres (cf. [30], and [22]), with exceptions which are easily explained by the history of the celestial bodies (e. g., escape of hydrogen from small bodies), without recourse to transmutations becoming necessary. Even if the earth was formed from ejected portions of the primitive sun (Chamberlain and Moulton), the relative abundance of the elements in it cannot correspond to equilibrium conditions inside the present sun. There seems to be no escape from the conclusion that the meteorites, the earth, and to all appearance the stellar atmospheres reflect the composition of primordial matter which must have been well mixed; present processes in stellar interiors may influence the abundance of the less abundant lighter elements (e. g., lithium, beryllium, boron in the sun, cf. [22], and Section 3. *d*, *f*). In some stars (if not in the sun), on account of imperfect mixing of the stellar material, the internal changes in the composition may but slowly (perhaps not at all) become reflected in their atmospheres (giant stars; "composite" adiabatic-radiative model, cf. below). The observed abundance of the more abundant lighter elements, such as oxygen, and (especially) the abundance of the heavy ones, must have its origin in conditions which prevailed in the universe before the present stars were formed (neutrons if formed at all can influence the heavy elements only to a limited extent).

d. White dwarfs.

White dwarfs, such as Sirius B, o^2 Eridani, represent cases where we are forced to conclude that the mixing of their material is rather inefficient. Possessing internal temperatures doubtlessly higher than those of the main sequence stars, and densities that are much higher, these stars should develop much more energy than they actually do, if there is a trace of hydrogen in the interior; hydrogen must be completely absent from the interior of the white dwarfs (where the temperature exceeds $\sim 7.10^6$ K); on the other hand, spectroscopic evidence points to a not inconsiderable abundance of hydrogen at the surface of these stars. We are forced to conclude that there is practically no mixing in these white dwarfs. If red giants exist for 3.10^9 years, the hydrogen in their central portions must have become exhausted (cf. below), whereas their atmospheres seem to show a rather abnormal abundance of hydrogen; thus mixing must be incomplete in the giants, too. The white dwarf A. C. + 70° 8247 according to Kuiper[65] is devoid of spectral lines; the writer has shown[66] that its colour as estimated by Kuiper implies an effective temperature of 12 700°; the absence of Balmer lines in such a case can be explained only by the absence of hydrogen; AC + 70° 8247 is thus an instance where the mixing has been complete (as in the sun) and where the internal exhaustion is reflected in the atmosphere.

e. Calcium.

The remarkable constancy of the relative abundance of calcium in stellar atmospheres[31] seems to speak in favour of the stellar atmospheres reflecting the composition of a primordial uniform mixture, although the equality of calcium content for stars of similar spectrum and absolute magnitude may be the result of similar conditions and history (the absolute amount of calcium cannot change much from transmutations, but its relative amount may change when the amount of hydrogen changes). The comparison of the relative abundance in giants and dwarfs made by the writer[31] generally leads to an ambiguous interpretation: the colour-absolute magnitude effect, partly or entirely due to a pressure broadening of Ca 4227, may partly, to an unknown extent, be due to a real variation of mean composition with absolute magnitude. However, there is one case when the ambiguity disappears: for a weak spectrum line, on the margin of appearance, the pressure effect must be absent. For Ca 4227, this corresponds to spectrum F0; the absence of the colour-absolute magnitude effect at this spectrum indicates that at least the atmospheres of F0 giants and dwarfs possess an almost equal relative calcium content (ratio of Ca to hydrogen *plus* all the other elements), within ± 5 per cent; such a coincidence of the expected and the observed disappearance of the colour effect is not very likely to be accidental; thus it appears highly probable that there is little systematic difference in the composition of the atmospheres of giants and dwarfs, although the internal composition is very likely to be different: Strömgren[21] finds for the interior of giants a smaller average hydrogen content than for the main sequence stars, whereas the data of the writer[31] (if the pressure effect is disregarded) would require an increased hydrogen content in the atmospheres of the giants (smaller relative abundance of calcium), except F0. The same was found by Russell[22] for red giants. Thus, whatever the interpretation of the observed colour-absolute magnitude effect, the conclusion is the same, namely, that the composition of stellar atmospheres is not always determined by the composition of the interior. The mixing of the stellar material may in some cases be rather inefficient.

f. Rotational currents and convection.

This conclusion appears to be at the first sight in conflict with certain theories requiring vertical circulation. Thus, in rotating stars convection currents inevitably arise, as shown by von Zeipel[32]. Biermann, Rosseland, Steensholt have shown that in stars generating energy by atomic synthesis, or generally by a process of a speed rapidly increasing with the temperature, the mathematical theory of the model leads to negative density gradients inwards (similar to the point-source model), which of course cannot persist as such, and give rise to convection currents instead[33,16]. Thus there cannot be the least doubt as to the existence of convection currents in stars. And, nevertheless, the mixing of the material may be incomplete. Following a suggestion made by Bjerknes, Eddington admits "that a circulation of this kind tends to become stratified, so that instead of one circulation between the centre and the outside we may have two or three layers of circulation. Each layer will then be thoroughly mixed, but there will be little interchange between consecutive layers" (cf.[1], p. 286). Rosseland[33] remarks, with regard to the central convective zone: "If ... the stars are convectively unstable even without rotation, the role of the rotation is less that of instigating convection than that of determining the type of the ensuing currents". In fact, the vertical currents in a rotating star are deflected horizontally by the difference of the linear velocity of rotation in different zones, exactly as happens to the wind on the earth; the deflection increases with decreasing friction (which is relatively much smaller in stars than on the earth), and is already considerable when the difference of rotational velocity (between the given and the "starting" point of the current) is comparable to the velocity of the current. For von Zeipel's effect Eddington[47] estimates a velocity of vertical convection less than 2.10^{-4} cm/sec for the sun, which would require about ten million years to travel from the centre to the surface if undeflected; appreciable deflection starts, however, already after a path of 100 cm only.

For the central unstable region the calculated negative density gradient is, of course, only a mathematical fiction; it means the breakdown of the assumption of radiative equilibrium there; adiabatic equilibrium and a heat transfer by convection supplementing radiation take place (radiative equilibrium may persist in the outer portions of some stars, giants for example; probably not in the sun). The region of the convectional transfer of heat may extend beyond the computed region of a negative density gradient, in some cases over the whole star (cf. Section 5). The equations of adiabatic equilibrium must be almost strictly fulfilled in the convective region because, as shown in Section 5, a sufficient transfer of heat takes place at such low velocities of the current that the ensuing deviations of pressure and density from their static equilibrium values are negligible. Indeed, we obtain a maximum stream velocity of convection by assuming that the transfer of heat is accounted for by the mere kinetic energy of the current, disregarding the heat transfer by an excess of temperature (only in the absence of viscosity or turbulence, and for adiabatic changes of state, our assumption would cease to be an overestimate); for the central region of the sun, the maximum velocity we find $< 10^4$ cm/sec (Section 5. *a*). An ascending current of $v = 10^4$ cm/sec will experience sensible deflection only after travelling the considerable fraction ~ 0.05 of the sun's radius; it is therefore probable that the entire inner core of convectional instability forms one system of circulation, with complete and rapid mixing. On the other hand, in the outer shell of radiative equilibrium (probably absent from the sun) only the weak currents due to rotation can exist; these must form a large number of superposed shells (thickness of the order of 10^3 cm) with a more or less complete circulation in each; the mixing in such a case is practically nil, if the interchange between surface and interior is considered. However, peculiarities of ionization $\left(\text{reducing the value of } \gamma = \frac{c_p}{c_v}\right)$ may produce local instabilities and convection zones (cf.[34]), such as we probably observe in sun-spots; probably all the mixing in the outer layers of the sun is due to such causes. Thus, rotation favours mixing only to a negligible extent, and prevents it efficiently by deflecting the more powerful central and ionization currents; however, in completely adiabatic stars (cf. Section 5) the role of rotation, by overcoming a certain "dead zone", may be important, leading to complete mixing.

For dwarfs with their slow rate of energy generation a considerable change in composition with age may be expected only among the brighter ones, of classes A and B; "composite" models (Sirius A, Procyon?, cf. Section 7) should show an unchanged original composition of their atmospheres, whereas complete adiabatic models should exhibit a considerably deviating distribution of the lighter nuclei (up to oxygen) (such as exemplified by the low abundance of lithium and beryllium in the sun). Giant stars with radiative shells conceal their inner composition. Perhaps giants of exceptional composition (such as R — N stars) are those where the mixing is (or has been more or less recently) complete, so that their atmospheres reflect the internal changes. In this respect we may suppose that the relative excess of carbon, as compared with oxygen, postulated by Curtiss and investigated by Russell in the N stars ("carbon" stars), is the result of atomic synthesis in the stellar interiors. The equlibrium of abundance, for intermediate members of the atomic synthesis, can be reached only when the life-time of the atom is short as compared with the life-time of both the star, and the hydrogen in it. The last nucleus which has a life comparable to, but greater than, the life of the hydrogen in the star, will accumulate from the more rapid transmutations of the lighter nuclei; the abundance of the next heavier nucleus, however, will not appreciably increase before the life of the star has approached the life of the nucleus. From Sterne's tables we derive the following figures, for $\varrho = 10$ gr/cm^{3} *, $q = 0.01$ (except hydrogen, for which $q = 1{,}3.10^{-19}$ is assumed, cf. above):

Life time for proton capture

	Hydrogen	Boron	Carbon	Nitrogen	Oxygen
$T_\varepsilon = 13{,}8.10^6$ K	10^{10} years	2.10^9 years	3.10^{11} years	10^{14} years	10^{16} years
$T_\varepsilon = 18{,}6.10^6$ K	10^9 „	2.10^7 „	1.10^9 „	2.10^{11} „	3.10^{13} „

Both temperatures are higher than the probable adiabatic temperature of the sun (cf. above); $T_\varepsilon = 18{,}6.10^6$ gives on the Li+H synthesis several hundred times more energy per gram than is produced by the sun, thus this case may correspond to a supergiant in its early, non-collapsed (main sequence) stage (cf. below); $T_\varepsilon = 13{,}8.10^6$ gives 35 times more energy per gram than the sun and may correspond to the case of a normal giant.

We see here that the lives of carbon and hydrogen are of the same order of magnitude; also, that for the normal giant (first case) boron falls below the limit of 3.10^9 years, whereas carbon exceeds considerably the limit and may be considered the last element accumulating in the chain of the synthesis, whereas nitrogen, and still more so oxygen are "inert". Nitrogen must accumulate in the supergiant. Thus, it is very likely that in the interior of giant stars carbon has

* For another value of ϱ, the figures little change if T_ε is chosen so as to keep ε, the rate of energy generation per unit mass constant.

increased in amount, whereas oxygen has remained unchanged; if the original mixture contained more oxygen, the final composition may show an excess of carbon; in stars where the mixing is efficient* the excess of carbon may extend to the atmospheres.

g. Neutron.

There have been attempts to attribute to the neutron an important role in the structure of all stars (W. Anderson, cf.[4]). In superdense cores the neutron as shown above cannot play a conspicuous role. Outside the superdense cores the neutron must possess a rather short life (cf. Atkinson[13]) and can never have a chance to accumulate; the penetrability of matter by neutrons has been largely overestimated by Flügge[36] (observed target $\sim 10^{-24}$ cm^2, instead of his 10^{-27} cm^2 for elastic collisions of fast neutrons; slow neutrons have a 100 times larger target), and his conclusions as to the possibility of a diffusional separation and concentration of the neutrons at "ordinary" stellar centres are untenable. Thus, the only role of the neutron in stellar interiors is that of a short-lived, highly active link in atomic synthesis; the amount of free neutron must be vanishingly small, just on account of its high activity.

Section 5.

The Composite Adiabatic-Radiative, and the Complete Adiabatic Stellar Models; Giant and Dwarf Structure.

a. Transfer of heat by convection.

It is known that the point-source model leads to a decreasing density towards the centre (cf.[1], p. 126); physically this means that convection currents arise, and that the calculated point-source state of equilibrium is replaced by another kind of equilibrium. Rosseland, Biermann, and Steensholt have

* But still incomplete; with complete mixing, according to our views exposed below, planetary nebulae and Wolf-Rayet stars are likely to be formed, instead of giant N stars. The mixing may perhaps be considered the result of some catastrophic event (as perhaps the "swallowing" of a companion by the expanding giant, cf. Section 7. *i*).

shown that, for the law of energy generation $\varepsilon \sim \varrho^k T^s$ (ϱ = density, T = temperature), there will be formed a convective core for as low a value of s as 3 (with $k = 0$)[33]. As Cowling[18] truly remarks, the transfer of energy in the interior of the star is in this case by convection, not by radiation. Actually convection takes place wherever the temperature gradient tends to exceed the adiabatic value, ξ_a (absolute values); in such a case adiabatic equilibrium sets in, with a slight excess of the gradient $\Delta\xi = \xi - \xi_a$, sufficient to keep convection going. The extent of the convective core is thus larger than would appear from the extent of the calculated negative density gradient.

The convective transfer of heat (per unit of time and cross section) between two surfaces may be set equal to

$$Q_c \sim v \varrho c_p \Delta T \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4),$$

where v is the velocity, ϱ the density, c_p the specific heat, ΔT the excess temperature of the current. The transfer by radiation is

$$Q_r \sim \frac{(T_1^4 - T_2^4)}{k \varrho x},$$

where T_1 and T_2 are the temperatures, k the coefficient of absorption, ϱ the density (supposed to be constant), x the depth. For surfaces separated by a large $k\varrho x$ the advantage of convection, as compared with radiation, is obvious; if the depth of the convection current is of the order of the radius of the star, convection is much more efficient than radiation. Take for the sun, at one-sixth of the radius from the centre, a net transfer of 5.10^{12} erg/sec per cm^2 of the convection current [this is supposed to give one-half of the energy output of the sun, provided by a central fraction of 0.03 of its mass, corresponding to a highly concentrated source of energy about $\varepsilon \sim \varrho T^{12}$ (cf. Section 3. d), if the rising current covers one-quarter of $4\pi r^2$]; further, take $\varrho = 10$ gr/cm^3, $c_p = 3.10^8 \dfrac{\text{erg}}{\text{gr. deg}}$. Formula (4) then gives: $v \Delta T = 1700$. *

* For negligible friction (viscosity and turbulence), such as must obtain in the case of a large-scale current in a star, v^2 is of the order of $2 c_p \Delta T$; this gives $v^3 \sim \dfrac{Q_c}{\varrho}$, or, with our adopted data, $v \sim 8000$ cm/sec, and $\Delta T \sim 0^\circ.2$: the deviations from adiabatic hydrostatic equilibrium are of the order of 10^{-8} for T, and about the same for the pressure.

For $\Delta T = 100^0$ (or $\sim 10^{-5} T_1$), $v = 17$ cm/sec. It is obvious that convectional transfer is very efficient, and that it requires such small deviations of temperature and pressure from the static values that it is legitimate to assume for them adiabatic equilibrium values. Thus, the convective region may be assumed to be built according to the polytrope $n = \frac{1}{\gamma - 1}$, where γ is the ratio of specific heats, for almost any concentration of the energy sources. An exception is presented by the true point-source, for which convection becomes inadequate at a small distance from the centre, at about 10^{-6} of the radius (for the sun). Also, in the outer layers of a star (the sun), when $\varrho < 2.10^{-5}$ gr/cm^3, convection may be incapable of transporting the net flux of heat.

b. The net flux of radiation in a polytrope.

Heat that has escaped from a shell of radius r inside a star, containing a fixed mass M_r, cannot get back (because free convection cannot transport heat in the direction of the gravitational force; convection forced by rotation is too slow, and too weak, to work against the excess of the adiabatic temperature gradient required for a reversal of the transport of heat). Also, there are no subatomic processes able to absorb energy at temperatures below 10^9 K. Therefore, all the net flux of heat (radiation + convection) which has once passed outside of r, must make its way through to the surface (with the exception of a mostly small or zero fraction spent upon the heating of an expanding star). On the other hand, the temperature gradient cannot perceptibly exceed its adiabatic value ξ_a; if radiation at the maximum possible value $\xi = \xi_a$ is incapable of transporting all the heat, convection comes into play to supply the difference.

The net flux of radiation passing outwards through a shell of radius r is

$$Q_r = \frac{4\pi ac\, r^2}{3k\varrho}\left(-\frac{dT^4}{dr}\right) \quad . \quad . \quad . \quad . \quad . \quad (5) \text{ (cf.}^1\text{, p. 101)};$$

here $\frac{1}{4}\, ac$ = Stefan's constant of radiation.

In the case of pure radiative equilibrium, this is also equal to the net flux of the energy, L_r; in the presence of convection, however,

$$Q_r < L_r.$$

For Kramers' law of opacity $k = k_0 \varrho T^{-\frac{7}{2}}$ . . . (6) (k_0 = intrinsic opacity depending upon composition, primarily upon the hydrogen content), and for a polytropic model

$$\varrho = \varrho_c u^n \quad . . . . (7),$$

where $u = \frac{T}{T_c}$ (ϱ_c and T_c = central density and temperature). Equation (5) becomes, after appropriate substitution:

$$Q_r = A\, u^{\frac{13}{2} - 2n} \left(-z^2 \frac{du}{dz}\right) \quad . . . . (8),$$

where

$$z = \frac{R'}{R} r, \; -z^2 \frac{du}{dz} = M' \frac{Mr}{M}, \text{ and}$$

$$A = \frac{16\, \pi\, ac\, R\, T_c^{\frac{15}{2}}}{3 k_0\, R' (\varrho_c/\varrho_m)^2 \varrho_m^2} \quad . . . . (9).$$

R is the radius, ϱ_m the mean density, M the mass of the star; R' and M' are constants of Emden's tables (final values of z and $-z^2 \frac{du}{dz}$), depending as well as $\frac{\varrho_c}{\varrho_m}$ upon the polytropic index, and

$$T_c = \frac{R'}{(n+1) M'} \frac{G}{\Re} \frac{\bar{\beta} \mu M}{R} \quad . . . . (10)$$

(cf. Eddington[1], pp. 79—85); μ is the molecular weight, G the constant of gravitation, $\Re$ the gas constant, β is the *effective* ratio of gas pressure to total pressure which for a given n differs slightly from the value given by Eddington's quartic equation (derived for a particular model, $n = 3$).*

* If β_3 is the value for $n = 3$, the average value for (10) is given by the following table:

n	3	2.5	2.33	2.0	1.5
$\frac{1-\beta}{1-\beta_3}$	1.00	0.82	0.82	0.83	0.90

c. Condition for convection to start at the centre.

For one and the same star A is a constant; equation (8) determines the distribution of energy sources in the star only in the absence of convection, in which case $\eta \sim \int \varepsilon \, dM \sim u^{\frac{13}{2} - 2n}$. In the case of convection, however, the distribution of energy sources is independent of Q_r, and is determined by L_r; for convection to start it is evidently necessary that

$$Q_r < L_r \quad \ldots\ldots\ldots \quad (11),$$

which means that radiation alone is unable to transport all the energy liberated inside a given shell. The inequality (11) determines the minimum degree of concentration of the true energy sources for producing convection*. Setting

$$L_r = \bar{\varepsilon} M_r, \quad Q_r \sim T^{\frac{13}{2} - 2n} M_r,$$

$$\text{and } \bar{\varepsilon} \sim \varrho T^s \sim T^{s+n}\,{}^{**}, \quad \text{for } n < 3.25,$$

(11) leads to the "minimum law of energy generation" for convection (for this purpose, ε must increase inwards faster than Q_r/M_r):

$$s > \frac{13}{2} - 3n \quad \ldots\ldots\ldots \quad (12).$$

(The use of a mean value, $\bar{\varepsilon}$, instead of ε, introduces little difference.)

For $n = 1.5$ (minimum value), $s > 2$ appears to be sufficient. Actually, for the mixture of ionized gas + radiation, $n = 1.63$ seems to be a fair estimate for the sun (with $T_c \sim 1{,}9.10^7$, $\mu = 0.98$, cf. Section 3. *b*); $s > 1.6$ is only required in this case. There cannot be any doubt that any kind of subatomic processes will satisfy this condition (Atkinson estimates $s \sim 20$), and that convection is inevitable in such a case.

* When $Q_r > L_r$ (subatomic), the balance is made up by gravitational contraction; the contraction soon stops, when the rapidly increasing L_r exceeds Q_r (which varies little).

** We notice that on account of convection the structure is almost exactly polytropic (cf. above), and an objection of J. Tuominen, Zeitschr. f. Astrophysik **9**, 260, 1936, against using $\varrho \sim T^n$ in the formula for ε is thus invalidated.

For gravitational contraction, $\varepsilon \sim T$ (cf.[1], p. 123); convection would start when $1 > \frac{13}{2} - 2\,n$, or

$$\frac{1}{\gamma - 1} = n > 2.75, \text{ or } \gamma < 1.364.$$

This can happen, except under peculiar conditions of ionization, only when $1 - \beta > 0.81$, thus in stars of exceptionally large mass; the gravitational source of energy for "normal" stars probably never leads to convection.

d. The luminosity of a polytrope.

Let us consider a complete polytropic model of constant n. For all values of $n < 3.25$, Q_r as given by (8) increases from the centre outwards to a certain maximum value, Q_{max}, at $r = r_0$, and then drops down to zero at the surface ($r = R$). By reason of our postulate (Subsection *b*, non-reversibility of the flux of heat), the luminosity of the star, L_0, must satisfy the inequality

$$L_0 \geqslant Q_{max} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13).$$

The minimum luminosity is $L_{min} = Q_{max}$. On account of the probable absence (weakness) of energy sources outside r_0, the true luminosity must be very near its minimum value. The mass-luminosity function of the complete polytrope is then determined by

$$L = Q_{max} = \frac{Q_{max}}{A} \cdot A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14),$$

where $\frac{Q_{max}}{A}$ is computed from (8) by the aid of Emden's tables (Gaskugeln, Leipzig u. Berlin 1907; cf.[1], loc. cit.); for a constant polytropic index, Eddington's mass-luminosity function results, with a certain divisor a of the luminosity (cf.[1], p. 124) depending upon the polytropic index. The following table contains the data for different polytropic indices:

Table 2.

Divisor of luminosity, a $[L \sim \frac{1}{a} f(M, R)]$.

$n =$	3.25	3.0	2.5	2.0	1.5	0.0
$r_0/R =$	1.00	0.457	0.378	0.375	0.383	0.433
$M_{r_0}/M =$	1.00	0.85	0.54	0.37	0.25	0.24
$Q\ max/A =$	1.96	0.996	0.480	0.302	0.211	0.309
$a =$	1.63	2.64	3.67	3.31	2.02	0.057
$T_c\,(M, R, \mu = \text{const.}) =$	1.12	1.000	0.822	0.705	0.632	0.584 *

For constant mass, radius, intrinsic opacity, and molecular weight, the luminosity is inversely proportional to a. The standard value of $a = 2.5$ chosen by Eddington (loc. cit.) fits well for $1.5 < n < 2.0$.

e. *The adiabatic model.*

Now it is proposed to prove in a rather simple way that the complete adiabatic polytrope ($n < 3.25$) may actually be a persistent form of stellar structure. For $r < r_0$, $Q < Q_{max}$, conditions (11) and (12) are fulfilled, convection takes place, and the state is one of adiabatic equilibrium. For $r > r_0$, $Q_r < Q_{max} < L_{min}$, the same holds as far as formula (8) is valid, or as far as a polytropic distribution may be used for a satisfactory (not necessarily exact) representation of the state of stellar matter. The extra heat, $Q_{max} - Q_r$, is transported by convection, and convection maintains the adiabatic structure almost rigorously. Thus, a star can be built completely on the adiabatic model with $n = \frac{1}{\gamma - 1}$ (n may, of course, be slightly variable), but for a thin outer layer which more or less "shines through to space", and where the density becomes too small for an efficient transport of heat by convection. At $r = r_0$, the radiative, and the adiabatic states of equilibrium coincide, and the transport of heat is by radiation only. Nevertheless, the mixing of the stellar material which is complete inside and outside of r_0, may be efficient also in the narrow "dead" zone at $r = r_0$, on ac-

* The central temperature has a minimum:

$n =$	1.5	1.0	0.5	0.0
$T_c =$	0.632	0.584	0.567	0.584

count of von Zeipel's rotational currents which, although weak for the star as a whole, may be sufficient to fill out the narrow link required for complete mixing of the whole stellar material. There may be cases, however, where the interchange of matter through the shell r_0 is weak, or absent; with progressing atomic synthesis in the interior the molecular weight becomes larger inside r_0 than it is outside, a circumstance of great importance in stellar evolution (cf. below).

f. The composite model.

Another possible form of the equilibrium of a star with a concentrated source of energy is an adiabatic-convectional core inside, surrounded by an envelope (of considerable mass and extent) in radiative equilibrium. Which form of equilibrium a model actually assumes can be settled only by computations of stellar models by the method of trial and error; some general principles and qualitative criteria are formulated below.

The condition of the stability of radiative equilibrium is

$$n > \frac{1}{\gamma - 1} = n_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15),$$

where n is the "local" polytropic index, n_0 the adiabatic polytropic index of the material. We choose to define n by

$$n + 1 = \frac{T}{P}\frac{dP}{dT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

(which follows from $P \sim T^{n+1}$), where P is the total pressure. If computations on the basis of the formulae of radiative equilibrium lead to

$$n < n_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17),$$

radiative equilibrium is unstable, and convectional-adiabatic "equilibrium" governs instead *.

The possibility of fitting a shell in radiative equilibrium on the top of a convective shell of radius r depends upon $\frac{dn}{dr}$ calculated from the formulae of radiative equilibrium: when

* In actual computations, the index $1 + \frac{1}{n} = \frac{\rho}{P}\frac{dP}{d\rho}$ . . . (16′) may be preferred as a criterion of convectional stability. On account of the presence of radiation, the two indices (16) and (16′) are not identical.

the derivative is positive at $n = n_0$, (15) holds, and radiative equilibrium is possible; when the derivative is negative, (17) holds just outside, and adiabatic equilibrium continues; a transition to radiative equilibrium at the given value of r for which (17) holds is impossible.

From (8), together with (16) and the well known equations of radiative equilibrium,

$$\frac{dP}{dr} = -\frac{G\,M_r\,\varrho}{r^2},\quad P = \frac{\Re\,\varrho\,T}{\beta\,\mu},\quad \frac{dM_r}{dr} = 4\,\pi\,\varrho\,r^2,$$

and $P \sim T^{n+1}$ (because the structure is polytropic with $n = n_0$ at the value of r chosen), we find

$$n + 1 = \frac{M_r}{\beta}\,T^{\frac{13}{2} - 2n_0} \times \text{const., and}$$

$$\frac{1}{(n_0+1)}\,\frac{dn}{dr} = \frac{(\frac{13}{2} - 2\,n_0)}{T}\,\frac{dT}{dr} + \frac{1}{M_r}\,\frac{dM_r}{dr} + \frac{\beta\, d\left(\frac{1}{\beta}\right)}{dr} \quad . \quad . \quad (18).$$

For $r = r_0$, $\frac{dn}{dr} = 0$, because there radiative and adiabatic equilibrium merge one into the other (cf. above). For $r < r_0$, calculations give $\frac{dn}{dr} > 0$, whereas for $r > r_0$, $\frac{dn}{dr} < 0$ ($n_0 < 3.25$).

Thus, for a star of uniform composition, a radiative equilibrium envelope can be fitted to an adiabatic core only when the radius of the core is smaller than the radius r_0 of the maximum radiation, Q_{max}. In such a case, for $r < r_0$, as convection is absent from the envelope, $L = Q_r = L_r < Q_{max}$; the luminosity of the whole star must be less than the luminosity of the complete adiabatic polytrope (of which the core represents the central portion).

The condition $r < r_0$ is necessary, but not sufficient for the fitting of a radiative equilibrium shell to an adiabatic core; this question will be discussed in more detail in another paper.

g. Regulation of luminosity for the adiabatic model.

For the complete adiabatic model the minimum luminosity is Q_{max}; on the other hand, a larger luminosity is also possible, because convection currents can transport an almost unlimited amount of excess energy to the surface. However, it is extremely improbable for a star to get into a state of being actually able to radiate into space more than Q_{max}. In the process of original gravitational contraction the star settles down at a certain equilibrium radius, when the subatomic energy sources exactly balance the amount directly required, Q_{max} (the total heat is balanced; the impossibility of detailed balancing of radiation and subatomic sources leads to convection). To obtain more, the star must be compressed further, so that T_c, ϱ_c and ε (subatomic) may increase, and the star must be kept in the "overcompressed" state until the convection currents transport the extra liberated heat to the surface, and until the extra potential energy of contraction $\left(\sim \frac{3\beta\, G\, M^2\, \Delta R}{2(5-n)\, R^2}\right)$ is radiated into space: if the latter does not happen, the star rebounds to its original equilibrium state $L = Q_{max}$. Now, the time during which a star can be kept without interruption in an "overcompressed" state is of the order of one-half of the period of its free pulsation. The extra amount which may be radiated into space during so short an interval (a few hours, or days) is extremely small, corresponding for δ Cephei to a relative decrease in the radius, or an increase of T_c of about 10^{-7}, which is too small to be of a perceptible influence upon the energy generation. There cannot be a cumulative effect for a whole pulsation period: the star sets its average luminosity in balance with the average energy generation during the whole pulsation. An equally important hindrance for a pulsating star to settle into an overcompressed state is the delay in the transport of heat to the surface: it is obvious from elementary mechanical considerations (even disregarding the numerical estimates of the velocity of the current as made above) that the period of the circulation of a convection current must be large as compared with the period of the pulsation of a star; therefore the extra heat of compression has no chance to get to the surface of the star. To obtain a permanent overcompression of 0.1 of the radius (which would increase the rate of subatomic energy generation $\varrho\, T^s$ by 1.0 — 2.5 mag, for $s = 6 - 20$), δ Cephei should be kept in the overcompressed state for 7 000 years, with free radiation into space allowed. Evidently, overcompressed stars are practically impossible; a monotone mass-luminosity relation ($L = Q_{max}$) for adiabatic stars of the same composition must hold, as it holds for the purely radiative model.

h. Model of non-homogeneous composition.

So far stars of uniform composition throughout have been considered. It is, however, not likely that all stars are of a homogeneous composition (we do not consider here, as of minor importance, the change of mean molecular weight due to the variable degree of ionization considered by Eddington). In the composite, but originally homogeneous model exhaustion of hydrogen leads to an increase of molecular weight in the core; for the purely adiabatic model (massive stars), the link between the central, and the marginal convective systems may in some cases be inefficient, leading to the same result. A core containing less hydrogen (condensation of meteoric material, cf. later on) than the envelope may appear in some stars from the very "beginning"; perhaps a very small core of such an origin is present in all stars. It is easy to show that quite a small differentiation of composition would prevent any further convectional interchange of matter between the core and the envelope, leading thus to further differentiation, the extreme case of which is a complete absence of hydrogen from the core. For the surface of demarkation between core and envelope (we assume schematically a sudden change of molecular weight), the conditions of mechanical stability and of the finiteness of the flux of energy require equality of temperature and total pressure (thus also of radiation pressure and β) on both sides of the surface; there is a discontinuity in density,

$$\frac{\varrho_i}{\varrho_e} = \frac{\mu_i}{\mu_e} \quad \ldots\ldots\ldots\ldots \quad (19),$$

and in the temperature gradient. Here the index i refers to the inner, e to the outer side of the surface of demarkation. Assuming first that there is no convection, and that the opacity is given by (6) (k_0 is the intrinsic opacity, depending upon composition), the ratio of the temperature gradients becomes

$$\frac{\left(\frac{dT}{dr}\right)_i}{\left(\frac{dT}{dr}\right)_e} = \left(\frac{\varrho_i}{\varrho_e}\right)^2 \frac{k_i}{k_e} \quad (20). \text{ Further,}$$

$$\frac{\left(\frac{dP}{dr}\right)_i}{\left(\frac{dP}{dr}\right)_e} = \frac{\varrho_i}{\varrho_e} \quad \ldots \ldots \ldots \quad (21).$$

Hence, from (16), the ratio of the polytropic indices is

$$\frac{n_i + 1}{n_e + 1} = \frac{\varrho_e k_e}{\varrho_i k_i} \quad \ldots \ldots \ldots \quad (22).$$

The intrinsic opacity, defined differently from the usual definition, for variable hydrogen content varies as follows*:

Hydrogen content, X	0	0.25	0.333	0.50	0.75	0.90	0.99
μ	2.24	1.21	1.063	0.836	0.631	0.546	0.500
$10^{-23}\ k_0$	288	146	116	71.3	26.3	9.27	0.853

The actual opacity is then given by

$$k = k_0\, \varrho\, T^{-\frac{7}{2}} + 0.2\,(1 + X) \quad \ldots \ldots \ldots \quad (23),$$

including the correction for electron scattering.

For 25 per cent hydrogen in the envelope and none in the core, (21), (22), and the above table give

$$(n_e + 1) = 3.6\,(n_i + 1).$$

In other words, $n_e \gg n_i$; as the latter cannot fall below the adiabatic value, the adjacent envelope is always in stable radiative equilibrium.

If the margin of the core has definitely settled to radiative equilibrium ($n_i \sim 3$), $n_e \sim 13$: the envelope starts almost isothermally.

If the core is adiabatic up to the surface of demarkation, $n_i > 1.5$, $n_e > 8$, which makes little difference as compared with the preceding case. In addition to the radiation emerging from the core, the adjacent envelope containing

* Computed from Eddington's data in [19],

$$k_0 = \frac{k_c}{g} \cdot \frac{T_c^{\frac{7}{2}}}{\varrho_c}.$$

25 per cent hydrogen is able to take up and transport by radiation a much greater amount of heat supplied by convection from the core, without starting convection itself *. Assuming $\gamma_i = \gamma_e \sim 1.5$, $n_i = n_e = 2$, the maximum amount which the envelope is able to transport by radiation alone equals 3.6 times the radiation from the core [cf. (16), $n_e + 1 \sim \left(\frac{dT}{dr}\right)_e^{-1}$, as $\left(\frac{dP}{dr}\right)_e = \text{const.}$]; thus, such an envelope may be fitted to an adiabatic core with $r > r_0$. Eventual convection currents reaching the boundary of the core cannot rise by inertia any farther into the envelope, because of the difference of density in the core and the envelope **. Our conclusions remain unchanged in principle also in the case of the existence of a gradual transition (stratification) from core to envelope, instead of that of a sharp boundary.

i. Collapse of the exhausted core of a composite model and giant structure.

A core devoid of hydrogen, thus presumably devoid of subatomic sources of energy, is doomed to collapse on a "Kelvin" time scale, i. e., with gravitation as the source of energy; high densities can be attained, and a super-dense core *** may be formed. The hydrogen-containing envelope cannot be sucked into the core as long as traces of hydrogen are present, because the corresponding immense increase of temperature and density would lead to an instantaneous release of the whole store of subatomic energy, sufficient to disperse all the envelope into space. Actually no such

* Actually such an extra supply of heat is necessary; as shown in another paper, the necessary condition (assuming Kramers' opacity) for a finite solution is $n_e < 3.25$.

** For a velocity $< 10^4$ cm/sec (cf. above), and $\frac{\varrho_i}{\varrho_e} = 2$, the height to which the current mouth intrudes into the envelope is of the order of $< 10-20$ meters, for the interior of the sun.

*** We avoid the term "centrally condensed", as it has been attached by Milne [23] to a certain mathematical model which is a mathematical, and probably also a physical impossibility, leading to the central singularity which can be removed only by an appeal to unknown physical properties created ad hoc, cf. [5].

catastrophe happens *, the contraction of the core being a gradual one; instead of blowing up, the envelope gradually expands and adjusts itself to such low values of the effective density and temperature that the release of subatomic energy remains more or less normal (it may be even less than the "normal", as the gravitational energy of the core supplies now a large fraction of the star's needs). In spite of the high gravitational force exerted by the core, the transition from the superdense core to the envelope of "normal" density and temperature is made possible by the peculiar distribution of the energy sources, and the smaller molecular weight of the envelope **; the presence of subatomic energy sources suddenly beginning to work outside the core creates radiation pressure that "blows away" the matter of the shell, leaving a small density of matter just sufficient for the subatomic sources to work. The conditions are similar to those in the mathematical point-source model (cf. [1], p. 126), except that here the subatomic source of energy is not concentrated in exactly one point, and that an additional point-source of energy and a considerable point-mass complicate the problem.

At a certain distance from the core adiabatic equilibrium may set in; it may be shown that for the distance R_ε, which halves the subatomic energy sources, convection is always effective, and adiabatic equilibrium takes place. We omit the proof, as this result obviously follows from the fact that R_ε and ϱ_ε are of the order of $R_\odot$, and $\varrho_\odot$, for which case the numerical estimates made at the beginning of this section apply. As to R_ε, the effective radius of energy generation, and ϱ_ε, the effective density, they may be estimated in the following way. Let L_1 be the energy output of the core, L_2 the additional energy from the subatomic energy sources outside the core; M_c and $M - M_c$, the mass in the core, and outside the core respectively; T_ε the temperature at R_ε. For constant β, the temperature represents the potential; for a small distance from the centre, and a considerable mass in

* Except for the "dissociative collapse" discussed in the preceding section, where this was shown to be insignificant for all actual stars.

** Without admitting such peculiar conditions, the central density and temperature of a star of fixed outer dimensions cannot exceed certain "moderate" limits, cf. Eddington [48].

the core, the potential is close to $\frac{M_c}{R_\varepsilon}$, thus $T_\varepsilon \sim \frac{M_c}{R_\varepsilon}$. The law of energy generation we assume to be ϱT^4 (cf. Section 3.*g*, where $s \sim 6.5$ is estimated for the deuteron synthesis at $T_\varepsilon \sim 1,1.10^7$; $s = 4$ seems to be better a value, allowing for higher temperatures $\sim 5.10^7$ and more in our present case, even when the deuteron synthesis is replaced by a reaction of higher order). Approximately, for a constant fraction of active mass, we have

$$L_2 \sim (M - M_c)\, \varrho_\varepsilon\, T_\varepsilon^4 \sim (M - M_c)\, \varrho_\varepsilon\, M_c^4\, R_\varepsilon^{-4}\,.$$

In Section 3.*d* we estimated for the active mass an effective density of $\varrho_\varepsilon = 10$ gr/cm³ ,and $R_\varepsilon = 0.16\, R_\odot$ in the sun. If mass and radius are measured in units of the sun, for a constant ratio of active mass to total mass we have

$$\varrho_\varepsilon = \frac{10\,(M - M_c)}{\left(\frac{R_\varepsilon}{0.16}\right)^3} = \frac{M - M_c}{25\, R_\varepsilon^3}\,.$$

This gives for L_2, in units of the sun's luminosity,

$$L_2 = \frac{(M - M_c)^2\, M_c^4}{25\, R_\varepsilon^7}\,\frac{\varrho_\varepsilon}{10};$$ hence we find

$$R_\varepsilon = 0.46\,(M - M_c)^{\frac{2}{7}}\, M_c^{\frac{4}{7}}\, L_2^{-\frac{1}{7}} \quad . \quad . \quad . \quad . \quad (24),$$

all in units of the sun,

$$\text{and} \quad \varrho_\varepsilon = 0.4\,(M - M_c)^{\frac{1}{7}}\, M_c^{-\frac{12}{7}}\, L_2^{\frac{3}{7}} \quad . \quad . \quad . \quad . \quad (25),$$

the density in gr/cm³.

For a typical case of $M_c = 0.25\, M$ (when the non-collapsed no-hydrogen core has approximately a luminosity equal to the luminosity of a hydrogen-containing star of mass M), and for extreme limits of L_2

$$0.01\, M^3 < L_2 < M^3$$ (the subatomic energy sources cannot give much more than the "normal" luminosity of a star which is $\sim M^3$ empirically); we have

$$0.25\, M^{\frac{3}{7}} > R_\varepsilon > 0.125\, M^{\frac{3}{7}}\,,$$

and

$$2\, M^{-\frac{2}{7}} < \varrho_\varepsilon < 16\, M^{-\frac{2}{7}}\,,$$

thus rather narrow limits for such a wide range (100:1) in the subatomic energy output, L_2. For $M = 1 - 10 \odot$ we have $R_\varepsilon \sim 0.2 - 0.6\, R_\odot$, thus large and almost invariable, as compared with the small and widely variable radius of a superdense core ($10^{-2} - 10^{-4}\, R_\odot$); $\varrho_\varepsilon = 2 - 5$ gr/cm^3. As shown in the following section, the outer radius of the star in such a case may be subject to considerable variation (for small variations in the luminosity), and may be large in some cases (when L_t is large) as compared with R_ε. A typical giant structure results, consisting of a vast extended envelope of low density in radiative or adiabatic equilibrium, an intermediate zone in adiabatic (convectional) equilibrium, of a density about the central density of main sequence stars, containing active sources of subatomic energy, and a contracting superdense core of zero hydrogen content and no subatomic energy. The intermediate zone, with active atomic synthesis, is supposed to contain a decreased amount of hydrogen and to get in this way definitely separated from the outer envelope (cf. above); if not, the whole outer mass except the core may be stirred by convection currents (as in the purely adiabatic model), and the outer radius becomes little sensitive to eventual changes in the luminosity (corresponding to the changing mass of the core which must increase with the progress of time from the exhausted material of the shell, and decrease as the result of energy losses)*, in which case an apparently "main sequence" star with a superdense core may result.

Section 6.

The Course of Stellar Evolution.

a. Presumptions.

Let us consider the course of stellar evolution determined by the most probable conditions which follow from our preceding discussion: atomic synthesis and gravitation as the only sources of stellar energy; absence of complete mixing in some

* Some kind of equilibrium for the mass of the core may result: increasing mass leads to rapidly increasing energy output and radiation pressure at the boundary of the core, which resists the flow of exhausted material inwards.

stars; complete mixing for all stars (without superdense cores) in a central portion of considerable extent, without necessarily an efficient interchange of material with the outer shell; origin from condensation of diffuse matter (nebula), which also determines the original composition.

b. Condensation from a diffuse state.

The first stage of a star's life consists of a comparatively short interval of contraction from a diffuse state; the structure of the star approaches closely Eddington's radiative model (polytrope $n = 3$; $\varepsilon \sim T$), the rate of generation of gravitational energy is automatically equal to the "prescribed" loss by radiation; convection currents are practically absent (the rotational currents are too much stratified); the central temperature increases during contraction inversely to the radius, and $\varrho_c = 54\,\bar{\varrho}$ (with slight uncertainty as to the definition of the boundary of a star); the first stage may last $\sim 10^7$ years for $M = \odot$, $\sim 10^5$ years for $M \sim 10\odot$.

c. Stage of atomic synthesis.

As the central temperature rises, processes of atomic transmutation come gradually into play; a second stage of the star's life starts when an outwardly steady state is reached, subatomic energy balancing the "prescribed" losses by radiation; contraction becomes extremely slow (just enough to balance exhaustion of hydrogen by an increase of the central temperature); for the sun, this stage may last for 10^{10} years, for $M \sim 10\odot$ perhaps 10^8 years.

d. Evolution of the adiabatic model.

If the star is of the completely adiabatic structure, with complete mixing (sufficient rotation to overcome the dead zone at $r = r_0$, cf. preceding section), it remains a "main sequence" star of more or less "normal" density; with the gradual exhaustion of hydrogen its luminosity increases (cf.[19,20] and Section 7). At the same time slow changes in the radius occur which may be estimated in the following way.

For the energy generated by atomic synthesis we assume further the expression

$$L \sim X^2 M \varrho T_c^s \text{ (cf. Section 3 and 5).}$$

With the aid of (10) and $\varrho \sim M R^{-3}$ this becomes

$$L \sim \frac{X^2 (\beta \mu)^s M^{s+2}}{R^{s+3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26);$$

here X is the relative proportion of hydrogen (for a composite model the formula holds when X refers to the core); the second power of X follows for the reaction $H^1 + H^1 \rightarrow H^2$; for the neutron synthesis the first power of X should be used.

On the other hand, with Kramers' law of opacity (disregarding electron scattering) we have

$$L \sim k_0^{-1} R^{-\frac{1}{2}} M^{5.5} (\beta \mu)^{\frac{15}{2}} \quad . \quad . \quad . \quad . \quad . \quad (27)\ *.$$

(26) and (27) lead to

$$R \sim k_0^{\frac{1}{s+2.5}} X^{\frac{2}{s+2.5}} (\beta \mu)^{\frac{s-\frac{15}{2}}{s+2.5}} M^{\frac{s-3.5}{s+2.5}} \quad . \quad . \quad . \quad (28).$$

With Eddington's quartic equation this becomes

$$R \sim k_0^{\frac{1}{s+2.5}} X^{\frac{2}{s+2.5}} (1-\beta)^{\frac{2s-15}{8s+20}} M^{\frac{2s+1}{4s+10}} \quad . \quad . \quad . \quad (28').$$

The deviation from the prescribed radius may be considered a measure of deviations from uniform homologous structure.

For the interval $0 < X < 0.50$, the intrinsic opacity as tabulated in Section 5. *h* is satisfactorily represented by $k_0 \sim (1-X)^2$.

Thus, for constant mass and changing hydrogen content the radius changes according to

$$R \sim [(1-X)X]^{\frac{2}{s+2.5}} (1-\beta)^{\frac{2s-15}{8s+20}}.$$

In Section 3.*g* we estimated $s = 6.5$ for the most probable process of atomic synthesis. With that, $1-\beta$ influences the radius but slightly $[\sim (1-\beta)^{-0.03}]$, and in the same direction as X. With sufficient approximation the change of radius for

* This equation is equivalent to Eddington's mass-luminosity relation; it is independent of the polytropic index, when homologous structure is presumed.

an adiabatic star is then $R \sim [(1-X)X]^{0.22}$. The change is rather slow. For $X \sim 1$ per cent, the radius is about one-half of its original value at $X = 33\frac{1}{3}$ per cent. Thus for $s = 6.5$ a slow contraction proceeds during the atomic synthesis; after its exhaustion, the star starts rapid contraction, relying upon gravitational energy alone. A superdense O-type or Wolf-Rayet star results.

For $s = 19$ (cf. Section 7), $R \sim [(1-X)X]^{0.10}\ (1-\beta)^{0.14}$; with increasing molecular weight $(1-\beta)^{0.14}$ increases faster than $X^{0.10}$ decreases, and the radius starts very slowly expanding; after reaching a maximum (for the sun, at $X = 0.069$, $R = 1.2\ R_{\odot}$) just before exhaustion, the radius begins to decrease and ends in a collapse as described before.

e. Evolution of the composite model.

If the star possessed originally a core of smaller hydrogen content, or should acquire such a core as the result of incomplete circulation and atomic synthesis, or if it originally settled into a compound radiative-adiabatic state, it will, during the second stage of its life, maintain the typical compound structure; in this stage the star is supposed to consist of a more or less extended convective core, built adiabatically according to a polytrope of $n = \frac{1}{\gamma - 1}$ (cf. Section 5), above which an outer shell in complete or partial radiative equilibrium is placed; at first no energy is produced in the outer shell. For such stars there is little, or perhaps no interchange of matter between the inner core and the surface. With the short time scale, composite main sequence stars of solar mass and less may at present still be in this stage of evolution; if larger masses also should be found in this stage (Procyon, cf. Section 7), this could be explained by their age being less than 3.10^{9} years.

With the exhaustion of hydrogen in the convective core the third stage of evolution for the compound model starts: the contraction of the core which gradually is transformed into a superdense nucleus. An inner core devoid of subatomic sources of energy assumes a structure very similar to an incomplete polytrope $n = 3$ (built up from the centre), generating gravitational energy according to $\varepsilon \sim T$; the persistent contraction of such a nucleus is unavoidable (the only non-collapsing form would be an isothermal structure, where the loss of energy is zero; this, however, could not maintain itself: with the first increase ΔP of the external pressure over its original equilibrium value the configuration departs from isothermity, and the net flux of energy which arises then stimulates progressive contraction and progressive departure from isothermity, until the polytrope $n = 3$ is approximately reached). The change of the radius r of the nucleus with time may be represented by

$$\frac{1}{r} = \frac{1}{r_0} + ct \qquad \text{(29)}.$$

Outside the nucleus the material is not exhausted; with the progress of the central condensation the temperature of the shell adjacent to the nucleus rises, and subatomic energy is released in an intermediate shell; the rapid increase of energy generation with increasing temperature and density in the intermediate shell prevents it and the rest of the star from being drawn into the overdense nucleus; on the contrary, if the outmost shell is in radiative equilibrium, by a process described below, it is forced to expand, and a giant star is formed. (For adiabatic equilibrium in the outer shell, a giant structure is also possible.) Let Fig. 2 show the scheme of a giant star; C is the exhausted nucleus, of radius r, mass M_c, and a net output of gravitational energy L_1 transported by radiation; A is the region of release of the subatomic energy and of convective circulation (at least in its outer portion), of radius R_1, mass $M_1 - M_c$, and a net output of energy L_2, transported to the top of the shell partly by convection; B is the region of undisturbed radiative equilibrium *, with a temperature T_1 at the bottom, extending to the surface of the star of radius R and mass M; no energy is generated in B (gravitational energy at eventual changes of radius being there negligible). The condition for secular stability is

$$L_1 + L_2 = L \qquad \text{(30)},$$

* The alternative of adiabatic equilibrium we do not consider here now.

where L is the luminosity. The violation of this condition, leading to an increase or decrease of the energy content, affects primarily the outer shell, B. The nucleus liberates automatically the practically fixed amount which it spends, and it is doomed to gradual collapse: there is no secular stability for the nucleus; but, if r/R is small, changes in r do not much reflect directly upon R, and the star may keep

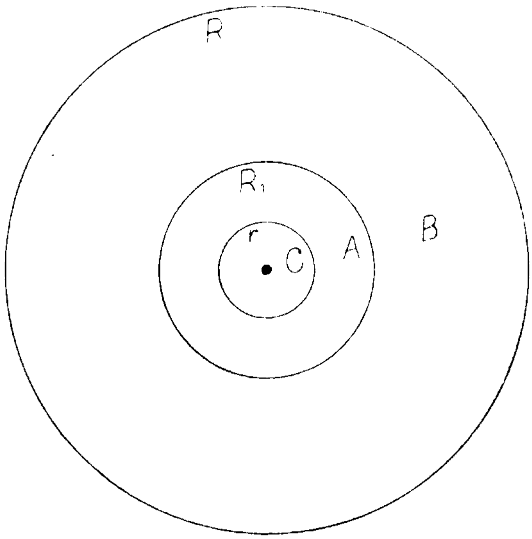

Fig. 2. Scheme of giant structure.

the outer appearance of being unchanged, which we describe as secular stability. The intermediate shell A, with convectional transport of heat, transports, under all circumstances, all the heat $L_1 + L_2$ to its top (cf. Section 5. *a*, *b*), and no accumulation is possible there. The shell B in a given fixed state, however, with its radiative transfer of energy, is able to transport, and actually does transport, a fixed amount, equal to L, and unless L is apt to vary with the radius, secular stability cannot take place. The expansion of B, of course, causes mechanically the expansion of A, and in a minor degree of C*. For homologous changes of structure, L can indeed change but slowly with the radius, the change being of a

* For C, only a slowing down of the contraction is actually imaginable.

non-stabilizing character ($\sim R^{\frac{1}{2}}$, thus opposite to the required direction, cf. [1]), so that secular stability can be attained only by an automatic adaptation of the energy sources. For our model, L_1 is not much apt to vary; as to L_2, although an increase of it is able to prevent collapse in the case $L_1 + L_2 < L$, it is unable to prevent expansion if alone $L_1 > L$. In spite of that, as shown below, secular stability may be attained, because the changes in our complex model are not homologous, and L may vary so as to fit almost an arbitrary amount of energy generated in the interior.

To derive the luminosity we apply a simple method sufficient for our qualitative purposes. The flux of radiation between two spherical surfaces R_1, M_1, T_1 and R_2, M_2, T_2, may be represented with sufficient approximation by

$$L \sim \frac{(T_1^4 - T_2^4)\ R_1\ R_2}{k\varrho\,(R_2 - R_1)} \quad \ldots\ldots\ldots \quad (31).$$

This follows from the equation of radiative transfer with $k\varrho = \text{const.}$, when sources of energy inside the shell $R_2 - R_1$ are absent; when such sources are present, the proportionality remains valid, with a variable proportionality factor depending upon $\frac{M_1}{M}$ and $\frac{M_2}{M}$ (relative internal masses), and upon the relative amount of energy developed inside the shell. For k, the coefficient of absorption, and ϱ, the density, certain mean, or effective values for the given shell must be assumed.

For stars of a homologous series, when $\frac{M_1}{M}$ and $\frac{M_2}{M}$ are kept constant, we have: $T_1 \sim T_2 \sim \frac{\beta\mu M}{R}$ (cf. [1]); $R_1 \sim R_2 \sim R$; $k \sim \varrho T^{-\frac{7}{2}}\mu^{-1}$ (Kramers); $\varrho \sim R^{-3}$; $1 - \beta \sim M^2\mu^4\beta^4$ (Eddington's quartic equation for the radiation pressure *); $L \sim R^2 T_e^4$; with these proportionalities, (31) is easily transformed into

$$L \sim M^{\frac{7}{5}}(1 - \beta)^{\frac{3}{2}}\, T_e^{\frac{4}{5}}\, \mu^{\frac{4}{5}},$$

which is exactly Eddington's mass-luminosity relation [37],

* Valid for homologous stars, with the coefficient of proportionality depending upon structure.

derived of course on the assumption $1 - \beta = \text{const.}$ throughout the star. This may be considered as a check of reliability of formula (31).

With the more general law of absorption

$$k \sim \varrho\, T^{-3+\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32),$$

the mass-luminosity relation for stars of a homologous series (not necessarily polytropic) becomes (the influence of the molecular weight upon k and luminosity is not considered now, as it is a more complicated function of the composition, cf. [19, 20] and Section 5; it cannot be represented by simple proportionality):

$$L \sim \beta^{7-\alpha} M^{5.5} R^{\alpha},$$

where β is an effective value for the whole star, depending upon its mass. For the Kramers formula $\alpha = -\frac{1}{2}$, for relativistic non-degenerate matter $\alpha = 0$. To degenerate matter the formula does not apply.

The "giant" model of Fig. 2 cannot undergo homologous changes; the nucleus, an incomplete polytrope ($n \sim 3$), when having reached a sufficient degree of compression, is practically independent of changes occurring in the outer shells. For an example take a nucleus of about the mass of the sun, $\mu = 2.1$ (no hydrogen), $T_c = 1,6.10^9$, $\beta \sim 1$; the radius of the complete polytrope is $0.025\ R_{\odot}$. The boundary of the incomplete polytrope coincides with the effective "centre" of the outer configuration*, for which a temperature of $1,6.10^7 < T' < 7.10^7$ may be estimated; for these "boundary" temperatures from Emden's tables ($n = 3$) we find: the radius of the incomplete polytrope equal to from 0.94 to 0.87 of the "complete" radius; the mass equal to 1.000 of the "complete" mass. If $T' \sim \frac{1}{R}$, our estimated figures correspond more or less to the following relation

* Which means that the central temperature and density of a polytropic configuration, devoid of a superdense nucleus, may be assumed to be of the same order of magnitude as the temperature and density are at the boundary of the superdense nucleus, when the mass and external radius in both cases are equal.

between the radius of the core, and the radius of the star:

$$r \sim R^{0.05}.$$

The exponent here decreases with increasing density of the core. Thus, from the standpoint of external changes, the nucleus is almost incompressible. On the other hand, for regions near the outer boundary of the star the changes of radius in a shell must follow closely the changes in R. Generally, for an intermediate shell like A, we may write the intermediate formula

$$R_1 \sim R^{1-p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33),$$

with $(0.05) < 1 - p < 1$. Substituting (33) and (32) into (31), with $R_2 = R$, $T_2 = 0$, $\varrho \sim (R^3 - R_1^3)^{-1}$, $T_1 \sim R_1^{-1}$ (the approximation is sufficient, as for inner points the major part of the potential is due to the central condensation, if it is considerable, thus $T_1 \sim \frac{Mc}{R_1}$), and assuming with sufficient precision $R^3 - R_1^3 \sim R^3$, $R - R_1 \sim R$ (R_1/R and p being small, justify this simplification), we get for the dependence of luminosity upon the radius:

$$L \sim R^{6.5\,p + a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34).$$

A comparatively small deviation p of the exponent in (33) from unity suffices to make our model secularly stabilized by changes in the radius; (34) must for this purpose give increasing luminosity with increasing radius, thus the stabilizing condition is

$$6.5\,p > -a.$$

For $a = -\frac{1}{2}$ (Kramers), $p > 0.077$ is required (for the non-degenerate relativistic coefficient of absorption $a = 0, p > 0$ *). For an increase of the luminosity in the ratio 2 : 1, different changes in the radius are met by different values of p as follows:

Ratio of R	2	10	50	∞
Ratio of density $\bar{\varrho}$	0.125	0.001	8.10^{-6}	0
p_1 ($a = -\frac{1}{2}$)	0.23	0.123	0.104	0.077
p_2 ($a = 0$; or $k =$ const.)	0.15	0.046	0.027	0.000
Ratio of R_1	1.70	7.5	33	∞

* The same condition $p > 0$ is valid for $k =$ const., which corresponds to a predominant role of electron scattering in the opacity (massive stars).

Thus, for a small value of p, or for a slight deviation from proportionality of R_1 and R, the outer radius may be extremely sensitive to changes in the energy generation; the doubling of the internal source of energy may produce a typical giant, of any degree of diffusion, from an originally dense star. Now, a progressive increase of the internal generation of heat, above its original normal (original regulated) value may be actually expected, when the exhausted central core starts its collapse. This core, practically a complete polytrope as far as mass and rate of heat generation are concerned, being devoid of hydrogen, radiates more energy than if hydrogen in the normal proportion were present (cf. [19, 20]) (up to 100 times more for a solar mass; the difference, however, is greatly reduced for large masses and high central temperatures, on account of electron scattering).

If L_0 is the "prescribed" luminosity of the original "main sequence" star, no great expansion of the radius can start before L_1 is a considerable fraction of L_0 (cf. formula (30)), because a moderate expansion reduces the subatomic energy L_2 and makes a balance; but when the energy output of the nucleus L_1 approaches, or even exceeds, L_0, the expansion of R must be large, and the star enters the giant stage (especially because L_2 can never drop to zero, or even to a very low value; at the boundary of the nucleus a rather peculiar zone exists where the exhausted material of the adjacent outer shell continually is driven into the nucleus, adding the released gravitational energy to L_2; the nucleus thus increases steadily, probably until a certain equilibrium size is reached, when the outwards directed resultant of the radiation pressure, being large on account of the sudden increase of the energy sources outwards, produces at the boundary of the central core a sufficiently small material density, so that the amount of inflowing material becomes equal to the radiation losses of the nuclear mass). For the exhausted core, containing no hydrogen, the amount L_1 may be read off from Eddington's table in[37] (or [1], p. 137), with the mass M_c of the nucleus as argument, and by making the bolometric magnitude 4.6 magnitudes brighter than tabulated (except when the mass is large; cf. [19, 20]). For a core of high density, and thus of large theoretical effective temperature T_e, the correction 2 log T_e is unnecessary on account of electron scattering. Take $M = 5.0\odot$ (Eddington's case of the point-source, cf.[1], p. 126), for which the central region of the convectional transfer of heat ($Q < Q_{max}$, cf. Section 5) during the original "main-sequence" stage may be estimated to reach to $\sim 0.3\,R$, including 0.25 of the total mass; taking this as the mass of the future central core, $M_c = 1.25\odot$, we have: "normal" absolute magnitude of the star, $m_0 = -0.8$; absolute magnitude of the core, $m' = 3.5 - 4.6 = -1.1$.

Thus, the original core alone, without further accretion of mass, may produce more energy per unit of time than the original non-exhausted star (the collapsed core may be even more efficient); in our case, $L_1 \sim 1.3\,L_0$, so that $L_1 + L_2 \sim 2\,L_0$, or more or less as in the above given table.

Our discussion is but qualitative: a more definite picture can be obtained only with the aid of laborious calculations. Nevertheless, it appears highly probable that the structure of the giants, and the riddle of their energy generation, can be explained as above, where only such physical laws as are known at least in principle have been applied. The picture does not change essentially if at high temperatures and pressures something like annihilation of matter occurs; this would put a limit to the collapse of the central core, whereas the external phenomena would remain the same. In the process of the collapse, there may indeed be opened up an auxiliary source of energy, by the transmutation of helium and other light nuclei into heavier ones (chiefly into the group of iron), which may yield perhaps 10 per cent of the energy of the original synthesis of hydrogen. After that, if not stopped by the interference of some unknown source of energy, the core enters the stage of perpetual and gradual contraction, without actual collapse: as shown above (Section 4. *b*), nuclear dissociation can never start appreciably; the contraction ends when the upper limit of density of matter and radiation is reached, if such a limit exists ($\varrho \sim 10^{15}$ gr/cm^3 may be such a limit, corresponding to a close packing of the atomic nuclei). The exhausted material of the envelope is gradually absorbed by the core, and there will be reached a stage when no envelope is left: Wolf-Rayet stars, or planetary nebulae with their overdense central stars perhaps represent the final stage of the giant evolution, too.

f. The imaginary catastrophic collapse.

Although we have shown that nuclear dissociation cannot be of importance in actual stars ($M < 200 \odot$), we might for a moment forget this conclusion, and try to imagine the consequences if dissociation existed. The stage of nuclear dissociation, as already mentioned above (cf. Section 4. *b*), must assume the form of an actual cataclysm; the subatomic energy radiated previously must now be paid back at the expense of the gravitational energy (all the preceding radiation into space being thus a "credit account" to be cleared at the collapse); the energy of contraction is absorbed by the nuclear dissociation, and the collapse stops only when all the matter is transformed into neutrons; the duration of the collapse must be very short, of the order of one second. As a consequence the non-exhausted outer shell A (Fig. 2) follows, the temperature and density suddenly increase there, so that an actual atomic explosion (sudden transformation of hydrogen into heavier elements) follows, releasing (in a small fraction of the mass, of course) almost instantaneously an amount of energy which normally might have lasted for millions of years; a sudden expansion of the outer shell starts, and a part of the shell is dispersed into space: thus, the phenomenon of a Nova may be well explained as a secondary effect, following the collapse of the inner nucleus. We notice that this picture of a Nova phenomenon bears some outer resemblance to Milne's theory [23], but actually the two conceptions have little in common. Unfortunately, by reasons explained in Section 4. *b*, this picture of a Nova must also be abandoned. Although we probably have to do here with a subatomic explosion, we do not find any better explanation than an old hypothesis of ours (cf.[10], p. 35 f.), according to which the "explosive" (hydrogen) mixture is driven into the interior by some external force, or by the upset of radiative equilibrium.

g. Effect of degeneracy.

All the above said refers to massive stars, for which the core is supposed to remain non-degenerate; for smaller masses, the core may become partly, or completely, degenerate, which in the first place means a decreased energy output of the core. The value of L_t approaches zero, and thus the cause which led to the formation of the diffuse envelope in massive stars is greatly reduced, or even absent, for the smaller ones. The chief reason for the absence of diffuse stars among small masses seems to be, however, a question of the speed of evolution. During 3.10^9 years, the hydrogen in dwarfs did not get exhausted, whereas in giants it did.

h. Semi-giant stage of the composite model.

After having sketched the supposed course of the evolution of a giant in broad outline, a few details may be considered. During the "main sequence" stage of a composite model, the gradual exhaustion of hydrogen in the central convective (adiabatic) region increases the mean molecular weight there, as the result of which the amount radiated outwards increases considerably (up to a maximum of about 100 times the initial value for a solar mass of the core; actually much less, because the imaginary mass of the "complete" polytrope ot the core decreases *); as shown in Section 5. *h*, the increased internal radiation never disturbs the radiative equilibrium of the outer shell; the excess of heat accumulating in this shell at first produces expansion and re-adjustment to the new conditions of radiative equilibrium. Formula (28) above was derived for a complete polytrope, but the formula practically represents the tendency of the change in radius for an incomplete polytrope, as well; thus, for $s \sim 6$ a very slow contraction with increasing molecular weight (exhaustion) follows; for $s = \frac{15}{2}$ the radius remains constant. We assume for a moment the constancy of the radius of the core, and imagine its molecular weight to have been increased suddenly in a ratio $\frac{\mu_2}{\mu_1}$; to support the external pressure the temperature in the entire core, thus also at the surface of demarkation, should increase in the ratio $\frac{\beta_2 \mu_2}{\beta_1 \mu_1}$. Thus, the

* A problem to be considered in another paper.

temperature at the bottom of the envelope (with unchanged $\mu = \mu_1$) increases, too, and the temperature gradient decreases [cf. Section 5. h, formula (22) ff.] by the condition of radiative equilibrium; to meet the change of temperature, the condition of mechanical equilibrium requires the density at the bottom of the envelope to decrease in the ratio $\frac{\mu_1}{\mu_2}$ (because the internal density of the core has remained the same), which by itself means an expansion of the envelope; but the expansion must proceed further, as the result of the decreased weight of the envelope (after the first imaginary expansion), which no longer balances the pressure at the surface of demarkation. As a result, the surface of demarkation must also expand somewhat, and a steady state will be reached when the temperature at the surface of demarkation exceeds by less than $\frac{\beta_2 \mu_2}{\beta_1 \mu_1}$ times its original value (which it had before the change of μ). Thus, with progressing exhaustion, the outer layers of the star show a definite expanding tendency, whereas the core cannot adequately follow (form. 28), because its subatomic energy sources work only at a certain degree of compression; the picture is in principle the same as that described for the giant model above. With a maximum estimated increase of $\sim 20:1$ in the heat output of the convective core due to exhaustion, and a corresponding expansion of the envelope and decrease in the mean density, "semi-giants" may be produced; the maximum luminosity*, for a given mass, of such stars may be higher by about 3 magnitudes than the "normal", or the apparent (computed) hydrogen content may be smaller (RT Lacertae, cf.[21]; Procyon, β Aurigae, cf. Section 7); of course, according to our conceptions, typical giants with a collapsing core may also show a similar excess of luminosity, though smaller (because of electron scattering) and depending upon the fraction of mass in the core.

With progressing exhaustion of the core the luminosity increases; therefore, exhaustion proceeds more and more rapidly

* The luminosity here is chiefly prescribed by the structure of the inner core without much influence of the surrounding envelope; a more limited case of the influence of an atmosphere has been considered by Eddington[49], with similar conclusions.

and the last transition phase towards a real giant, when the luminosity excess is conspicuous, cannot last for long, and the representatives of the "semi-giants" cannot be very numerous *. When the giant stage finally is reached, the luminosity may decrease again (although the extent of the envelope, with decreasing r and p, may increase considerably), because the core, now an almost complete polytrope, has a smaller net output of radiation than the former, very incomplete, adiabatic polytrope, which radiates almost at the rate of a much larger mass, of which it is an imaginary portion.

i. White dwarfs.

To explain the low rate of energy generation in white dwarfs, we are forced to conclude (as Atkinson does, cf.[12]) that their interior is devoid of hydrogen (and of neutron, too, of course); the hydrogen observed in their atmospheres must be a superficial feature, and cannot reach into regions where the temperature exceeds 10^7 K (cf. Section 4. *d*). On the short time scale it would be hard to understand how a star of less than the solar mass could ever get exhausted, unless it contained from the beginning a relatively smaller proportion of hydrogen (about 26 per cent of original hydrogen content for Sirius B, according to Table 4 below, which would have made the star originally by 1.0 mag. more luminous than the sun, thus speeding exhaustion; o^2 Eridani B would require an initial amount of 16 per cent of hydrogen). In double stars the components might have got such widely different proportions of hydrogen (Sirius A > 0.40; Sirius B $= 0.26$, etc.) perhaps as the result of the unequal differentiation of meteoric material at an early stage of the primordial nebula (cf. Section 7. *l*).

Another suggestion is that the white dwarfs are perhaps remnant cores of composite models after Nova explosions, where the greater portion of the original mass (the hydrogen containing shell) had been thrown away.

* The same must hold for the complete adiabatic model of an advanced degree of exhaustion, before its final collapse.

Section 7.

Theory and Observation.

a. Hydrogen content and mass-luminosity relation.

A mass-luminosity relation based on Kramers' general opacity corrected for electron scattering according to (23), may be written as follows:

$$L = \frac{C\left(\frac{M}{M_\odot}\right)^{\frac{16}{3}}\beta^{\frac{15}{2}}}{\left(\frac{\varrho}{\varrho_\odot}\right)^{-\frac{1}{6}} + F\beta^{\frac{7}{2}}\left(\frac{M}{M_\odot}\right)^{\frac{7}{3}}} \cdot \frac{2{,}5}{\alpha} \quad . \quad . \quad . \quad . \quad (35),$$

where C and F depend upon composition and structure. For $F = 0$, (35) corresponds exactly to Eddington's mass-luminosity relation; the formula differs from Eddington's by the factor $\dfrac{1}{1 + \dfrac{0.2(1+X)}{\bar{k}}}$, which is mostly close to 1 except for massive stars. For the mean opacity of a star, the following expressions may be assumed: $\dfrac{\bar{k}}{k_0} = 1.31\, \varrho_c T_c^{-\frac{7}{2}}$ for $n = 3$, and $2.42\, \varrho_c T_c^{-\frac{7}{2}}$ for $n = 2$. When the sun is taken as the unit of luminosity ($m_{bol.} = 4.65$), the constants are (in agreement with [19]): for $X = 0$ (no hydrogen), $n = 3$, $\alpha = 2.64$ (gravitational collapse), $C = 107$, $F = 0.033$; for $X = 33\frac{1}{3}$ per cent hydrogen, $n = 2$, $\alpha = 3.31$ (adiabatic model), $C = 1.02$, $F = 0.0054$. It is understood that the formula refers to homologous models of homogeneous composition, more or less of a polytropic structure; it may be applied to centrally situated incomplete polytropes, in which a correction factor $\dfrac{Q}{Q_{max}}$ (cf. Section 5. *b*) is required, when $r < r_0$ and none, when $r \geqslant r_0$. Thus, for the collapsing core formula (35) applies (until the relativistic change of the absorption coefficient comes into play) with $X = 0$; in this case $\left(\dfrac{\varrho}{\varrho_\odot}\right)^{-\frac{1}{6}}$ may be neglected, and the limiting formula for the luminosity of the collapsing core at high density becomes (in solar units of luminosity):

$$L_1 = 2{,}63.10^3 \left(\frac{M}{M_\odot}\right)^3 \beta^4 \quad . \quad . \quad . \quad . \quad . \quad (36),$$

where β may be taken from Eddington's table in[37]; the relativistic absorption coefficient at $T \sim 2.10^{10}$ leads to practically the same result. The luminosity may be reduced by the increased opacity due to "Paarbildung" and increased electron scattering, which, however, may have only a small influence, because the density of radiation remains small as compared with the density of matter (verified by computations of nuclear dissociation, cf. Section 4.*b*). Thus it appears that the collapsing core exhibits an enormous luminous efficiency, and that only a small core can persist without making the outer shell expand to infinity (cf. Section 6.*e*; as explained already, the size of the core is probably subject to automatic regulation). To get a total luminosity of the order of the empirical, or more or less like Eddington's mass-luminosity curve, the size of the core (M_c) according to (36) must be as follows:

M	1	2	4	9	20 $\odot$
$L_1 \sim$	1	13	100	600	2500 $\odot$
M_c	0.08	0.19	0.33	0.61	0.99 $\odot$
$M_c : M$	0.08	0.095	0.08	0.07	0.05

Thus, the core, if it exists and if it is not degenerate, can amount to only a few per cent of the stellar mass. It must be emphasized that Chandrasekhar's criterion of degeneracy[5], for our complete polytropic core which behaves almost like an independent star, applies to the mass of the core, not to that of the whole star. These cores are so small that they can be degenerate, even when increasing in mass ($M_1 < 1.6$); L_1 must be in this case much smaller, formula (36) applying only to a transition phase (contraction before degeneracy is reached). We notice that Biermann[50] has considered somewhat similar stellar models.

Below are given some examples (all computed with the same $\alpha = 2.5$ for the sake of simplicity) of the application of formula (35). For ζ Aurigae br. (not in Strömgren's list), an eclipsing binary consisting of a K1 supergiant and a B companion, $T_e = 3360^0$ according to the colour-index (cf.[55], Table I, *H. R.* 1612, $C = 1.66$ = corrected colour of the K1 star), and $\varrho/\varrho_\odot = 2{,}1.10^{-6}$ have been adopted.

Star	$\odot$	Sirius A	Algol	U Oph. br.	U Her. br.	Y Cygni br.	ζ Aur. br.
$m_{bol.}$ observed . .	4.6	0.8	−1.1	−2.0	−3.6	−5.3	−4.5
Mass	1.0	2.45	4.7	5.4	10.0	17.3	15.5
$m_{bol.}$, $X = 0\%$ . .	0.0	−3.8	−5.7	−6.2	−7.7	−8.8	−7.2
$m_{bol.}$, $X = 33^1/_3\%$.	4.6	−0.2	−3.3	−4.0	−6.4	−8.0	−6.1
Difference	4.6	3.6	2.4	2.2	1.3	0.8	1.1
$X\%$, Strömgren[21]	37	40	53	49	(70 ±)	(80 ±)	. . . .
Spectrum	G0	A0	B8	B5	B3	O9.5	K1

Star	*H. D.* 1337 br.	Trumpler's typical O star *
$m_{bol.}$ observed	−8.5	−6.0
Mass	36	91
$m_{bol.}$, $X = 0\%$	−9.2	−11.6
$m_{bol.}$, $X = 33^1/_3\%$ . .	−8.6	−12.9
Difference	0.6	−1.3
$X\%$, Strömgren . . .	(30 ±)	. . .
Spectrum	O8.5	O9

For massive stars, the influence of hydrogen content upon calculated luminosity is small (cf. also Strömgren[21]), and even may change the sign (Trumpler's star): the hydrogen content for massive stars cannot therefore be determined with confidence; mostly the observed luminosity is found to be rather low for the mass, which requires a high hydrogen content (Y Cygni, sp. O9; V Puppis, sp. B1, cf. Strömgren[21]; the K1 supergiant ζ Aurigae behaves in the same way, opposite to what Strömgren finds for the fainter giants in his list). Certainly no hydrogen content can satisfy Trumpler's O stars; there must be some reason for the strongly reduced luminosity of very massive stars (at least of those of early spectra), perhaps an increase of opacity from an unforeseen

* Cf.[51]. The typical star is just an estimate, on the basis of the seven O stars of large mass. Trumpler's average mass is greater than our adopted ypical.

source ("Paarbildung" in a collapsing core?), of which Trumpler's stars present an extreme case; the apparently high hydrogen content found by Strömgren[21] for a number of massive eclipsing binaries (*H. D.* 1337 br. for which data are given above is an accidental exception to the general rule, within the observational uncertainty) may therefore be illusory, the depressed luminosity of these stars presenting perhaps the start (at $M = 5\odot$ already, cf. Algol in the preceding table) of a phenomenon which for Trumpler's extreme masses already amounts to 6—7 magnitudes. Nothing forces us to accept the suggested high hydrogen values of other massive stars, and the most simple hypothesis seems to be at present the following: the stars are originally built of a material containing ~ 40 per cent hydrogen; the amount may decrease with time, but it can never exceed the original value (except by stratification in the original nebula, cf. below condensation of meteoric material).

For the few giants (semigiants) occurring in Strömgren's list[21], the hydrogen content falls below the normal value:

Star	Capella A	Capella B	U Sge ft	Z Vul ft	RS Vul ft	TV Cas ft	Z Her ft	RT Lac ft	RT Lac ft
Sp. . . .	G 0	F 5	G 2	(F 2)	(F 4)	(F 5)	(G 5)	(G 9)	(K 0)
$m_{bol.}$ . .	−0.4	+0.2	+1.5	+0.2	+0.3	+1.9	+2.8	+2.7	+2.9
$R : R_\odot$.	12.6	6.6	5.4	5.6	6.0	2.9	3.3	4.9	4.9
Mass . .	4.2	3.3	2.0	3.0	1.7	1.2	1.3	1.0	1.9
$X\%$ . .	30	31	19	27	3	7	15	2	28

[However, as already mentioned, ζ Aur. br., mass $15.5\odot$, requires a high hydrogen content ($\sim$ 70—80 per cent).]

In other words, the luminosities of most of these giants are too high as compared with the usual one for their masses; this exactly may be expected from our conception of the giant structure, where the outer shell is forced to expand by excessive luminosity from an "independent" core; in which case, however, the mass-luminosity formula does not hold any more: these giants, therefore, may — or may not — contain a more normal amount of hydrogen; apparently we find again that there is no reliable method of determining the hydrogen content in giants. Only "normal" main sequence stars, built according to a more or less polytropic model, permit of the determination of the hydrogen content from the observed mass-luminosity relation. How many such stars exist? In Strömgren's list[21] we find the following figures for main sequence stars fainter than absolute magnitude zero:

$m_{bol.}$ . .	0.8	4.6	5.3	4.8	2.9	1.1	1.4	0.5	0.9	1.3	2.2	2.6
X% . .	40	37	28	37	22	32	28	24	24	35	34	32

The spread is comparatively small, which explains the fact that normal dwarfs fit excellently into a single-valued mass-luminosity relation; in these stars, with their moderate luminosity, the exhaustion of hydrogen cannot as yet have proceeded very far, whence their relative uniformity of apparent composition. Thus, for dwarfs the small spread around the mass-luminosity relation is explained by the shortness of the time scale. If the more massive stars "do not play havoc" with the mass-luminosity relation, it is because the influence of the hydrogen content upon luminosity decreases with increasing mass (cf. above); if all stars are of the same age (3.10^9 y.), an equal degree of exhaustion of hydrogen may be expected for equal masses, which would result in an almost exact mass-luminosity relation for the massive stars as well. This apparently is not the case; the apparent hydrogen content is variable (cf. Strömgren's data[21], also above; unfortunately, for $M > 10\odot$, the variability cannot be well detected), thus these stars show a sensible spread around the mean mass-luminosity relation. Either the stars are not all of the same age; or the deviations are due to difference of internal structure.

The effect of variable hydrogen content upon the dispersion of the mass-luminosity relation is not so great as it seems at the first glance. If the change in hydrogen content is the result of evolution, in a steady statistical state the number of the representatives of a given mass must be inversely proportional to the luminosity; therefore, large deviations of the luminosity, corresponding to a small hydrogen content, are rare. For a solar mass starting with 36 per cent hydrogen we have:

X%	36 — 30	30 — 24	24 — 18	18 — 12	12 — 6	6 — 0
$m_{bol.}$	4.6	4.2	3.4	2.5	1.6	0.5
relative frequency .	100	63	33	14	6	2

The arithmetical mean luminosity is $m_b = 4.0$, and the individual mean deviation from this luminosity is only ± 0.59 mag.; actually a solar mass cannot have passed through such a complete evolution during the short time scale, but larger masses can. For these the mean deviations from an average empirical mass-luminosity relation, due to evolution on a hydrogen synthesis basis, are:

$M/M_\odot$	2.5	5	10	20
deviation . . .	± 0.46	± 0.28	± 0.17	± 0.09

The deviations are so small that Eddington's fear that a widely variable hydrogen content might "play havoc" with the mass-luminosity relation is not justified. It is a trick well known to observers: the probable error of the observations seems to be surprisingly small as compared with the extreme range of the measures.

b. Atomic synthesis and stellar structure.

M. Schwarzschild[52] recently made a purely formal attempt to explain the stellar energy generation by a single process, in correlation with the polytropic ($n = 3$) central temperature, density, and the apparent hydrogen content. He sets $\varepsilon \sim M^p X^q \varrho^m T^n$ (X = hydrogen content) and derives empirical correlations from forty stars (Strömgren's data[21]):

$$p = 2.29 + 1.34\,m - 0.05\,n;$$
$$q = -0.77 - 1.64\,m - 0.32\,n.$$

If we abandon the formal procedure, and consider the problem from the physical standpoint, it is almost beyond doubt that $p = 0$ and $m = 1$. Hence $n = -73$, and $q = -25$ (!). Now, q also should be equal either to 2,1, or to 0. The result for q is absurd. Further, from these values of the exponents a ratio of luminosity of Capella : sun $= 10^{-24}$ (!) results, which needs no further comment. If we set $q = 1$ as a more reasonable value, we get two values for n: $n = +73$, and $n = -11$; thus enormously contradictory results for the temperature dependence of ε. In view of such catastrophic discrepancies* we do

* Which cannot be removed by other, even unreasonable combinations of the exponents: the observational data are intrinsically contradictory.

not think that even pure mathematical reasons can justify the absence of the most primitive physical insight from the above mentioned paper. There is, nevertheless, one useful conclusion (which the author failed to draw): for a single process of energy generation the stars cannot be homologous polytropic structures; the calculated polytropic central temperatures may differ considerably from the true central temperatures.

We may invert the problem; the laws of atomic synthesis are perhaps better known than the internal structure of the stars; having adopted a law of energy generation we may pick out stars of similar structure, and mark those of a different one.

For stars of the main sequence, especially for the less massive ones, we may expect a p r i o r i a more or less uniform structure resembling a polytropic one. The radius in this case is given by formula (28), wich holds for homologous stars when the law of energy generation ϱT^s, and Kramers' opacity are valid. In (28) the main change in radius is due to the mass, whereas for any probable value of s the influence of X is very small, and that of $\beta\mu$ is also small. Further, for non-massive main sequence stars ($M < 5$), β is so near to 1 ($\beta > 0.98$), and the hydrogen content and μ change so little (cf. Strömgren's data[21], and above), that the mass alone determines the radius. We may thus write

$$R \sim M^{\frac{s-3.5}{s+2.5}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37),$$

for the "main sequence".

Densities of visual binaries furnish the most extensive statistical material for the test of relation (37). A list of such densities based upon our actual knowledge of the colour temperatures has been published by Gabovitš and the writer[58]. As the density of a visual binary is very sensitive to the adopted colour, deviations from a normal spectral energy distribution due to line or band absorption may introduce systematic errors into the calculated densities. For the spectral interval F0-M0, no serious influence of line absorption upon colour index (λ 440—550 μ) exists, and for this interval the directly determined highly accurate colour temperatures are certainly to be preferred to average estimated values (such as those given by Russell-Dugan-Stewart, partly based on the ionization temperatures which contain the hypothetical element of pressure; besides, the effective temperatures of Russell-Dugan-Stewart are somewhat too high for giant stars, which has led to not inconsiderable systematic errors in computations based upon them) (cf. [31, 53]). For spectra later than M0, the effect of TiO absorption has been taken into account in [53] (cf. also Gabovitš, [54]); for spectra earlier than F0 the effect of the crowding of the hydrogen Balmer lines (wings of the lines) towards the violet must produce a depression of the colour index, which, as well as the space reddening of the distant B stars, is not taken into account in [53] (the effect of the Balmer lines is practically the only one to be considered in the case of the brighter A stars). Systematic corrections are estimated from the following data:

H. D. Sp.	F 0	A 5	A 3	A 2	A 0	B 9	B 8	B 5
Observed colour, C_0	0.19	0.04	0.03	−0.07	−0.14	−0.18	−0.24	−0.25
Assumed T_e	7300	8700	8900	10000	11000	12000	13000	15000
True colour, C_1 . .	0.09	−0.12	−0.15	−0.27	−0.36	−0.43	−0.49	−0.59
ΔC	−0.10	−0.16	−0.18	−0.20	−0.22	−0.25	−0.25	−0.34
$\Delta \log \varrho$	+0.17	+0.27	+0.31	+0.34	+0.38	+0.43	+0.43	+0.58

Here the second line gives the mean observed colour index (in a special system) for naked-eye stars (mostly brighter than 5.0 mag, cf. [55], p. 50); the third line — the adopted "true" radiation temperature, assumed to be 15000° at B5, and made to approach gradually the colour temperature towards F0; the fourth line gives the "true" colour index, $C_t = \frac{9730}{T} - 1.24$ (cf. [31, a], p. 180); the fifth, $\Delta C = C_t - C_0$; the last line gives the resulting correction of the *log* density of a binary computed with the apparent colour:

$$\Delta \log \varrho = -1.71 \, \Delta C \quad \ldots \ldots \quad (38)^*.$$

After applying these corrections, the mean density logarithms and other data for main sequence visual binaries, according to [53], Table IV, are as follows:

* Cf. [53], p. 4, form. (8).

Table 3.

Mean Densities and Radii of Main Sequence Visual Binaries.

Mean sp.	M 2	K 4	G 9	G 4	F 9	F 5	F 1	A 5	A 1	B 3
n	6	13	9	13	31	28	24	10	21	2
$\log \varrho/\varrho_{\odot}$	0.22	0.09	0.27	0.25	−0.21	−0.24	−0.55	−0.19	−0.55	(−0.67)
$\log M/M_{\odot}$	−0.40	−0.22	−0.12	−0.05	0.04	0.15	0.20	0.28	0.36	(0.74)
$\log R/R_{\odot}$	−0.21	−0.10	−0.13	−0.10	0.08	0.13	0.25	0.16	0.30	(0.47)
p. e.	±0.04	±0.03	±0.03	±0.03	±0.02	±0.02	±0.02	±0.03	±0.02	±0.07

The last line gives the observational probable error in the mean log R. The data of the table are well represented by the linear correlation

$$\log R = (0.72 \pm 0.06) \log M + 0.03 \quad . \quad . \quad . \quad (39)^*.$$

The individual spread around this correlation is considerably greater than expected from the observational probable error, indicating real causes of the deviation ("inflation" of the atmospheres at F1, "deflation" at G 9, G 4, and A 5). Comparing (37) and (39), we find $s = 19$, within the probable limits from 15 to 25, thus an exponent for the temperature variation of subatomic energy close to the value suggested by Atkinson, but in conflict with the hypothesis of $H^1 + H^1 \rightarrow H^2$ as the basic process, which requires $s = 6.5$.

With this value of the exponent ($s = 19$), formula (28) is transformed into

$$R \sim [(1-X)X]^{0.10} (\beta\mu)^{0.54} M^{0.72} \quad . \quad . \quad . \quad . \quad . \quad . \quad (40).$$

Also, as

$$(\beta\mu)^4 \sim (1-\beta) M^{-2}$$

(Eddington's quartic equation), we have

$$R \sim [(1-X)X]^{0.10} (1-\beta)^{0.14} M^{0.45} \quad . \quad . \quad . \quad . \quad . \quad (40').$$

Fig. 3 represents the correlation; in addition to the mean data of Table 3, individual values for five nearby binaries and the sun are given; for Sirius A the correction of log ϱ for Balmer wings is applied. The radii for Sirius A, Procyon, and α Centauri A are based on directly observed colour indices, and

* We take the opportunity to point out that the correlation of mass and radius has been already successfully studied by K. Lundmark, cf.[67].

for α Centauri B on a mean adopted colour index. Further, in Fig. 3, all "main sequence" individual components of eclipsing binaries are plotted which occur in Strömgren's list [21] (for equal components, one point representing the average is plotted): for these the masses and radii are directly observed quantities, independent of the adopted temperature and the data are especially valuable for the correlation *.

As shown by Fig. 3, the individual data including even the massive stars V Puppis and Y Cygni, agree excellently with the linear correlation derived from the visual binaries. Some stars, such as Procyon (F 5), Sirius A (A 0), β Aurigae (A 0), TV Cas br (B 9), u Her ft (B 8.5) show definitely inflated radii (from 25 to 60 per cent, and 2.5 times for u Her ft), which perhaps represent a transition toward giant structure. A slight depression at large masses (B stars) is indicated, which however is well accounted for (with Strömgren's data) by the factor $(\beta\mu)^{0.54}$ which becomes important at these masses; however, we did not make a correction for this factor, because we do not consider the hydrogen content, nor μ and β, as being well established for the massive stars (cf. above). Trumpler's typical O star falls decidedly below the line of correlation, which, however, must be explained by the failure of our luminosity formula (with Kramers' + electron scattering opacity), in which case formulae (28) or (40) are no longer valid.

Thus, practically all main-sequence stars from $M = 0.2$ to $M = 20\odot$ appear to form one continuous sequence, corresponding to a homologous structure and to a law of energy generation with $s = 19$. In such a case the hypothesis of $H^1 + H^1 \rightarrow H^2 + \beta_+$ being the starting reaction of the atomic

* The hydrogen content, X, computed by Strömgren [21] is based on luminosities derived from mean adopted temperatures, and, therefore, X has lost somewhat of its individual value. If we keep T_e constant for a given spectrum, a larger radius (for eclipsing binaries) leads to a greater luminosity and thus to an apparently smaller hydrogen content; such a correlation is prominent in Strömgren's data, and part of it may be spurious: for constant mass, a larger radius means smaller pressure, and lower T_e for a given spectrum; thus, the luminosity is overestimated, X underestimated in such a case.

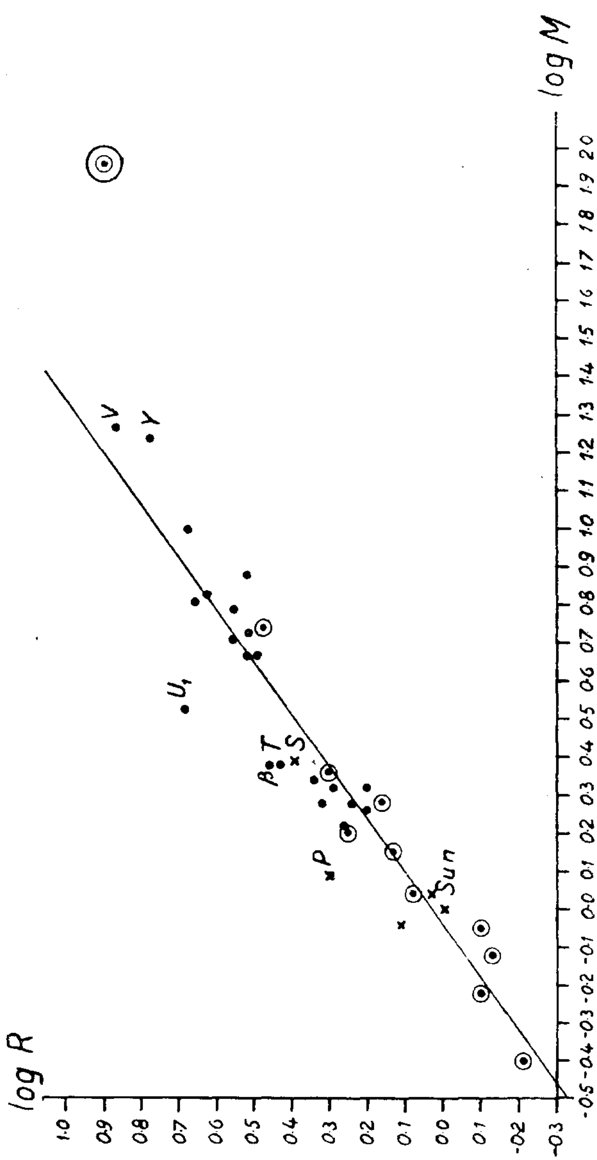

Fig. 3. Correlation of *log* radius and *log* mass for main sequence stars. The straight line is $\log R = 0.72 \log M + 0.03$; encircled dots = group mean values for visual binaries; crosses = individual stars of large parallax (Sun, Procyon (P), Sirius A (S), α Centauri A and B); dots = eclipsing binaries ($\beta = \beta$ Aurigae; T = TV Cas br; U_1 = u Her ft; V = V Puppis; Y = Y Cygni); double circle with dot = typical O star.

synthesis must be abandoned; for the main sequence, this reaction requires $s \sim 6.5$; assuming this, we should have, instead of (40), the correlation:

$$R \sim [(1 - X) X]^{0.22} (\beta \mu)^{-0.11} M^{0.33} \quad . \quad . \quad . \quad . \quad . \quad \text{(a)},$$

which cannot be made to satisfy the correlation of Fig. 3. An escape may be found in the assumption that the correlation of R and M in Fig. 3 is the combined effect of a law of energy generation with $s \sim 6.5$ and a progressive inflation of the radius with increasing mass; the cause of the inflation may be the formation of an exhausted core causing an expansion of the envelope, as considered in Sections 5 and 6. The sudden increase of log R at about the sun's mass (cf. Table 3 and Fig. 3) may perhaps be regarded as an indication of the starting of such a core, which would seem to be in agreement with estimates of the time ($\sim 3.10^9$) required for the exhaustion of a limited central region (cf. below). If a superdense core, it need not be large, a few per cent of the total mass, and probably cannot be very large (cf. the table in Section 7. *a*). Such a small core need not be in contradiction to the small effective degree of concentration of mass $\left(\frac{\varrho_c}{\varrho_m} \sim 16\right)$ derived by Russell[46] for Y Cygni, or by Walter[56] and Kopal[58] for eclipsing binaries from their ellipticity, and by Walter[57] from librations in β Lyrae and W Ursae Majoris; if most of the mass is little concentrated, the influence of the small core is imperceptible. The observed small apparent concentration of mass in the B type eclipsing binaries indicates for them either a complete mixing with complete adiabatic structure throughout, or practically the same with a small superdense nucleus at the centre. An effective polytropic index $n \sim 2$ would result in a good agreement with Russell's data for Y Cygni[46].

The interpretation of Fig. 3 as partly the result of "inflation" is not very attractive; it has been proposed to save the hypothesis of hydrogen — deuteron synthesis by setting $s = 6.5$, but the postulated superdense core becoming an independent source of energy makes this escape illusory: formula (28), as based on a purely subatomic source of energy, loses its meaning in such a case, and the regular correlation shown by Fig. 3 cannot bear the theoretical interpretation which we have given; in this case the value of s would probably have little influence upon the correlation of R and M, which should be chiefly determined by the properties of the core.

A few words with regard to Kopal's paper[58] as containing the most numerous data referring to the ellipticities of eclipsing binaries. It has been pointed out rightly[59] that Kopal's material is not homogeneous. Indeed, little definite meaning can apparently be attributed to the conception of the mean ellipticity of a binary with largely differing radii, masses, and luminosities. Nevertheless, when retaining only the homogeneous data in Kopal's list, the change of the calculated concentration of mass with spectral class remains the same as found by him from the entire material: the concentration increases for the later spectra. As the later eclipsing variables are chiefly giants, this fact seems to be in agreement with our conceptions of giant structure (formation of a superdense core and inflated envelope). Kopal's absolute values of the effective polytropic index seem to be systematically in error, as for the early type stars he finds persistently $n = 0$ to 0.5, which is less than the minimum adiabatic value ($n = 1.5$), and would lead to catastrophic convection; we have seen that convection in stellar interiors (except quite near a superdense core) is of such high efficiency in transporting heat that no perceptible deviation from adiabatic equilibrium can occur. Therefore Kopal's figures can be regarded only as of a more or less qualitative character. To get reliable absolute data for the ellipticities, the elements of the eclipsing variables should be rediscussed with such a special purpose in view; the limb darkening should be taken as variable according to effective temperature, as follows from Schwarzschild's theory of the radiative equilibrium of the outer layers of a star.

Our above result with respect to the probable value of the exponent $s = 19$ (15 to 25) forces us to attempt some revision of our former a priori views concerning the basic process of subatomic energy generation. The hydrogen — deuteron synthesis requires $s \sim 6.5$ only; further, it must have as small a probability per collision as $q \leqslant 1{,}3.10^{-19}$, to work reluctantly enough at the actual adiabatic (minimum!) temperatures of the main sequence stars, and it can never be detected in the laboratory; if the probability of the reaction is even smaller than that — the reaction loses its importance in the stellar energy budget (the reaction is theoretically possible, but the probability may be too small to exert a practical influence). From considerations put forward in Section 3.g, the $He^4 \rightarrow Li^6 \rightarrow Li^7 + H^1 \rightarrow He^4$ regenerative process cannot very well be considered as the starting point of the atomic synthesis (although it may occur as a lateral reaction), because it requires deuterons or neutrons which must be supplied by a reaction of higher order which is split necessarily into different branches and yields, therefore, too small a number of deuterons for $He^4 + H^2 \rightarrow Li^6$. $s = 15 - 25$ corresponds, according to the theory of atomic synthesis, to proton capture of a probability $q \sim 0.1$ to 0.01 (as observed in the laboratory) by a nucleus of charge $z \sim 4 - 8$, thus from beryllium to oxygen. The process must be regenerative in Atkinson's sense, if the original abundance of the starting phase is not very great. Existing physical experimental data (we cite from a compilation made by Fleischmann and Bothe[60]) seem favourable to this suggestion — indeed, there are a number of observed reactions which must occur with great intensity; stars in any case cannot settle down to higher central temperatures before the possibility of these reactions is exhausted.

Observed proton captures with a release of energy:

Reaction	$Li^6 + H^1 \rightarrow He^4 + He^3$ $Li^7 + H^1 \rightarrow 2\,He^4$	$Be^9 + H^1 \rightarrow B^{10}$ $Be^9 + H^1 \rightarrow Li^6 + He^4$
s for $T_\varepsilon = 1{,}4.10^7$ . . .	12.5	15.0
T_ε required for the sun .	$1{,}1.10^7$	$1{,}8.10^7$

Reaction	$B^{11} + H^1 \rightarrow 3\,He^4$ $B^{11} + H^1 \rightarrow He^4 + Be^8$	$C^{12} + H^1 \rightarrow N^{13}$ $N^{13} \rightarrow C^{13} + \beta +$	N^{14}, O^{16} . . .
s for $T_\varepsilon = 1{,}4.10^7$ . . .	17.2	19.6	. . .
T_ε required for the sun .	$> 2{,}4.10^7$	$2{,}1.10^7$	. . .

In the last line, the temperature $T_\varepsilon = 0.94\ T_c$ is given, for which the reaction alone is able to cover the radiation of the sun, on the assumption of a relative abundance as found by Russell[22] for the solar atmosphere. Of course, the abundance may be quite different in the interior; if carbon is 100 times more abundant in the interior, T_ε becomes $1{,}9.10^7$ for the 4th reaction, instead of $2{,}1.10^7$. The abundances having been assumed as for the above table, only the carbon reaction represents an important store of energy, lasting for about 5.10^8 years in the case of the sun; if this is insufficient even for the short time scale, it is important to realize that the $C^{12} + H^1$ reaction makes the atomic synthesis an observed reality, and not a mere speculation based on probabilities. The stars **must** have gone through this stage of atomic synthesis, at least. The central temperature of the sun for 37 per cent hydrogen content and adiabatic structure with $n = 1.63$ is $1,2.10^7$ (cf. Section 3.*d*), thus too low for the carbon, boron, and beryllium reactions; if these are to happen in the main sequence stars, the temperatures of the latter must be higher, which can be accounted for only by attributing to them a non-homogeneous structure (some degree of condensation towards the centre). It is disappointing to realize that at the temperatures of these reactions no exothermic processes of generation of neutrons or deuterons can occur, except the $H^1 + H^1 \rightarrow H^2$ reaction, and that without the help of deuterons and neutrons no experimental continuous chain of atomic synthesis can be traced; the above reactions stand thus at present isolated; neither are regenerative processes not leading to helium indicated.

With helium, and without the intervention of neutrons or deuterons, we must admit still higher temperatures; a small central core, due to slight exhaustion of hydrogen and complete exhaustion of the C + H reaction (if no regenerative process exists) may be formed, with temperatures of $\sim 3,5.10^7$ which may render the following reaction important (cf. Atkinson):

$$_2He^4 + {_2He^4} \rightarrow {_4Be^8} + 0.36 \text{ Mev.}$$

It has been the belief that this reaction is endothermic, and that Be^8 is therefore unstable (cf. Kronig [61]); however, with the more accurate atomic weights (cf. Section 3.*g* and [27]) the result is different. The observed reaction $_4Be^9 + 1,3.10^6$ volts (γ radi.) $\rightarrow {_4Be^8} + {_0n^1}$ gives $_4Be^8 = 8.0074$, against $2_2\,He^4 = 8.0078$: thus it appears that the nucleus of Be^8 is stable, after all, with a small binding energy of $360\,000 \pm$ volt. At $T \sim 3,5.10^7$, Be^8 leads to a rapid cycle of reactions (those not definitely observed are in brackets, and not observed nuclei in parentheses) (all exothermic):

$$[_2He^4 + {_2He^4} \rightarrow {_4Be^8}];\ [_4Be^8 + {_1H^1} \rightarrow (_5B^9);\ \ (_5B^9) \rightarrow {_4Be^9} + \beta_+];$$

hence the principal branch follows:

$$_4Be^9 + {_1H^1} \rightarrow {_5B^{10}};\ [_5B^{10} + {_1H^1} \rightarrow {_6C^{11}};\ {_6C^{11}} \rightarrow {_5B^{11}} + \beta_+];$$

$$_5B^{11} + _1H^1 \rightarrow 3_2\,He^4;\ _5B^{11} + _1H^1 \rightarrow _4Be^8 + _2He^4;$$

$$[_5B^{11} + _1H^1 \rightarrow _6C^{12}];\ _6C^{12} + _1H^1 \rightarrow _7N^{13} \rightarrow _6C^{13} + \beta_+,\ \text{etc.};$$

latera branch:

$$_4Be^9 + _1H^1 \rightarrow _3Li^6 + _2He^4;\ _3Li^6 + _1H^1 \rightarrow _2He^3 + _2He^4.$$

Thus, the cycle involves an efficient regeneration of helium (cf. Atkinson), and heavier elements are also formed through C^{12}.

If this mechanism does not work, at still higher temperatures observed reactions involving α-particles and resulting partly in neutron synthesis come into play, such as

$$_4Be^9 + _2He^4 \rightarrow _6C^{12} + _0n^1 + 5.5 \text{ Mev (at } T > 10^8);$$

direc neutron synthesis, however, should also begin at this stage, and with neutrons the atomic synthesis is well provided for.

From all the preceding it is clear that the synthesis of heavier elements out of hydrogen must inevitably occur at a certain stage of the evolution of the star; observed reactions are able to absorb all the hydrogen content of a star when the central temperature is allowed to rise over 10^8. However, the apparent homogeneity of structure of the main sequence (correlation and Fig. 3) does not seem to favour such high temperatures and condensations; it is more likely that the important cycle of processes takes place at $T_c \sim 2.10^7$; on account of lack of experimental data we cannot decide yet upon this question. After all, it is not impossible that $H^1 + H^1 \rightarrow H^2$ is still the basic reaction, and that our correlation of R and M reflects chiefly the effect of a progressive inflation of the stellar envelope with increasing mass, so that $s \sim 6.5$ may still be valid.

Another escape able to remove some contradictions may be proposed. The direct reaction $H^1 + H^1$ may be prohibited for the protons in their ground states, but may become possible when one of the protons is excited to a certain level of H volts. The fraction of excited nuclei is $f = 10^{-\frac{5040\,H}{T}}$ (thermodynamic equilibrium); therefore, the rate of the reaction is equal to the rate computed from the absolute abundance of hydrogen multiplied by f. The apparent probability of capture, $q = 1{,}3.10^{-19}$, which we estimated in Section 3.g, is in this case equal to the product qf.

Thus, $q.10^{-\frac{5040H}{T}} = 1{,}3.10^{-19}$. With $q = 0.01$, $-\frac{5040\,H}{T} =$ $= -16.9$. This holds upon the assumption that the energy generated in the sun at $T_\varepsilon = 1{,}1.10^7$ equals its observed heat output. Thus, with $T = 1{,}1.10^7$, $H = \frac{16{,}9.1{,}1.10^7}{5040} = 36000$ volts. Such should be the hypothetical nuclear excitation potential of hydrogen to account for the energy generation of the sun (adiabatic model) and of the main sequence stars, and which is in harmony with the observed absolute abundance of lithium in the sun: the hypothetical rate of the deuteron synthesis in the sun is then in equilibrium with the rate of the Li + H reaction, calculated from experimental data.

The rate of the reaction as given by formula (3) must be multiplied by f in this case. For $T = 1{,}1.10^7$, the effective value of the exponent in the temperature dependence of ε is then $s = 44$, which is now too high for the trend of Fig. 3. With a smaller value of q, s may be reduced; thus, for $q = 10^{-8}$, $H = 24\,000$ v., $s = 32$; for $q = 10^{-16}$, $H = 6300$ v., $s = 13$. An agreement with the "observed" value ($s = 19$) may be obtained by a suitable choice of the constant q. Although the procedure is somewhat artificial, and the question quite problematic, the importance of the above speculations consists in demonstrating that an apparent disagreement between the "observed" and the theoretical values of s cannot be a reason for denying the possibility of the direct deuteron synthesis in stellar interiors. It might be worth while to attempt the synthesis in the laboratory, in a hydrogen medium "activated" by radiation of 10 000—50 000 volts (for the lower limit, the yield of the reaction may be too small to be detected in the laboratory if our calculations as to q are correct).

c. Stellar statistics and stellar evolution.

Under the above heading fifteen years ago the writer published an attempt to explain the observed frequency function of stellar luminosities on the basis of a recurrent cycle of stellar evolution, by assuming statistical equilibrium between the "rate of cooling" and the frequency of Nova catastrophes, which were supposed to bring the star back to the start of a new evolutional course with high initial luminosity and gradual "cooling". At present it stands without doubt that such a conception does not represent the state of our present stellar universe: the stars do not change much in luminosity (as once considered by Eddington, on the basis of a "Van der Waals degeneracy" of dwarf stars), except the white dwarfs; for an evolution of mass, the time scale is too short (studies of double stars by the writer[62], cf. also[2]). The Russell-Hertzsprung diagram is not a diagram of evolution, but a diagram of stellar structure.

Now, from the fact that the hydrogen content is smaller in giants than in main sequence stars, Strömgren[21] concludes that the course of evolution with practically constant mass is at right angles to the main sequence line of the diagram: main sequence stars change into giants. Our preceding theoretical discussion points to the same possibility. Below, a comparison with statistical data seems to impose certain restrictions upon this type of evolution also: some main sequence stars become giants; many, however, cannot.

d. Evolution of the sun and geologic temperatures.

Evolution with decreasing hydrogen content requires increasing luminosity, and in the case of the sun some vague information regarding such an evolution can be obtained from the geological history of the earth. The mean temperature of the earth must be chiefly determined by the intensity of solar radiation; unfortunately, local conditions during past ages obscure the general climatic picture too much; but certain conclusions can nevertheless be drawn. There seems to be no doubt that, during the Cambrian and Ordovician, the mean temperature was about the same as it is now; it was considerably warmer since the Silurian, up to the late Tertiary. The last Diluvial relapse of temperature, the ice age, which lasted with interruptions for about half a million years, is so short as compared even with the Tertiary (60 million years) that its influence upon the mean geological temperature is negligible; we may consider $+20^{0}$ C at present as the normal mean temperature of the earth, corresponding more or less to the conditions prevailing at the end of the Tertiary; the present actual temperature ($+15^{0}$) is still below the normal, not having as yet recovered from the relapse of the ice age. Allowing for the cooling effect of an ice + snow-covered area, an ice age with glaciation reaching to about 45° latitude (zero annual isotherm with sufficient snow in winter) should correspond to as high a mean temperature of the whole earth as + 9° C. There can be hardly any doubt that the last ice age, which was bipolar, was caused by a decrease in solar radiation (speculations upon the eccentricity of the earth's orbit cannot produce a bipolar effect, and the unipolar effect must be rather small even for the affected hemisphere), and probably most preceding ice ages were, too [there were mighty glaciations * in the Algonkian (South Australia; synchronous in South Africa and Canada), in the lower Cambrian (bipolar, Australia and Greenland), some glaciation in the Middle Ordovician, in the lower Devonian, and an enormous glaciation in the Permian (perhaps the late Carbon already), apparently restricted to the southern hemisphere, with traces left in Australia, Africa, East India, Brazil, and the Falkland islands; absence of Permian glaciation from the northern hemisphere may perhaps be explained by special circumstances of the distribution of continents and mountains]. Between the Permian and the Diluvian there are no ice ages known. Although Wegener's theory of continental drift, postulating a corresponding displacement in latitude, cannot explain the last ice age, and the amount of the drift required for such a purpose is not verified by astronomical observations, there can hardly be any doubt about the reality of large-scale horizontal displacements (Alpine foldings) in the earth's crust; these displacements cannot reach the scale of Wegener's theory, but, nevertheless, they summon to caution with respect to a generalization of local peculiarities of a geological climate: the latitude where at present a fossil is found may considerably differ from the latitude of its origin. With due allowance for all such circumstances, the general trend of temperature seems to be an increase, highly irregular, interrupted by sudden minima of short duration, the ice ages; the apparent absence of these from the Mezozoicum and Tertiary is perhaps due to the increase of the "normal" temperature of the earth (or of

* Professor A. Öpik, geologist, has checked upon these geological data.

the mean solar radiation), so that only rare exceptionally deep minima, such as the Diluvial, lead now to glaciation; whereas in the Palaeozoicum and Praecambrium, with their lower normal temperature, moderate and therefore more numerous minima of the solar radiation already produced glaciation; thus, the relatively greater frequency and extent of glaciation in these early periods seems to be in agreement with the gradual warming up of the earth following the increase of solar radiation with the gradual exhaustion of hydrogen. Judging from the oldest Archaic rocks, of an age $\sim 2.10^9$ years, in spite of changes from metamorphism, it seems to be certain that a permanent ice age could not have taken place even at this early age: the traces of ice in the Archaicum are scarce, probably mostly destroyed, but still suggestive of intermittent ice ages, as observed in later ages *. A minimum estimate for the mean "normal" temperature of the earth.

Table 4.

Luminosity of the Sun (Adiabatic Model, Complete Mixing), and Terrestrial Temperatures.

Hydrogen content, X %	40	39	38	37,5	...	...	...	...
Age, 10^8 years	−31.5	−20.2	−9.7	−4.8 (Ordovician)	−3.2 (Silur.)	−1.2 (Jurassic)	−0.6 (Palaeocene)	−0.2 (Miocene)
$m_{bol.}\odot$, computed	4.84	4.75	4.67	4.62	...	...	...	...
t^0 C Earth, computed	$+3^0$	$+9^0$	$+14^0$	$+17^0$	$+18^0$	$+19^0$	$+20^0$	$+20^0$
t^0 C Earth, geological estimate	...	$\geqslant +9^0$	$\geqslant +9^0$	$+15^0$:	$+22^0$	$+17^0$	$+25^0$	$+20^0$

Hydrogen content, X %	...	37,0	37	36	30	24	18	12	6	0
Age, 10^8 years	−0.01 (Diluv.)	0.00	0	+9.0	+49.0	+74.0	+87.2	+92.8	+95.2	+96.0
$m_{bol.}\odot$, computed	...	4.58	4.58	4.49	3.93	3.28	2.53	1.58	0.58	−0.47
t^0 C Earth, computed	$+20^0$	$+20^0$	$+20^0$	$+26^0$	$+62^0$	$+117^0$	$+191^0$	$+305^0$	$+458^0$	$+655^0$
t^0 C Earth, geological estimate	$+8^0$	$+15^0$	—	—	—	—	—	—	—	—

* Here a most important problem for Archaean geology presents itself.

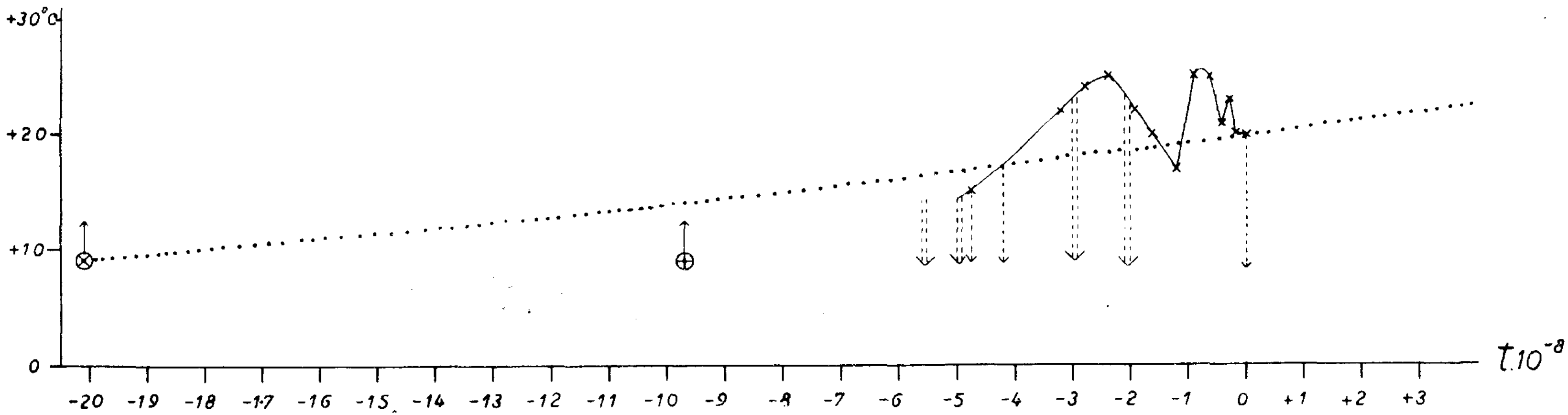

Fig. 4. Mean temperature of the earth (ordinates), and geologic time (abscissae, unit = 100 million years). Crosses and full curve = estimated mean temperature in the past; broken arrows directed vertically downwards = ice ages; crossed circles with arrows directed upwards = estimated minimum mean temperature during the Archaean. Dotted line = theoretical "normal" temperature of the earth, for complete mixing and atomic synthesis in the sun.

2.10^9 years ago may thus be $\geqslant +9^0$ C. Above, the table gives the theoretical change of the luminosity of the sun on a hydrogen synthesis basis with complete mixing, with the computed and "observed" mean terrestrial temperatures. The present hydrogen content is assumed to be 37 per cent, conventionally, and the present "normal" absolute magnitude (4.58) by 0.07 brighter than the present observed one (4.65). The data for the past and the "near" future are represented in Fig. 4; the theory is not contradicted by the observational evidence; the latter hardly admits of a more rapid increase in luminosity than the computed one, but the actual increase cannot be determined with certainty. Thus, on the basis of the complete adiabatic model the sun must have started $\sim 3.10^9$ years ago with a hydrogen content $X = 40$ per cent, and has used up to present 3 per cent of it; the accelerated rate of evolution on the atomic synthesis basis will come to an end after $9{,}6.10^9$ years, but life on earth will be destroyed by heat long before that. The total life of the sun on the hydrogen basis is $12{,}6.10^9$ years, after which a collapse follows towards the white dwarf stage (when the temperature of the earth may fall to -180^0 C).

From the table we also infer that a solar mass in complete adiabatic equilibrium containing originally less than 26 per cent hydrogen must already have become, within 3.10^9 years, a white dwarf (Sirius B? cf. Section 6. *i*).

The recurrence of the ice ages — i. e., of short minima of solar radiation — seems to be rather difficult to explain on the completely adiabatic model of the sun, with its complete mixing. A composite adiabatic-radiative model, with its changing extent of the adiabatic region, progressive stratification at the boundary of the convective core as the central exhaustion increases, and the formation of a small collapsing kernel, is more likely to lead to fluctuations of luminosity. The answer can probably be obtained from numerical computations. Qualitatively, however, it seems likely that the collapsing kernel (~ 7 per cent of the mass, cf. Subsection *a*, thus small enough to become exhausted during $\sim 10^9$ years), passing through different stages of degeneracy, and gradually acquiring mass from exhausted material of the outer shell, may produce an uneven variation in the luminosity. For example, let us suppose that the adaptation to the changing condition of equilibrium requires an intermittent expansion of the outer shell; during the expansion, a fraction of the heat output is stored in the form of potential energy of gravitation, and the radiation into space is temporarily reduced by a corresponding amount. To explain in this way an ice age of one million years' duration with a depression of the terrestrial temperature by 11^0 C, a total expansion (of the shell containing $\sim$ 90 per cent of the solar mass) of less than one per cent of the radius is required for an unchanged internal heat output. The plausibility of such an explanation adds some probability in favour of a complex model of the sun.

In the case of a superdense core, however, the luminosity cannot be calculated in such a simple manner as was done above; the degenerate core will not add much to the heat output, thus the atomic synthesis will be still the main source of energy; the luminosity, however, will not rise much with exhaustion, because the exhausted material joins the degenerate core. A longer age on the atomic synthesis basis must result.

e. *Duration of evolution for adiabatic models.*

Here we cannot as yet decide the question of the probable structure of the sun and the main sequence stars. As a standard of comparison for probable ages we below consider the complete adiabatic model for main sequence stars, with an initial hydrogen content of 40 per cent as found for the sun (cf. Table 4, $X = 40\,^0/_0$ at $t = -3{,}15.10^9$ years).

Allowing for the unknown cause of a depressed luminosity in massive stars (which Strömgren [21] ascribed to a large hydrogen content, and which we as yet hesitate to accept, for reasons given above), we take an empirical mass-luminosity relation, or assume luminosities of actual typical stars as corresponding conventionally to $X = 33\frac{1}{3}$ per cent, and use the hydrogen content only for a differential correction of the luminosity in Table 5. We need not bother about the rigour of these assumptions, as they are needed only for our order-of-magnitude comparison. Our table in any case corresponds

Table 5.

Duration of Atomic Synthesis Stage, Complete Mixing.

	$M/M_\odot$	1.0	1.5	2.0	2.5	3.0	4.0
$m_{bol.}$	begin., m_0	4.8	3.5	2.2	1.2	0.4	— 0.6
	mean, $\overline{m}$	4.0	2.7	1.4	0.4	— 0.4	— 1.4
	end, m_1	— 0.5	— 1.5	— 2.4	— 2.9	— 3.3	— 3.8
	Relative duration	1.000	0.479	0.208	0.115	0.0730	0.0437
	Duration, years .	$1{,}3.10^{10}$	$6{,}2.10^{9}$	$2{,}7.10^{9}$	$1{,}5.10^{9}$	$9{,}5.10^{8}$	$5{,}7.10^{8}$
	$M/M_\odot$	5.0	7.0	10 *	17 **	36 ***	90 ****
$m_{bol.}$	begin., m_0	— 1.4	— 2.3	— 3.3	— 5.1	— 8.4	— 6.0
	mean, $\overline{m}$	— 2.1	— 2.9	— 3.8	— 5.5	— 8.7	— 6.0
	end, m_1	— 4.1	— 4.4	— 4.9	— 6.1	— 9.1	— 6.0
	Relative duration	0.0301	0.0219	0.0148	0.0060	0.0007	(0.021 :)
	Duration, years .	$3{,}9.10^{8}$	$2{,}8.10^{8}$	$1{,}9.10^{8}$	$7{,}8.10^{7}$	$9{,}1.10^{6}$	$2{,}7.10^{8}$

* u Her. br.; ** Y Cygni; *** *H. D.* 1337 br.; **** Trumpler's typical O type star; no effect of hydrogen content assumed.

to the actual luminosities of the stars. The relative duration is computed with sufficient precision from the formula

$$t_{rel.} = \frac{5.34}{(m_0 - m_1)} \frac{M}{M_\odot} \cdot 2.512^{-(4.8 - m_0)} \left[1 - 2.512^{(m_0 - m_1)}\right],$$

and the mean bolometric magnitude as in Subsection *a* from

$$\overline{m} = m_0 + \frac{1}{t} \int (m - m_0)\, dt = m_0 - 1.085 + \frac{m_0 - m_1}{2.512^{m_0 - m_1} - 1} + \varDelta,$$

where $\varDelta$ is a small correction $= 0.2 - 0.0$ mag, and where m_0 and m_1 are the initial and the final bolometric magnitudes; the first formula implies a linear change of the magnitude with X (hydrogen content), which is not quite correct, but sufficient for the relative duration; the absolute duration for $M = 1$, however, is taken from Table 4 where it was computed step by step, without simplifying assumptions. The most interesting feature of the table is the still considerable relative duration for massive stars: the smaller increase of luminosity with exhaustion partly balances here the greater initial speed of evolution. Nevertheless, for masses exceeding 2.0, there seems to be a shortage of the subatomic energy source even with the short time scale. There exists, of course, a possibility of increasing the figures: allowing for uncertainties in the internal structure, the initial hydrogen content may be increased to about 50 per cent, which lengthens the duration of the "dwarf" stage by 25 per cent in proportion to the store of hydrogen (the theoretically fainter luminosity for the increased hydrogen content does not lengthen our time scale, because we started from the observed luminosity of the sun, and computed the rest in a differential way). On the other hand, for incomplete mixing (composite model), only a fraction of the mass is involved in atomic synthesis, so that the duration of the dwarf stage may be shorter. The composite model may be supposed to enter the giant stage after the exhaustion of perhaps 0.25 of the internal mass, thus after one-quarter of the time interval of Table 5. With 3.10^9 years as the maximum age, the minimum mass of a giant equals in this case $1.2\odot$, which more or less corresponds to the observed limit (RT Lacertae, 1.0 and $1.9\odot$; ZHer ft, $1.3\odot$; etc.).

f. Semi-giants.

It is tempting to identify the Mt. Wilson *s* (sharp) and *n* (nebulous) classification of B and A type stars [58] as corresponding to our two subdivisions of stars without collapsed cores: *s*, about 0.8 mag brighter than *n* (for the same spectrum), with the "sharp" lines corresponding to a smaller surface gravity, may be identified with the "semi-giant" stage, and *n* with the dwarf stage. From certain complicated considerations here omitted we estimate the difference of luminosity for equal mass, at $0.8 \times \frac{3}{2} = 1.2$ mag. From Table 5 it appears that the duration of the non-collapsed stage at $M \sim 2.5\odot$ (A stars) is certainly shorter than 3.10^9 years for the composite model (~ 0.25 of the tabulated durations); as non-collapsed stars are still observed, we must suppose that these stars are continually born in the place of those which have become giants; in such a case, the relative number of the *n* and *s* stars must be proportional to their relative lives, or inversely proportional to their luminosities, or 3 : 1 for $\Delta m = 1.2$, as found above. For northern stars brighter than the fifth magnitude (complete selection) we find indeed 69 stars B8*n* — A3*n*, of a mean absolute magnitude 1.0, against 24 stars B8*s* —

A3s, $\overline{\mathrm{m}}_{abs.} = 0.5$ *; for equal volumes of space, the number of s stars must be counted to $m = 4.5$ and is 17; this requires a ratio of the relative lives about $n:s = 4:1$, or slightly greater than the theoretical ratio (3 : 1). The agreement in relative number and luminosity is hopeful indeed. If A and B stars can be identified as non-collapsed, i. e., as dwarfs or semi-giants, their presence in the Galaxy, when confronted with the figures of Table 5, and allowance made for the most generous increase of these figures, would require a steady supply of such stars, to replace those entering the giant stage. The production of new stars (not Novae) cannot, of course, be limited only to these classes, but must be in this case a more general phenomenon.

The Mount Wilson s and n subdivisions may be looked upon also as different stages of exhaustion of the adiabatic model; from formula (40′), for $M = \text{const.}$, $R \sim [(1-X)X]^{0.10}(1-\beta)^{0.14}$, we find for $M = 2.5$, and an interval of luminosity $\varDelta m = 1.5$ mag, $X_1 = 0.40$, $X_2 = 0.21$, $\mu_1 = 0.92$, $1-\beta_1 = 0.0145$, $\mu_2 = 1.26$, $1-\beta_2 = 0.042$, and $\log \dfrac{R_1}{R_2} = -0.049$. This gives an effective relation $L \sim R^{12}$, thus a slow increase of radius with luminosity (for X near 0 the radius begins to decrease again). In such a case, the difference of luminosity between $n - s$ is 0.8, practically the same for equal mass, as it is for equal spectrum, and the relative number of $n:s$ to be expected is 2 : 1, thus a greater deviation from the observed ratio (4 : 1); the difference, however, is not serious. More serious is the question of surface gravity which for the adiabatic model varies as $g \sim L^{-0.16}$, and requires a ratio $g_n:g_s = 1.12$ only, too small to influence sensibly the appearance of spectral lines; whereas for the "semi-giant" theory of the As stars the ratio of surface gravity we estimate at $g_n:g_s = 2.3$, which may be considered as sufficient **. It is, therefore, possible that the Mount Wilson As stars are "semi-giants", on the point of becoming giants. The An stars may thus be supposed to

* The difference $s - n = 0.5$ mag is smaller than the difference for a given spectrum (0.8 mag), because of a preponderance of early n subclasses and late s subclasses in our sample.

** Computations referring to the semi-giant model will be given in another paper.

contain: all complete adiabatic models, and the younger composite models; whereas As contain only the advanced composite models; in such a case the relative number $n:s$ should be greater than the expected, 3:1, as actually is the case (4:1).

g. Statistical equilibrium of evolution.

Let us consider now the possibility of an evolution from the main sequence towards giants with a continuous supply of main sequence stars. Taking 3.10^9 as the age of our universe, and assuming a uniform rate of the generation of new stars (not Novae) — the order of magnitude may be right although more stars may be supposed to be generated at the beginning — further assuming that the dwarfs after the lapse of a time interval t_d become giants, — the relative number of giants and dwarfs (including semi-giants) of a given mass will be proportional to

$$\frac{n_g}{n_d} = \frac{3.10^9 - t_d}{t_d} \quad \ldots \ldots \ldots \quad (41),$$

where t_d is the total duration of the non-collapsed stage; the formula holds for $t_d < 3.10^9$ only, and for the case of an infinite (sufficiently long) life of the giants; for $t_d > 3.10^9$, $n_g = 0$.

If, however, giants are also doomed to a short life, t_g, (41) holds for $t_g > 3.10^9 - t_d$, whereas for $t_g < 3.10^9 - t_d$ we have

$$\frac{n_g}{n_d} = \frac{t_g}{t_d} \quad \ldots \ldots \ldots \quad (42).$$

The hypothetical "disappearance" of a giant, at the end of its life, may consist in it becoming a Wolf-Rayet star (similar to a nucleus of a planetary nebula) of high effective temperature and density; the difference of visual *minus* bolometric magnitude for high temperatures runs as follows:

Table 6.

Visual *minus* Bolometric Magnitude.

T_e	3.10^4	6.10^4	10^5	2.10^5	3.10^5	5.10^5	10^6
$m_v - m$	$+2.3$	4.3	5 6	7.9	9.3	10.9	13.2

Thus, if the surface temperature of the former giant attains 10^5 (such a temperature has been actually observed for nuclei of planetary nebulae, cf. [39]), and if its bolometric magnitude remains unaltered, it will appear fainter by about five magnitudes, so that less than one per cent of the former giants can be observed. Still higher effective temperatures may occur; for $T_e > 10^6$ this would mean complete disappearance of all former giants from our catalogues (such high temperatures have not yet been observed, but perhaps for the same reason — they cannot be observed; the high temperature stars are practically invisible).

Table 7 contains data which may be used as a crucial test for the question of the persistence of the giants. The first half of the table refers to the distribution in our Galaxy, based on the data derived by the writer and his collaborators [40] from proper motions of the Boss catalogue; the original figures, referring to a fixed limiting apparent magnitude, are reduced to equal volumes of space with the aid of Kapteyn's mean density function. The second half of the table represents the mean distribution for three globular clusters, derived from Shapley's data [41]. The "main sequence" is supposed to include the dwarfs and the semi-giants. Actually, the giants are distinguished by their mean density, and there is a possibility for the main sequence to contain collapsed stars with a giant structure, but of high mean density. The line of demarkation is not very definite, but this introduces little uncertainty into the data for the Galaxy, because stars with transition spectra are few in number there. For the globular cluster, the stars near $m_{bol.} = 0$ are most numerous for transition spectra, and the relative numbers depend more upon the choice of the line of demarkation between giants and main sequence stars.

On the assumption that all main sequence stars are transformed into giants within time intervals comparable to about one-quarter of those of Table 5, the distribution "around the sun" in Table 7 can be satisfied only by assuming comparatively short lives for the giants; a trial-and-error solution gave a rough representation, according to formulae (41) and (42), of the observed frequency of giants, with the following assumptions: $t_d = t_g = 0.3$ times the total ages of Table 5; $\bar{m}_d = \bar{m}_g$ *.

* The approximate equality of the average luminosity of a giant and a

Table 7.

$m_{bolom.}$	Distribution of Bolometric Absolute Magnitudes							
	Around the sun, $D < 125$ parsec				Globular cluster (Shapley)			
	Main sequence		Giants		Main sequence		Giants	
	Spectrum	Number	Spectrum	Number	Colour class	Number	Colour class	Number
−8	O	0.018	B0 — M	0.005	. . . .	0	. . . .	0
−7	O	0.045	B0 — M	0.047	. . . .	0	. . . .	0
−6	O	0.10	B0 — M	0.11	. . . .	0	. . . .	0
−5	O — B2	2.6	B3 — M	0.75	. . . .	0	b5 — m	0.7
−4	O — B5	2.5	B8 — M	2.5	. . . .	0	„	12.7
−3	O — B5	18	B8 — M	23	. . . .	0	„	18.0
−2	B0 — A0	105	A2 — M	51	< a0	0.3	a0 — m	46
−1	B0 — A5	122	F0 — M	420	< f0	8.1	f0 — m	109
0	B3 — A5	460	F0 — M	910	< f0	143	„	174
+1	A0 — F2	1380	F5 — M	890	< f0	60	„	155
+2	A0 — G0	4400	G5 — K5	2400	. . . .	. . .	. . . .	. . .
+3	A3 — G5	34000	K0 — K5	4300	. . . .	. . .	. . . .	. . .
+4	F5 — G5	8000	. . . .	0	. . . .	. . .	. . . .	. . .

The same assumptions, however, do not in the least fit the distribution in the globular clusters; unless all these are less than 6.10^7 years old (thus less than the time of one revolution or oscillation in the Galactic system), the existence there of red giants, unmatched by main sequence stars, can be explained only upon the assumption of a great persistence of the giant stars, $t_g > 50\, t_d$. The discrepancy can be removed only by rejecting our first assumption: we conclude, that not all main sequence stars can be transformed into giants; this conclusion agrees with our theoretical considerations (Sections 5 and 6: complete adiabatic structure).

The absence of bright main sequence stars in globular clusters may be explained by assuming that no "new" stars are born in the cluster, all its stars being of equal age; in such a case the more massive stars have either become giants, or collapsed into superdense Wolf-Rayet stars which have become

main sequence star of the same mass is an observed fact, cf. ζ Aurigae and Y Cygni above; the smaller hydrogen content of the giants is partly a computational result counterbalancing Eddington's correction $-2 \log T_e$.

invisible on account of their high effective temperature. For an age of 3.10^9 years, Table 5 sets the limit of collapse for the adiabatic model at $M/M_\odot = 1.9$ (thus above Chandrasekhar's limit of degeneracy for zero hydrogen content), initial magnitude $m_0 = 2.3$; the bolometric magnitude at the end of the main sequence stage, at complete exhaustion, then is: $m_1 = -2.3$ (cf. Table 5). Shapley's data according to Table 7 indicate a limit at -2.0; a better agreement one could hardly expect. Assuming a monotonous distribution of the initial magnitudes, the theoretical distribution of the final magnitudes of main sequence stars in a globular cluster is found to be as follows (cf. Tables 5 and 4; the magnitude excess $m - m_0$ as a function of relative age is computed with the aid of the latter table, with a reduction of the difference in a proportion of $\frac{m_1 - m_0}{5.3}$; only the adiabatic model, i. e., the durations of Table 5 are assumed, as only this model determines the "top" of the residual frequency of luminosities in the clusters, the composite model main sequence stars having changed into giants much earlier):

Frequency of Main Sequence Magnitudes (bol.) in Globular Clusters, and Final Magnitudes.

Initial mag., m_0 . .	3.8	2.8	2.7	2.6	2.5	2.4	2.35	2.325	2.31	2.30
Assumed frequency	. . .	51	46	42	38	19	10	6.4	3.6	
Duration of main sequ. stage, t_0 =	$7,1.10^9$	$3,8.10^9$	...	...	...	...	...	...	...	$3,0.10^9$
Relative age, $(3.10^9)/t_0$ = . .	0.42	0.79	0.84	0.88	0.92	0.96	0.98	0.990	0.996	1.000
Final (present) hydrogen content, X %	34	25	23	21	19	14	10	7	4	0
Final (present) mag., m = . .	3.3	1.5	1.2	0.9	0.5	−0.2	−0.75	−1.28	−1.69	−2.3

The resulting distribution, arranged according to the magnitude limits of Table 7, is:

Bolom. mag. m, limits . . .	1.5 ... 0.5	0.5 ... −0.5	−0.5 ... −1.5	−1.5 ... −2.5	< -2.5
Number, computed	139	48	22	6.6	0
Number, observed . . .	60	143	8	0.3	0

The disagreement between the computed and observed distributions lies chiefly in the observed excess about zero magnitude which is near the median magnitude of the cluster type variables (cf. Shapley [41]); if this maximum is smoothed out, the agreement between observed and computed distributions may become complete; the maximum, and perhaps the cluster type variability, may be intimately connected with some peculiarity in stellar structure where the exhaustion of hydrogen has attained a certain degree (about 15 per cent, as follows from the table).

Thus, the presence of massive and luminous main sequence stars in the Galactic System in general we choose to ascribe to stars being continually formed in the place of those which become giants, or which collapse. The distribution of the ages of meteorites (probably mostly interstellar) as determined by Paneth [64] (cf. also [2]) from the helium-radium ratio is suggestive of a continuous condensation of diffuse matter at a uniform rate (no matter whether the meteorites are products of direct condensation, or fragments of larger bodies):

Age, millions of years	0—500	500—1000	1000—1500	1500—2000	2000—2500
Number of meteorites	4	4	6	3	4

Age, millions of years	2500—3000	> 3000	—	—
Number of meteorites	3	0	—	—

Thus, the assumption that stars in the Galaxy are also born at a uniform rate does not seem quite arbitrary; in any case, there seems to be enough diffuse matter still left for such a purpose. Our other assumptions, based upon all the preceding theoretical and observational evidence, are: that all stars start as main sequence objects with a conventionally constant hydrogen content of 40 per cent; that a fraction a of them are adiabatic models, to which the figures of Table 5 apply, and which disappear observationally after the exhaustion of hydrogen and the following collapse (Wolf-Rayet stars for $M > 1.6 \odot$, cf. Table 6; white dwarfs for $M < 1.6 \odot$); that a fraction $1 - a$ of them are compound adiabatic-radiative models, which after an intermediate semi-giant stage of shorter duration (cf. above) become giants after the lapse of 0.3 the durations of Table 5, with a mean luminosity equal to $\overline{m}$ of that table (this is an empirical fact, although the decimal of the mean magnitude is not warranted); that the giant stage for any giant lasts for more than 3.10^9 years (cf. globular clusters).

Upon these assumptions, and with $3{,}0.10^9$ years as the maximum age, the frequency table for the Galactic System (Table 7, 1-st half) is analysed below. Unlike the globular clusters, where all stars are of the same age, all ages are represented in the Galactic System, and it is therefore permissible to use the average magnitudes ($\overline{m}$ of table 5) as representative of the average masses. The statistical equilibrium number n_1 of compound main sequence stars is computed from the observed number of giants by formula (41) ($n_d = n_1$, $t_d = 0.3\ t$); the number n_2 of existing adiabatic models is $n_2 = n_{dw} - n_1$, where n_{dw} is the total number of observed main sequence stars; the number of collapsed (invisible, in any case not counted in Table 5) adiabatic models n_c is given by the same formula (41), with $t_d = t$ (duration from Table 5), and with n_c for n_g, n_2 for n_d. The total number of adiabatic models which have come into existence since the creation of the Galactic System is $N_a = n_2 + n_c$; the corresponding number of composite models is $N_r = n_1 + n_g$. The relative frequency of composite models born is

$$1 - a = \frac{N_r}{N_a + N_r},$$ whereas the relative frequency of composite models among observed main sequence stars is $q = \frac{n_1}{n_{dw}}$. On these lines, Table 8 has been computed.

The most remarkable feature of this table is, for $\overline{m} \geqslant -1.0$, the steady behaviour of $1 - a$ (last line) fluctuating around 0.5, and the sudden drop for greater luminosities. If our interpretation is correct, this means that masses below $4.3 \odot$ have an equal chance to become adiabatic, or composite; whereas for larger masses the probability to become composite is small, about 0.06; the considerable frequency of luminous giants is, from this standpoint, explained by their longer life; all, of course, depends upon our assumptions.

As to the residual small fraction $(1 - a)$ of composite models for $M > 4.3\, M\odot$, it may be due to an original difference in composition between the central region and the rest of the star (cf. Section 5. *h*, and below).

Table 8.

Analysis of the Frequency of Main Sequence and Giant Stars in the Neighbourhood of the Sun.

$D < 125$ parsec. $1 - a$ = fraction of compound radiative-adiabatic models born (a = fraction of adiabatic models).

m_{bol} (= $m_{observed}$)	4.0	3.0	2.0	1.0	0.0	—1.0
$M/M_\odot$	1.0	1.4	1.8	2.2	2.8	3.6
t, years	$1{,}3.10^{10}$	$7{,}1.10^{9}$	$3{,}8.10^{9}$	$2{,}2.10^{9}$	$1{,}2.10^{9}$	$7{,}4.10^{8}$
n_g	0	4300	2400	890	910	420
n_1	(< 8000)	12600	1400	250	120	33
n_{dw}	8000	34000	4400	1380	460	122
$q = n_1/n_{dw}$	. . .	0.37	0.32	0.18	0.26	0.27
n_2	< 8000	21400	3000	1130	340	89
n_c	0	0	0	410	510	270
N_a	< 8000	21400	3000	1540	850	359
N_r	< 8000	16900	3500	1140	1030	453
$1 - a$	. . .	0.44	0.54	0.42	0.55	0.56

$m_{bol.}$ (= $m_{observed}$)	—2.0	—3.0	—4.0	—5.0	$\leqq -6.0$
$M/M_\odot$	4.9	7.3	10.8	14.9	25:
t, years	$4{,}3.10^{8}$	$2{,}8.10^{8}$	$1{,}7.10^{8}$	$9{,}7.10^{7}$	$5{,}6.10^{7}$
n_g	51	23	2.5	0.75	0.16
n_1	2.3	0.66	0.04	0.008	0.001
n_{dw}	105	18	2.5	2.6	0.16
$q = n_1/n_{dw}$	0.022	0.037	0.016	0.003	0.006
n_2	103	17	2.5	2.6	0.16
n_c	650	160	40	75	8
N_a	753	177	42	78	8.2
N_r	53	24	2.5	0.8	0.16
$1 - a$	0.066	0.12	0.056	0.010	0.020

With respect to the Mount Wilson n and s subdivisions of A — B stars, Table 8 opens up a different prospect; for the typical A star, the table suggests a ratio of $n_2 : n_1 \sim 3:1$, or approximately equal to the observed ratio of $n:s$ (cf. above); in such a case it may seem that the s subdivision comprises the entire composite model class, whereas n corresponds to the adiabatic model. It is, however, not advisable to go more deeply into these details.

Our general results seem to be in favour of our original assumptions made in connection with Table 8; it seems that these assumptions may be used as a working hypothesis in future research. As to the correlation of R and M found in Subsection b, in view of the apparently homogeneous material for the most important range of the correlation (for $M < 4.3 \odot$, constant small fraction $\bar{q} = 0.28$ of composite models in Table 8), corresponding chiefly to the typical adiabatic model, the conclusions drawn with respect to the law of energy generation ($\bar{s} = \sim 19$) may be considered corroborated: for the adiabatic model a considerable progressive inflation of the radius with mass does not seem possible. The chances for the reaction $H^1 + H^1 \rightarrow H^2$, with the protons in their ground states governing the atomic synthesis are thus rather low.

h. Probable structure of the sun and the main sequence stars.

With respect to the structure of the sun and the main sequence stars somewhat more definite statements can now be made. From Table 8 it appears probable that of the main sequence stars born about 50 per cent remain adiabatic models, the other half being transformed into composite models. Now, a composite model for a given mass and radius must possess a higher central temperature than the adiabatic model (cf. Table 2: the complex model must have a central temperature between the temperatures corresponding to $n = \frac{1}{\gamma - 1} \sim 1.7 - 2$, and $n = 3$); on the other hand, for a given luminosity (constant, or approximately constant) the subatomic energy sources require an almost constant central temperature; the composite model adjusts itself, therefore, from the beginning, to a radius by 30—40 per cent larger than the radius of an adiabatic model of the same mass, and progressive inflation with the gradual exhaustion of the core increases the difference (semi-giant stage). In fig. 3 the radius of the sun is 7 per cent less than the average, thus the sun is apparently an adiabatic model, and Table 4 probably refers to the actual sun; the ice ages may perhaps be explained by assuming a perturbing effect of a very small central core which is too small to disturb the general adiabatic equilibrium. A number of individuals in Fig. 3 may be classified as composite models:

Star	Procyon	Sirius A	β Aurigae av.	TV Cas. br.
Mass	1.2	2.4	2.4	2.4
Excess of radius, per cent	66	23	44	38

α Centauri B seems to belong to the same class, but this result depends upon estimated colour temperature and is therefore uncertain. The radius of Procyon agrees more or less with Strömgren's figure, whereas Sirius A differs on account of the discrepancy between the measured and Strömgren's mean adopted temperatures.

i. Close binaries and expanding envelopes.

If a main sequence component of a binary is going to become a giant, its expanding envelope may touch and enclose the companion; the event must lead to catastrophic consequences (even when the period of rotation has time to settle itself equal to the period of revolution), perhaps to some kind of a Nova phenomenon. The frequency cannot be great: about one-tenth of the total number of giants in the Galaxy per 3.10^9 years, or about one in 100 years. Distant companions of large eccentricity may be also "swallowed" by the expanding giant, in which case the effect may be especially violent (revolution and rotation cannot be equalized). Perhaps Supernovae may be explained in such a manner (atomic synthesis explosion stimulated by the collision).

k. Coexistent faint giants.

Thus, main sequence stars may become giants, but need not. The considerable number of eclipsing binaries with an early primary, and a considerably fainter typical giant secondary, in spite of the strongly favoured selection of such eclipsing systems, is still an argument against the supposed evolution of the more massive main sequence stars either into giants or into collapsed nuclei: during the time when the fainter component has become a giant, the brighter must have long ago finished its course of evolution. The argument is, however, a weak one: from among visual binaries, from which selection of this kind is absent, the main sequence — giant pairs are also practically absent (the rule being: primary = giant; secondary = main sequence); the eclipsing binaries may represent exceptional cases. Perhaps for these the giant structure of the companion has developed at an early date of the star's life, the giant structure being due to the formation of a core with little hydrogen content at the very beginning, as considered below.

l. Central core of meteoric material.

A giant structure, i. e., an extended outer shell, and with an excessive concentration of mass towards the centre, can be produced by an increase of the mean molecular weight towards the centre (cf.[1], p. 130), and by a peculiar distribution of the energy sources, produced by a central exhausted core (Sections 5 and 6); both conditions are attained early by making hydrogen originally less abundant near the centre than in the outer regions of a star. Let us inquire into factors other than exhaustion which might have led to a non-uniform distribution of hydrogen inside the star.

The diffusion of electrons, with the electrostatic field forcing protons to follow (cf. Rosseland,[43]), can hardly be made responsible for the differentiation, simply because of rotational mixing alone; at an early stage of the contracting nebulosity, when the rotation must have been slow (conservation of angular momentum), the differentiation might have taken place, but the time intervals in the most favourable case of feebly ionized matter (largest μ and temperature, large free path of the electrons undeflected by ionic fields) are still too large; we find the time of relaxation (cf.[1], p. 277 f.; $\bar{\mu} = 20$): $t \sim 3.10^{11} \sqrt{\frac{M}{R}}$, thus decreasing with increasing radius; for $M = 10^{34}$ gr ($5 \odot$), $R = 10^{18}$ cm (one light year, $T_c = 100^0$ only), $t \sim 3.10^{19}$ sec $= 10^{12}$ years: during the short early history of the contracting star no electrostatic differentiation could have taken place.

But, at the low temperature of the primordial nebula, meteor particles may start condensing and increasing in size (cf. Lindblad,[44]); as shown by Jung[45], electrostatic forces due to ionization in interstellar space would actually resist such an accretion of meteoric mass except when the gas density ϱ exceeds $\sim 10^{-22}$ g/cm^3. In our example, $\varrho \sim 10^{-21}$, and increases with contraction, thus the process is quite possible. The meteoric dust particles, instead of continuing their rotation with the rest of the gas, are forced by gravitation to fall towards the centre (because they are no more supported by gas pressure, and because the velocity of rotation of a nebula which is able to contract at all must be negligible as compared with the circular orbital velocity); resistance of the medium (proportional to the mass acquired by the meteor) decreases the major axis and the ellipticity of the originally very elongated ellipse; as a result, the condensed meteoric dust (it may also have been partly captured from interstellar space) collects near the centre of the future star (our present solar meteors are the last remnants of the nebula; they no longer suffer much from the resistance of the medium, that is why most of them have still almost parabolic orbits). Now, as taught by the observed composition of meteors and by chemical considerations, in the process of condensation hydrogen (free or water vapour), helium, and nitrogen are not included, whereas oxygen is partly chemically bound (stone meteors) and collects with iron, nickel, and other metals near the centre. Thus, in the process of condensation, a strong differentiation in the required direction must take place.

It is probable, therefore, that a nucleus poor in hydrogen and soon exhausted is present in each star from the very beginning; but the nucleus is mostly so small that the radius, luminosity, and evolutional trend of the star remain practically the same as they would be without the nucleus; in rare cases ($\leqslant 6$ per cent of all, judging from $1-a$ of Table 8 for $M > 4.3\odot$), the original nucleus is large enough to force upon the star an evolution towards the giant model, which in some cases may start from the beginning. Our conclusion is that the stars from the very beginning may possess somewhat distended atmospheres and concentrated central masses, due to the deficiency of hydrogen at the centre; rotational mixing, which sets in powerfully enough at an early stage of contraction (when the central convectional currents have not yet started), is apparently unable to upset the differentiation (cf. Section 4. *f*, and 5. *h*), especially as the larger molecular weight at the centre increases the convectional stability of the distribution. Further, as the result of concentration and greater molecular weight, the central temperature must be higher than for uniform composition. We further call the model a differentiated one. In the differentiated model it is convenient to distinguish schematically a core of greater molecular weight (~ 2), surrounded by an outer shell of $\mu \sim 1$. Let us inquire whether giant structure can be produced without the intervention of the subatomic energy. If subatomic sources of energy are absent, and rotational convection is slow, the whole differentiated model is in radiative equilibrium (cf. Section 5. *h*); the energy production is chiefly determined by the central core which is an incomplete polytrope. The outer shell, fitted to it, takes little part in the energy production. If a large proportion of the mass is in the core, the luminosities of the stars must be much greater than observed, unless we assume the hydrogen content in the core as high as now assumed for the envelope ($\sim$ 40 per cent); a small core does not imply such a difficulty (cf. Subsection *a*). The energy is furnished by gravitational contraction; subatomic sources being absent in- and outside, there is no such obstacle to the outer shell following the collapsing inner core as considered in Sections 5. *f* and 6. *e*. Therefore the contracting model may reveal only moderate inflation (semi-giant structure), and the origin of diffuse giants remains a mystery as before.

Thus, we are forced to accept the efficiency of the subatomic sources of energy even in the case of the originally differentiated model; exhaustion of the central source, and intense subatomic energy starting only at a certain distance from the centre, give rise to distending forces through which an actual giant comes into being *.

Tartu, October 19, 1937.

* These qualitative considerations will be discussed mathematically in a paper to follow, where examples of the corresponding stellar models are given.

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