

# **METEOROLOGICAL ELEMENTS CHARACTERIZED BY FREQUENCY-CURVES**

BY

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In order to determine the climate of any district we must deduce from the meteorological observations the average, the deviations, the absolute and average max. and min. of the meteorological elements, and the average number of days characterizing the annual variation of air-temperature, precipitations, etc. We thus get a large number of different figures which require many diagrams for their representation.

In discussing the distribution of the meteorological elements of any district it is of great advantage to apply frequency-curves as a climatical characteristic. In the following paper the distribution of temperature in Tartu for every month is discussed by means of frequency-curves. As an example we give the frequency-curves for air-temperature in January and July for Thorshavn and Barnaul, these being representatives of typical marine and continental climates. The three daily observations of temperature have equally been taken into consideration. One degree C° is taken as a unit for the division of temperature observations into groups. For Thorshavn we have applied the observations for the years 1873—1925, for Barnaul — 1855—1911. In diagrams 1, 2, 3, 4, the distribution of temperature for the months of January and July for Thorshavn and Barnaul is represented by a broken line. The abscissa gives the temperature in whole degrees C°, the ordinate — its frequencies in percentage of the whole number of observations. The vertical lines represent the median and mode. Applying Pearson's method we get the equations for the frequency-curves. From the four equations given below, three belong to Pearson's system type I; one (July, Thorshavn) to type IV.

Barnaul, January:

$$y = 4.286 \left(1 + \frac{x}{48.98}\right)^{5.774} \left(1 - \frac{x}{15.63}\right)^{1.843} .$$

Barnaul, July:

$$y = 7.780 \left(1 + \frac{x}{14.37}\right)^{5.145} \left(1 - \frac{x}{34.44}\right)^{12.33} .$$

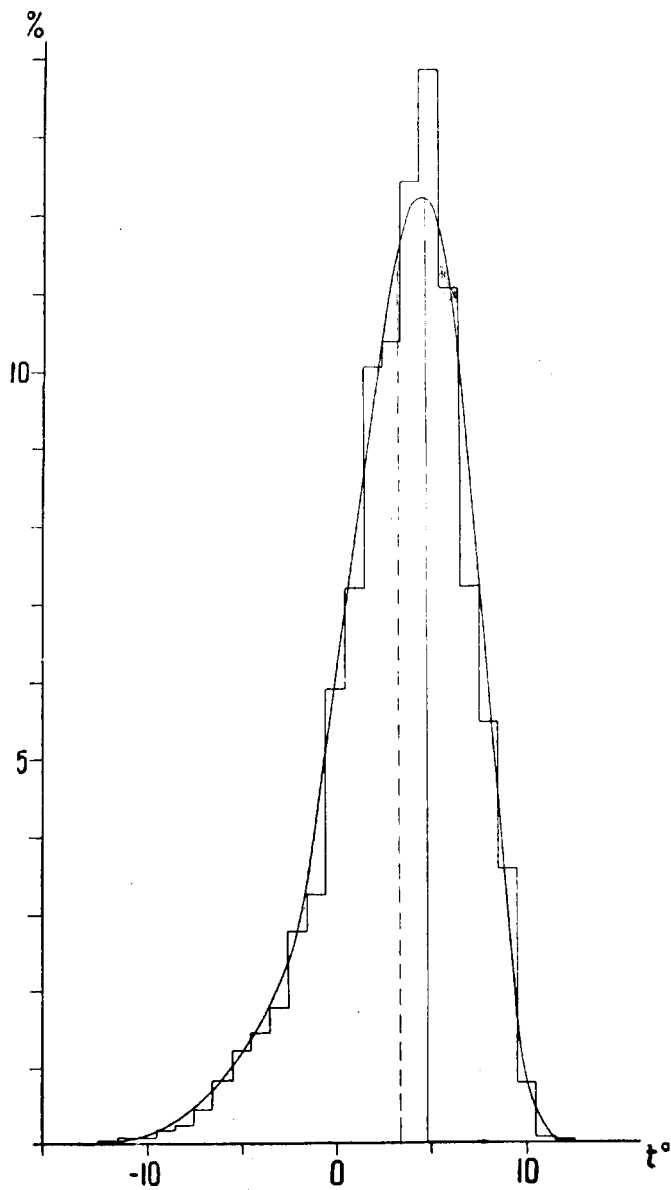


Fig. 1. Frequency-Curve for  
Thorshavn, January.

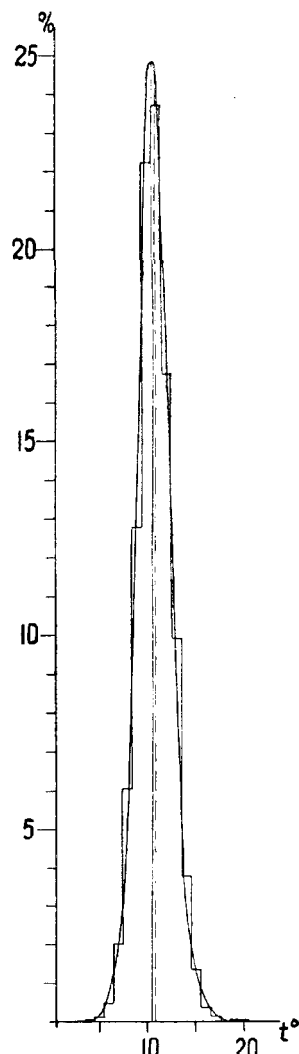


Fig. 2. Frequency-Curve for  
Thorshavn, July.

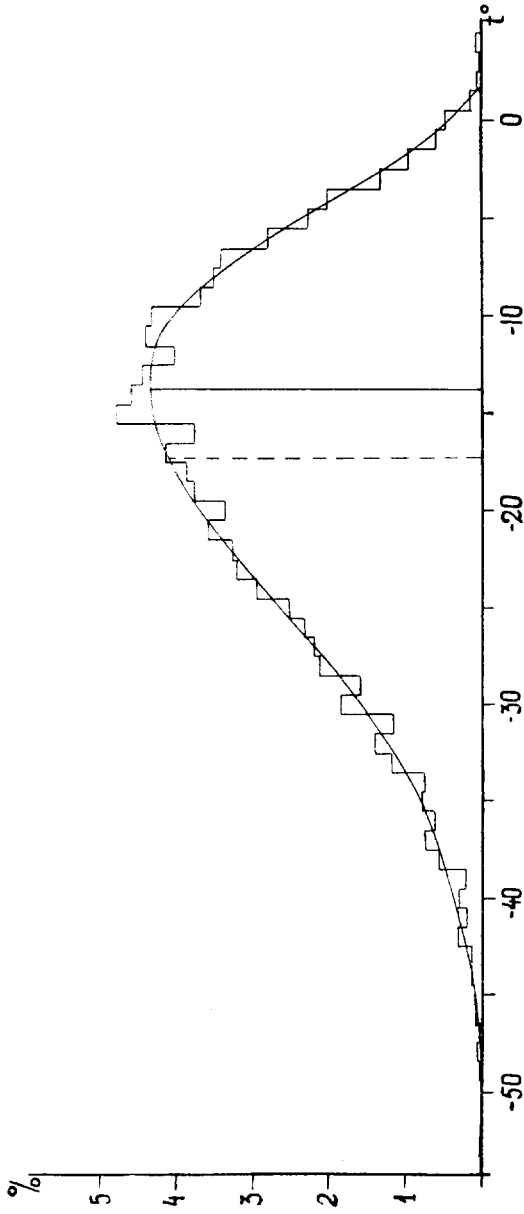


Fig. 3. Frequency-Curve for Barnaul, January.

Thorshavn, January:

$$y = 12.22 \left(1 + \frac{x}{88.33}\right)^{52.26} \left(1 - \frac{x}{6.74}\right)^{3.988}$$

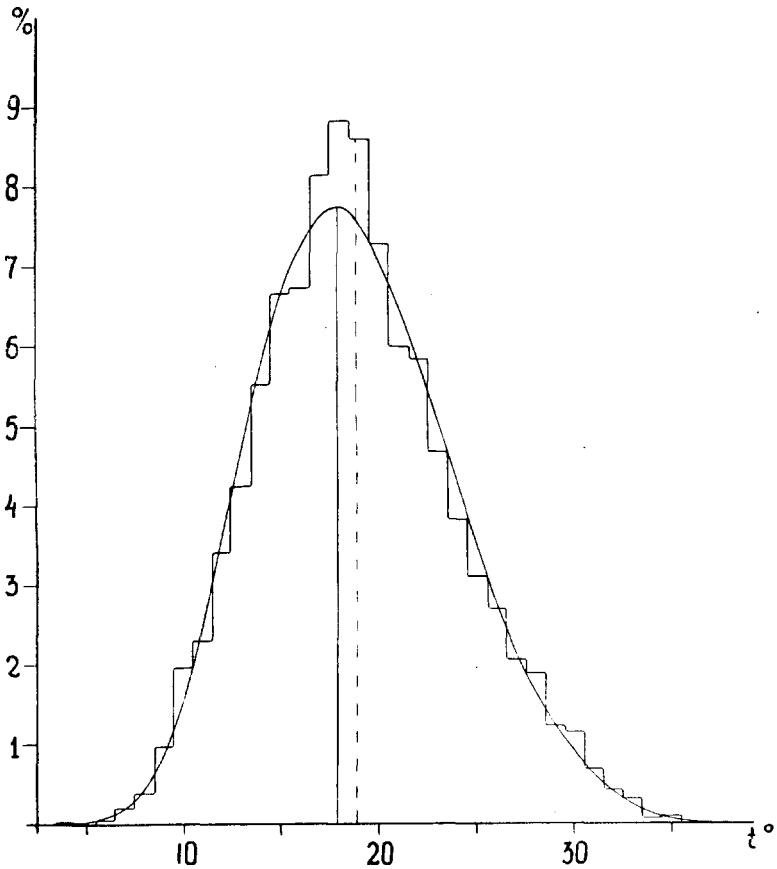


Fig. 4. Frequency-Curve for Barnaul, July.

Thorshavn, July:

$$y = 24.69 (\cos \Theta)^{9.215} e^{0.7244 \Theta},$$

where:

$$\Theta = \arctan \left( \frac{x}{4.351} \right).$$

In the diagrams 1—4, the frequency-curves drawn according to the above equations are given in full lines. Comparing the frequency-curves of Barnaul with those of Thorshavn, we see that the distribution of temperature for Barnaul has far broader limits, than that for Thorshavn. Further it must be noted

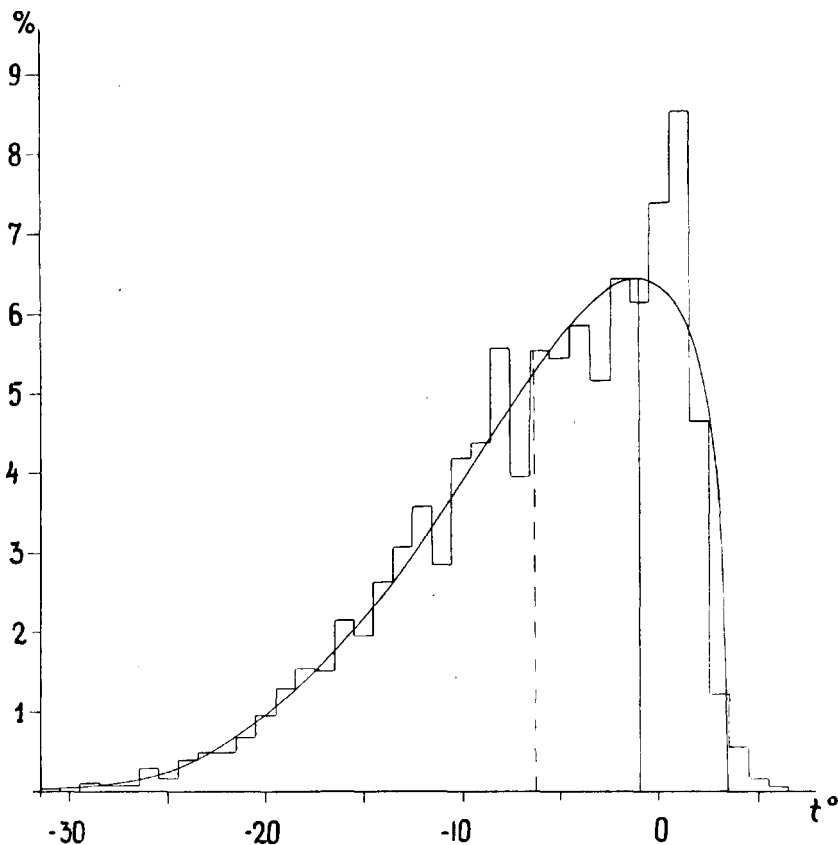


Fig. 5. Frequency-Curve for Tartu, January.

that the mode is characterized in Thorshavn by a much higher percentage than in Barnaul. The skewness of the curves for both stations is more strongly expressed in January than in July, with reverse signs in both months.

The following table shows the characterizing data of the frequency-curves, viz. Standard Deviation ( $\sigma$ ), Skewness ( $S$ ), and Excess ( $E$ ), reckoned by means of moments.

Table 1.

	$\sigma$	$S = \frac{\mu_3}{\sigma^3}$	$E = \frac{\mu_4}{\sigma^4} - 3$
Barnaul I	9.05	-0.50	-0.13
„ VII	5.01	0.34	-0.11
Thorshavn I	3.45	-0.76	0.75
„ VII	1.75	0.19	1.49

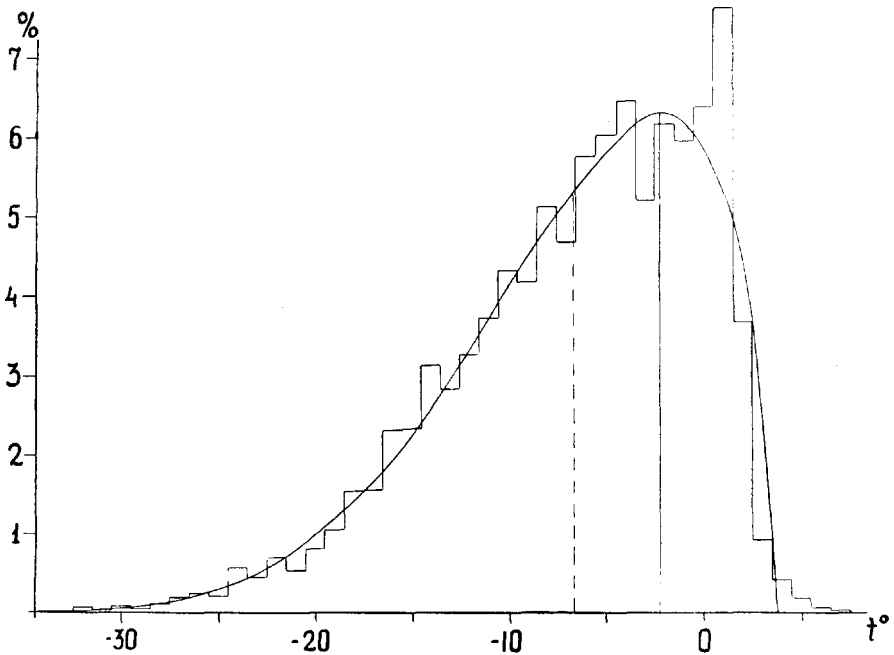


Fig. 6. Frequency-Curve for Tartu, February.

In the same way on the basis of the three daily observations ( $7^h$ ,  $13^h$ ,  $21^h$ ) we have obtained the frequency-curves of air temperature for Tartu for the years 1870—1933. In the following diagrams (5—16) the abscissa also represents the temperature, the ordinate — its frequency in percentage of the whole number of observations for the corresponding month.

One degree C<sup>o</sup> has been taken as unit for the division of temperature into groups. In the figures the dotted line denotes

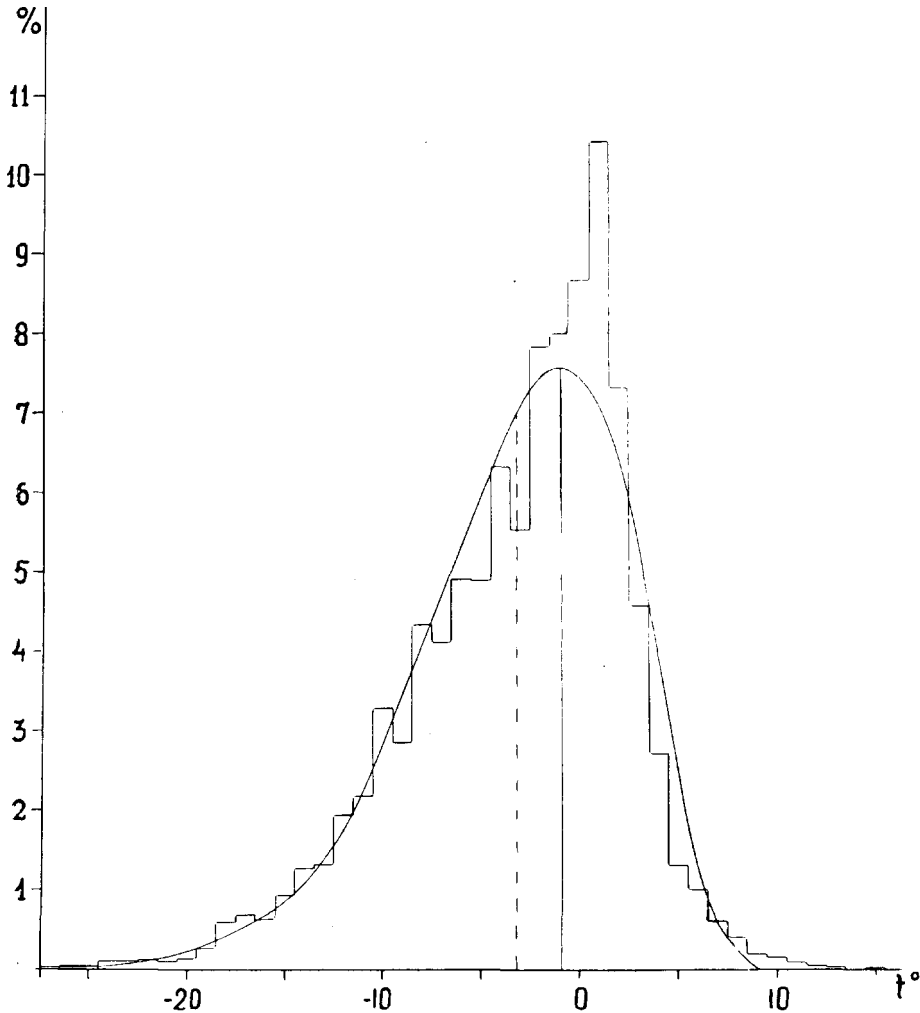


Fig. 7. Frequency-Curve for Tartu, March.

the observed frequency, the full lines — the frequency-curves drawn according to the following equations found by means of Pearson's method.

$$\text{January: } y = 6.443 \left(1 + \frac{x}{32.06}\right)^{2.801} \left(1 - \frac{x}{4.46}\right)^{0.390}.$$

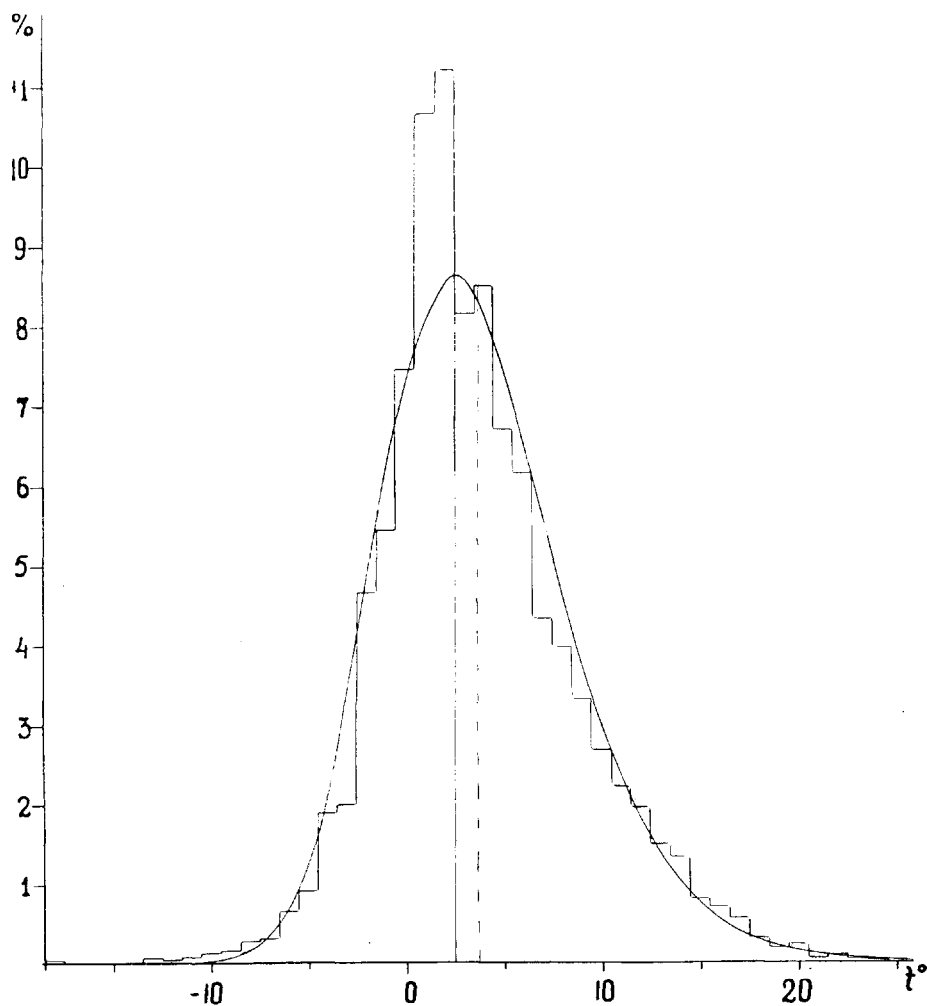


Fig. 8. Frequency-Curve for Tartu, April.

$$\text{February: } y = 6.346 \left(1 + \frac{x}{37.53}\right)^{4.441} \left(1 - \frac{x}{6.12}\right)^{0.724}$$

$$\text{March: } y = 7.544 \left(1 + \frac{x}{70.92}\right)^{21.75} \left(1 - \frac{x}{10.02}\right)^{3.074}$$

April  $y = 0.000554 (\cos \theta)^{\frac{28.09}{e}} \frac{24.53}{e} \theta$

where:  $\theta = \arctan \left( \frac{x}{17.98} \right)$

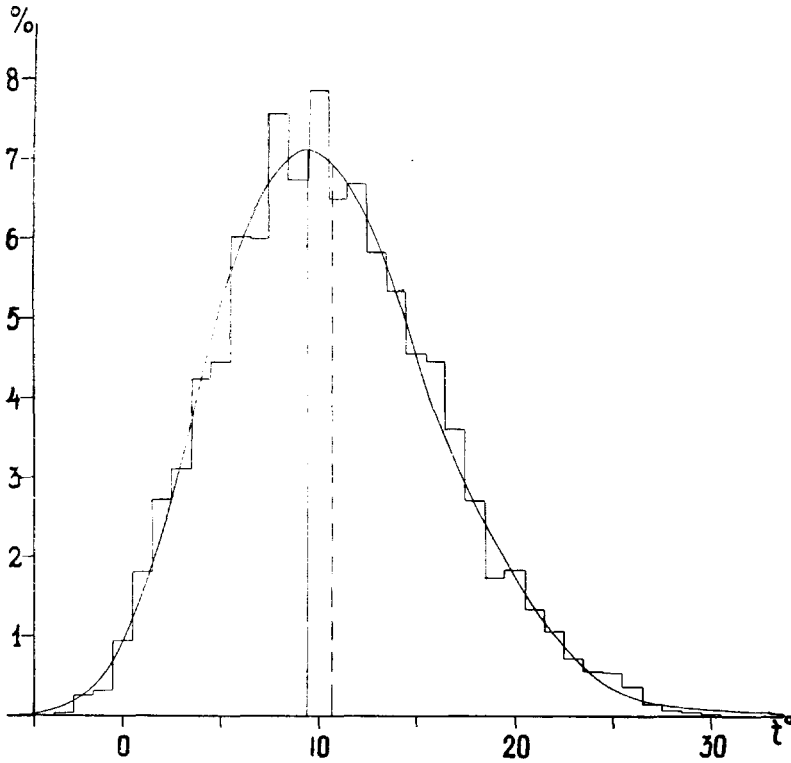


Fig. 9. Frequency-Curve for Tartu, May.

May:  $y = 7.087 \left( 1 + \frac{x}{14.68} \right)^{4.473} \left( 1 - \frac{x}{36.68} \right)^{11.18}$

June:  $y = 8.583 \left( 1 + \frac{x}{17.53} \right)^{10.50} \left( 1 - \frac{x}{55.44} \right)^{33.22}$

July:  $y = 9.918 \left( 1 + \frac{x}{9.26} \right)^{4.03} \left( 1 - \frac{x}{37.26} \right)^{16.21}$

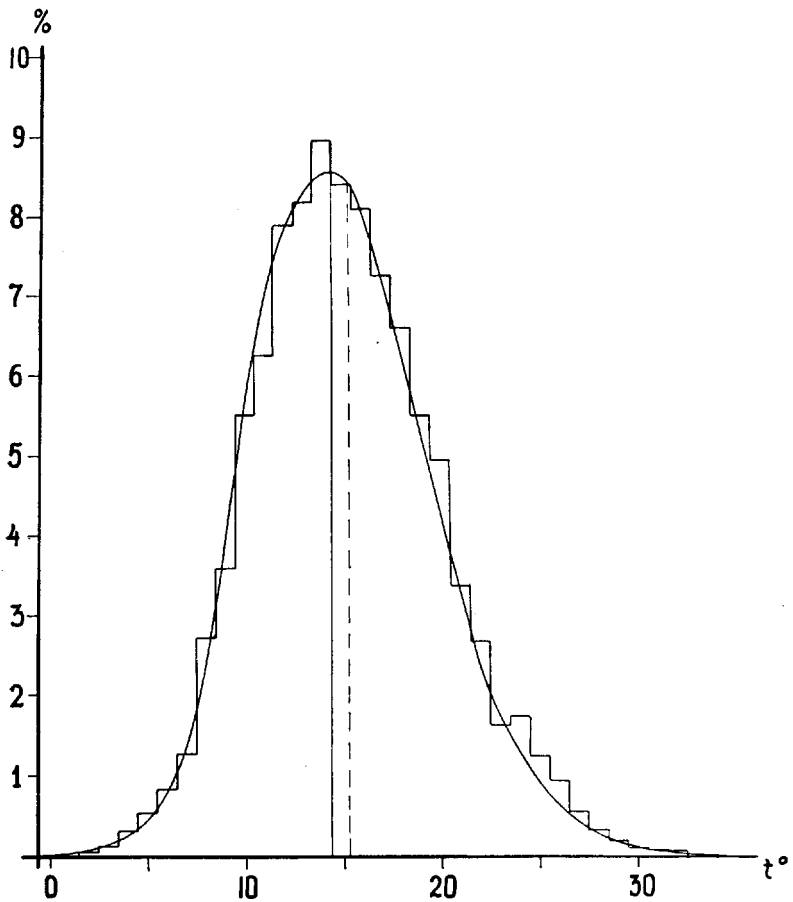


Fig. 10. Frequency-Curve for Tartu, June.

$$\text{August: } y = 10.34 \left(1 + \frac{x}{9.22}\right)^{4.985} \left(1 - \frac{x}{69.80}\right)^{37.74}$$

$$\text{September: } y = 3.133 (\cos \theta)^{\frac{46.01}{e}} \frac{10.22}{e} \theta$$

$$\text{where: } \theta = \arctan \left(\frac{x}{26.84}\right).$$

$$\text{October: } y = 8.646 \left(1 + \frac{x}{51.25}\right)^{51.36} \left(1 - \frac{x}{37.63}\right)^{37.71}$$

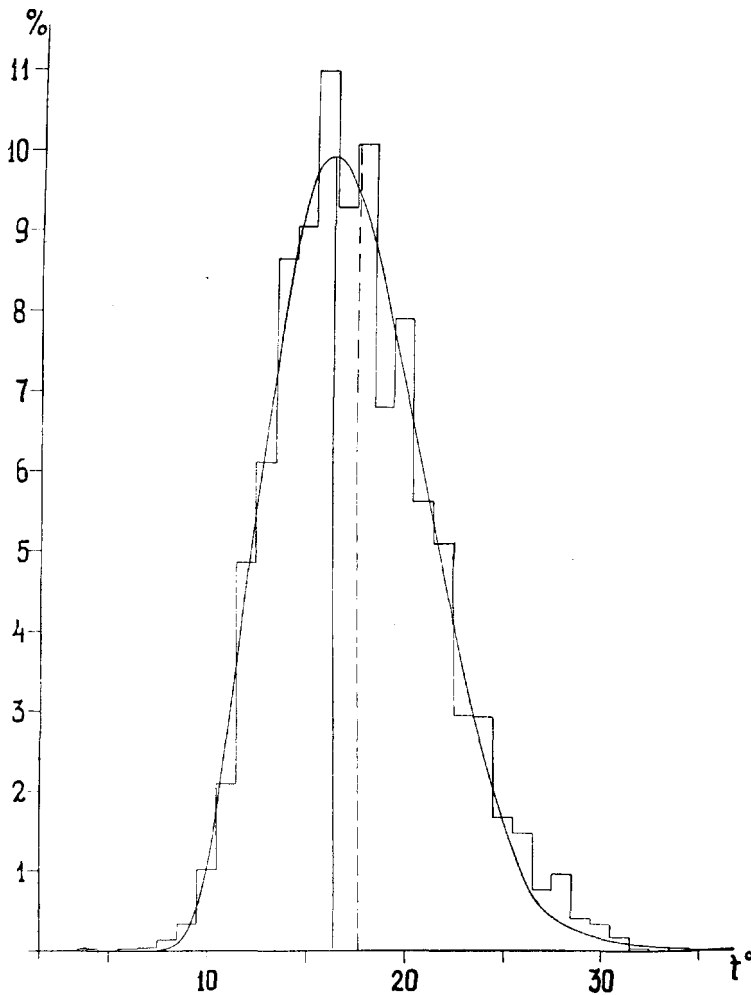


Fig. 11. Frequency-Curve for Tartu, July.

$$\text{November: } y = 8.530 \left(1 + \frac{x}{61.67}\right)^{25.34} \left(1 - \frac{x}{10.88}\right)^{4.471}$$

$$\text{December: } y = 7.279 \left(1 + \frac{x}{55.30}\right)^{8.968} \left(1 - \frac{x}{5.34}\right)^{0.866}$$

Table 2 gives the characteristic of the frequencies for each month viz. the Standard Deviation ( $\sigma$ ), Skewness ( $S$ ), and Excess ( $E$ ).

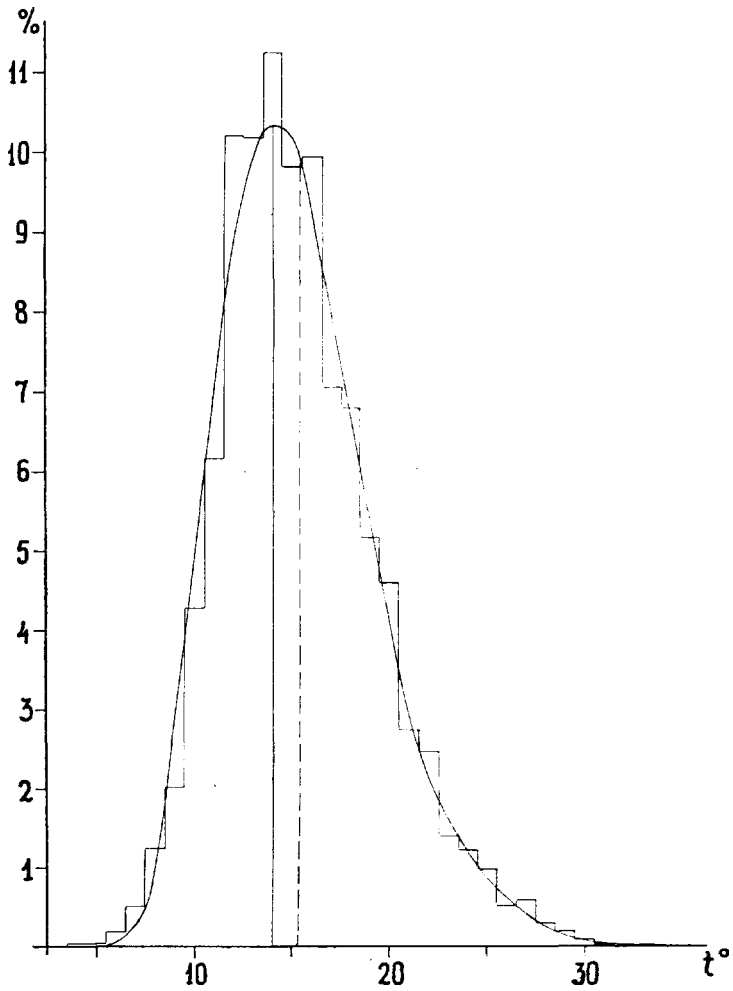


Fig. 12. Frequency-Curve for Tartu, August.

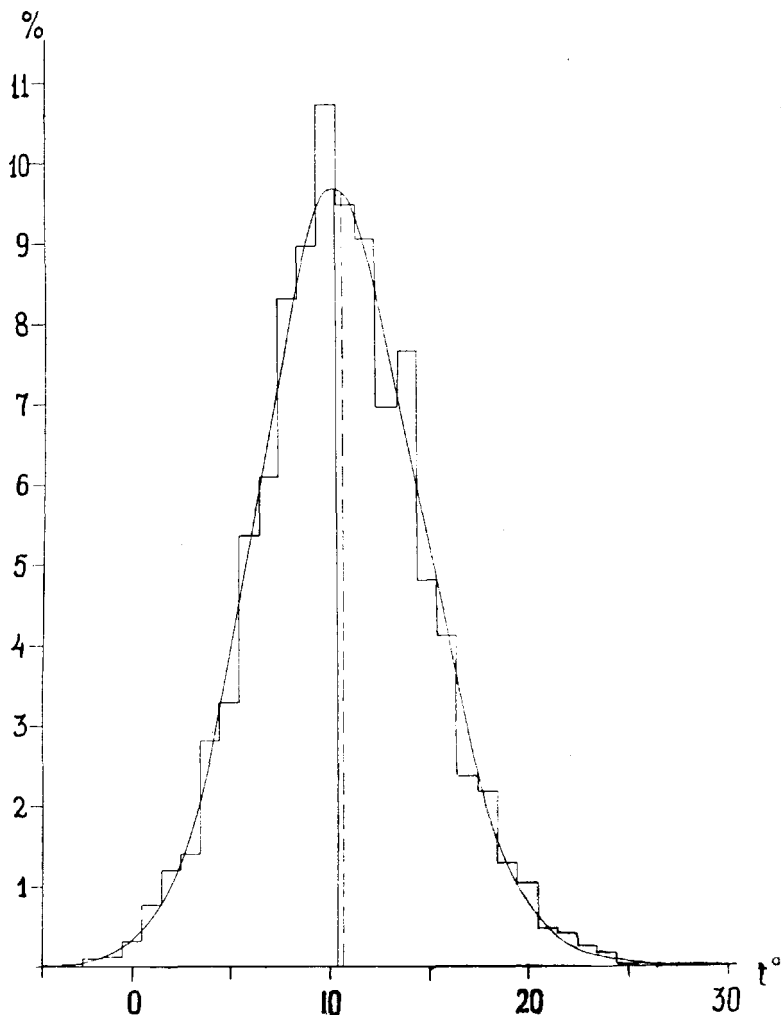


Fig. 13. Frequency-Curve for Tartu, September.

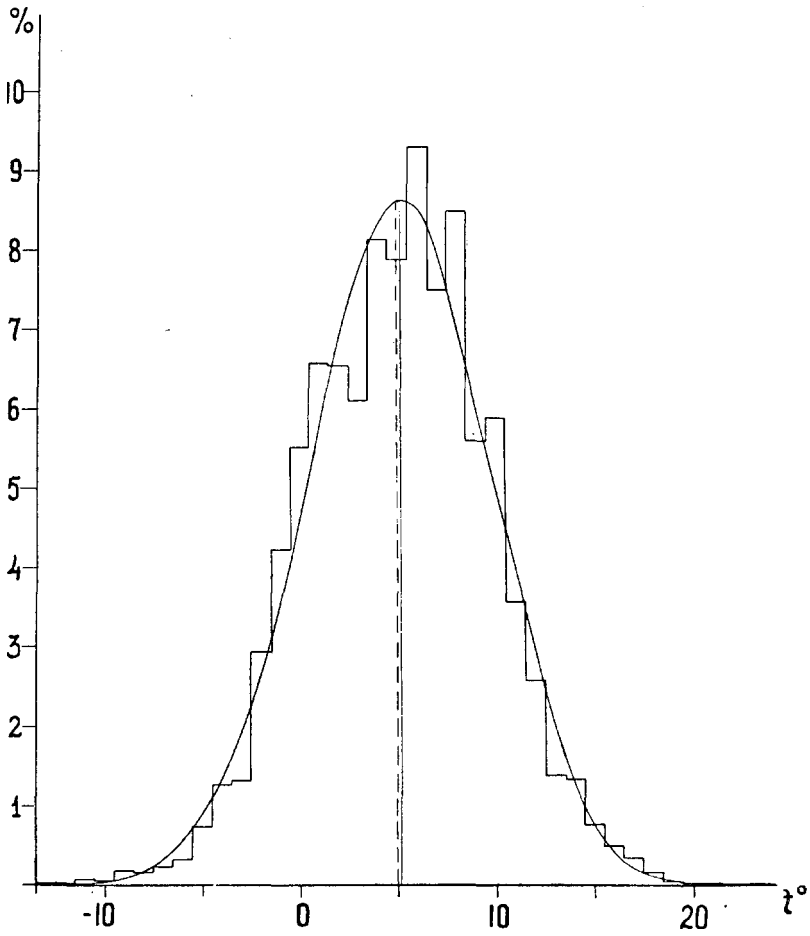


Fig. 14. Frequency-Curve for Tartu, October.

Table 2.

	$\sigma$	$S = \frac{\mu_3}{\sigma^3}$	$E = \frac{\mu_4}{\sigma^4} - 3$
I	6.50	-0.73	-0.04
II	6.52	-0.76	0.18
III	5.50	-0.72	0.57
IV	4.96	0.57	0.77
V	5.50	0.36	-0.11
VI	4.63	0.33	0.03
VII	4.04	0.52	0.15
VIII	3.98	0.62	0.44
IX	4.20	0.14	0.17
X	4.58	-0.06	-0.06
XI	4.78	-0.59	0.34
XII	6.21	-0.96	0.92

A closer discussion of the frequency-curves from month to month reveals the following changes. During the cold period of the year, from November till March, the range of low temperature has a greater extension than that of high temperature. During the warm season the distribution of temperature is enclosed within narrower limits than in the winter months, whereas the frequencies of high temperature are comparatively increased and the skewness is therefore inverse to that of the winter months. In September and October the curves are almost symmetrical.

Frequency-curves may also be successfully applied in discussing many problems connected with climatology. Thus we have used them in determining the changes that the climate of Tartu has undergone during the last 64 years (1870—1933). For this purpose the distribution of air temperature has been considered separately for two periods, of which the former includes the years 1870—1900, the latter — the years 1901—1933. As in the first part of this paper, the frequencies have also been expressed in percentage of the whole number of observations for each period. Both temperature-distributions are graphically represented for each month by broken lines. The full line denotes the frequency for the years 1870—1900, the dotted line — that for 1901—1933. The mean error of frequency for each group has been

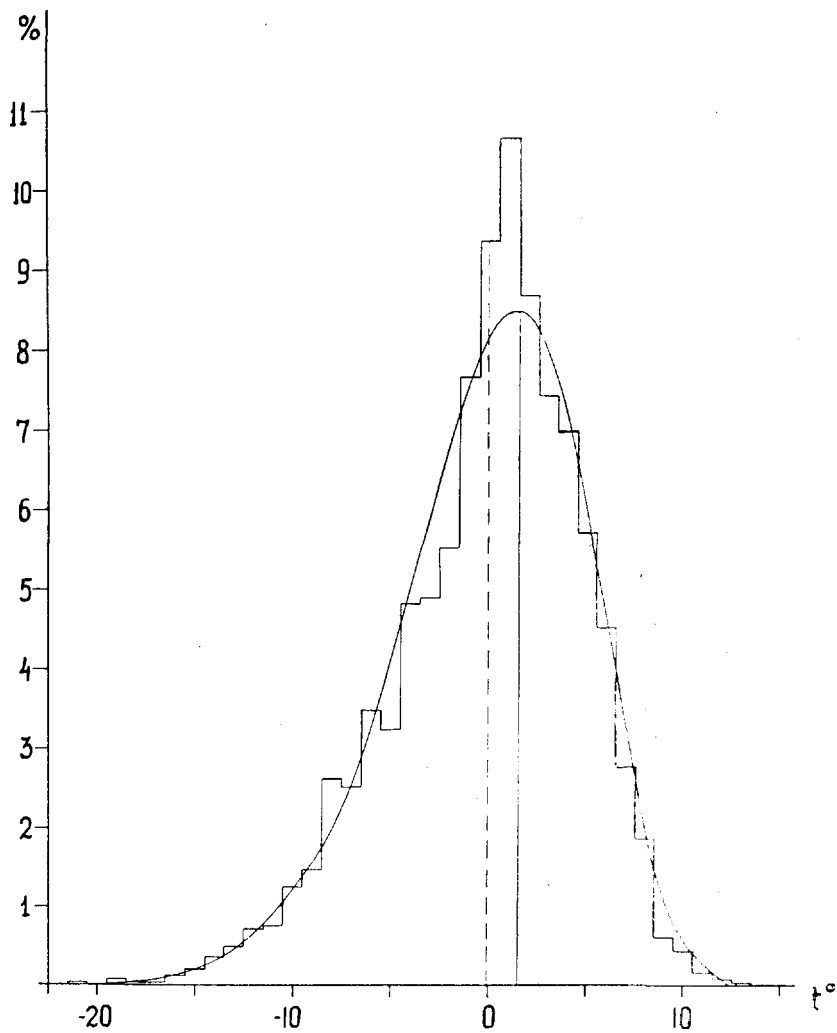


Fig. 15. Frequency-Curve for Tartu, November.

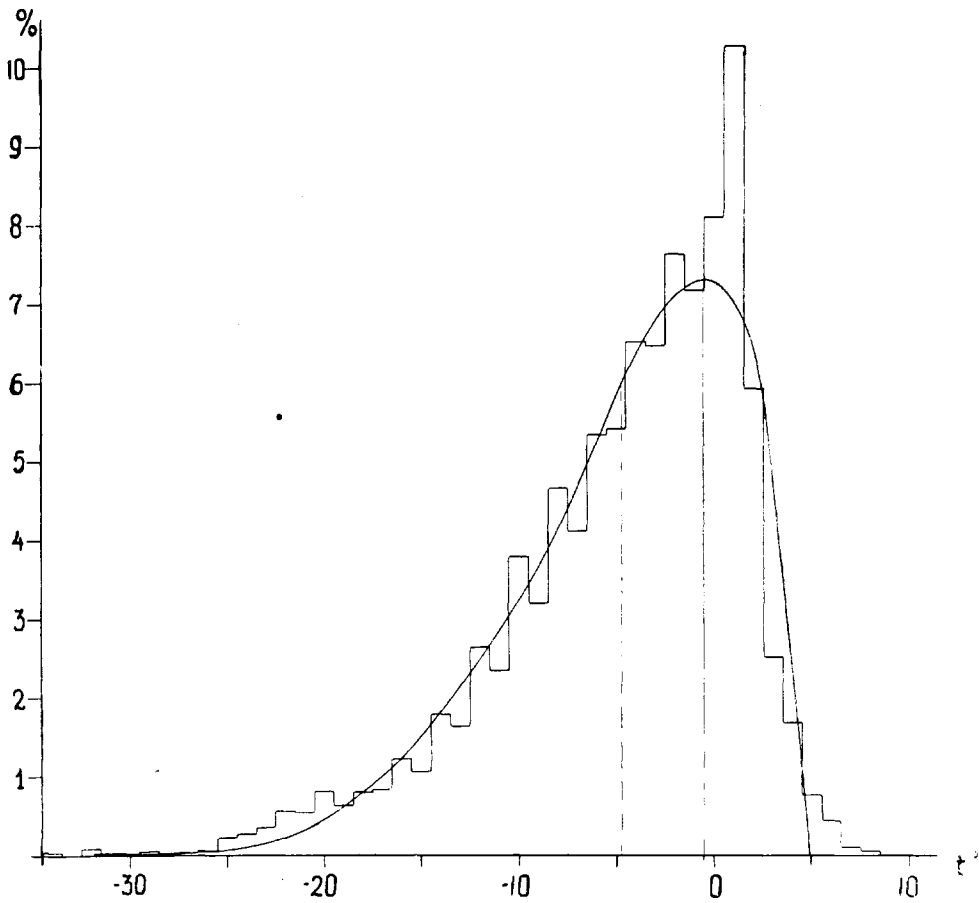


Fig. 16. Frequency-Curve for Tartu, December.

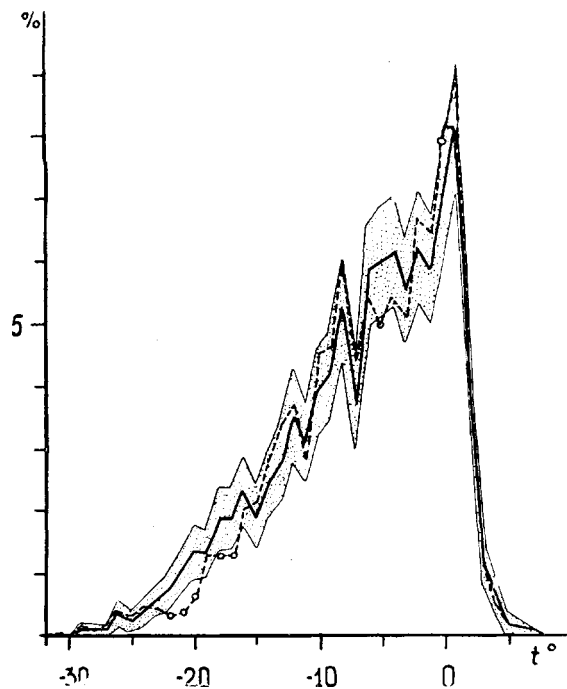


Fig. 17. Distribution of Temperatures for Tartu, January.

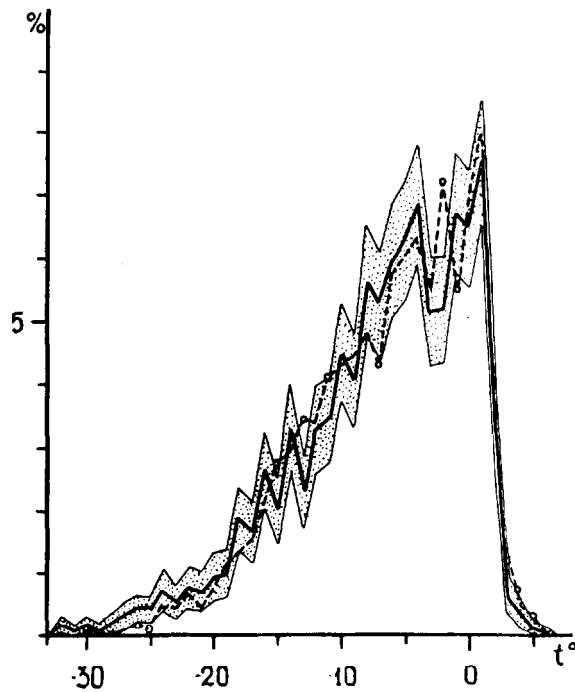


Fig. 18. Distribution of Temperatures for Tartu, February.

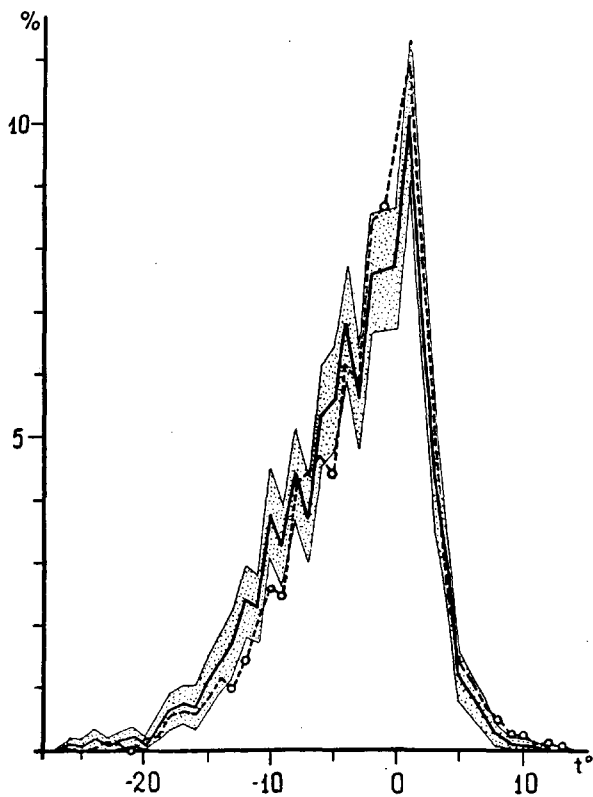


Fig. 19. Distribution of Temperatures for Tartu, March.

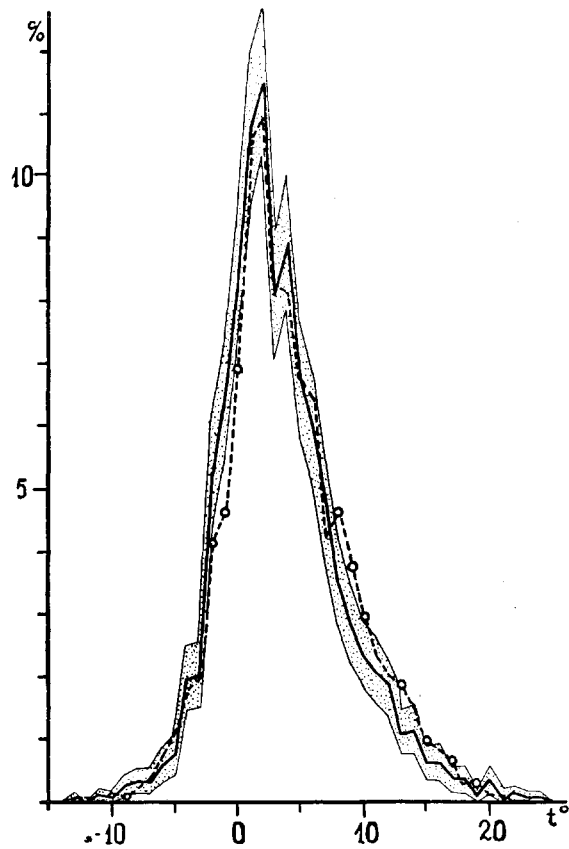


Fig. 20. Distribution of Temperatures for Tartu, April.

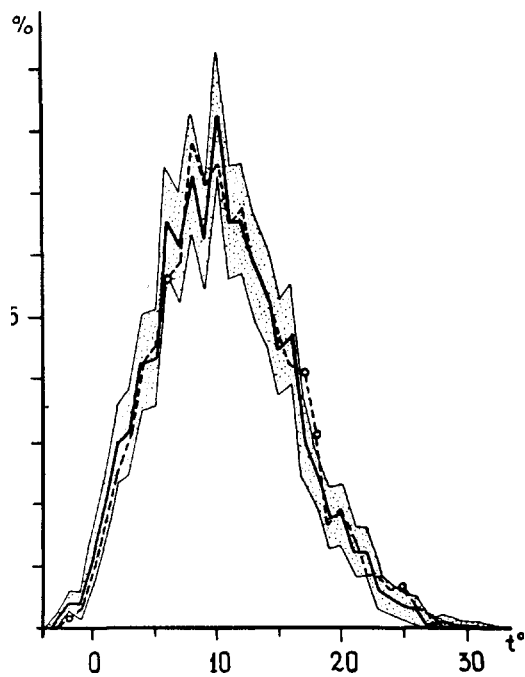


Fig. 21. Distribution of Temperatures for Tartu, May.

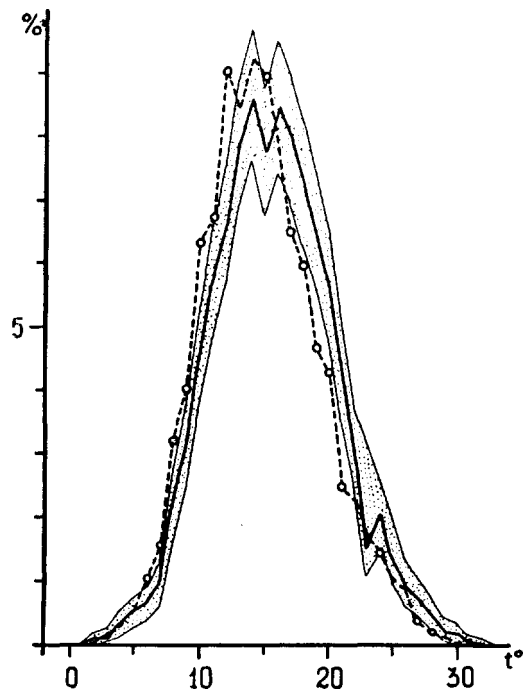


Fig. 22. Distribution of Temperatures for Tartu, June.

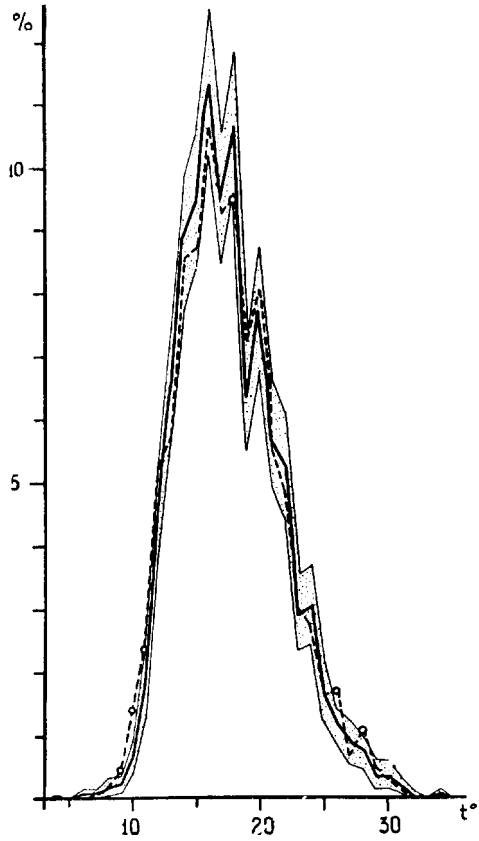


Fig. 23. Distribution of Temperatures for Tartu, July.

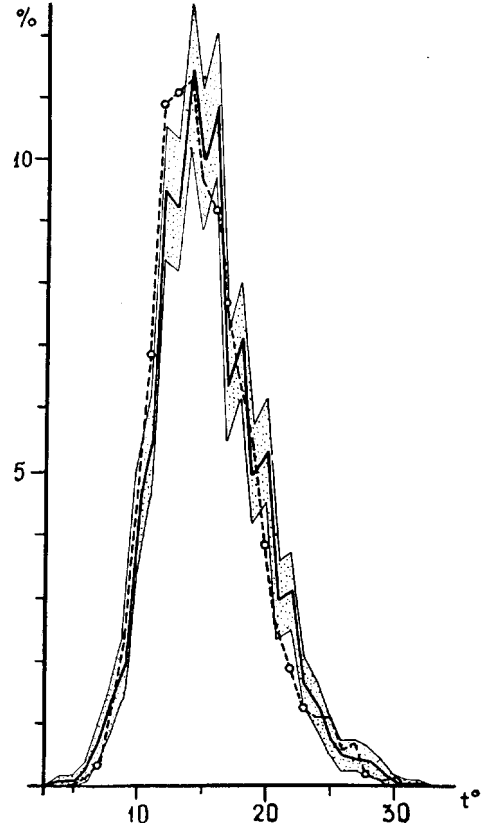


Fig. 24. Distribution of Temperatures for Tartu, August.

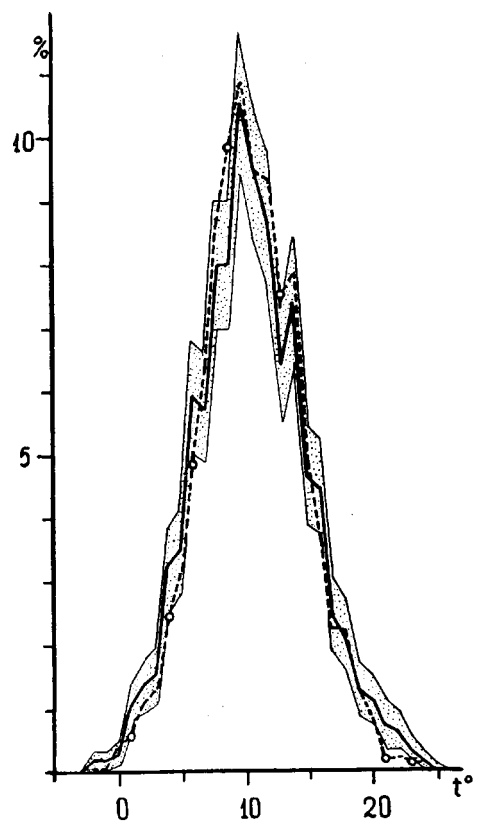


Fig. 25. Distribution of Temperatures for Tartu, September.

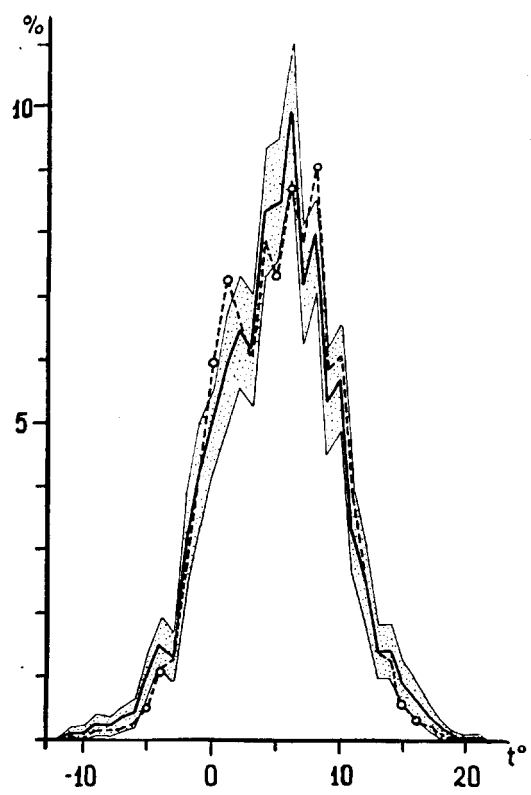


Fig. 26. Distribution of Temperatures for Tartu, October.

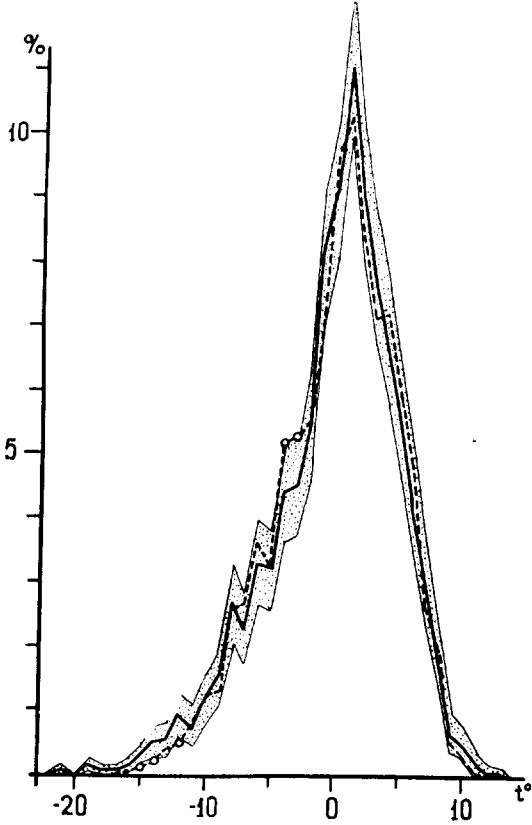


Fig. 27. Distribution of Temperatures for Tartu, November.

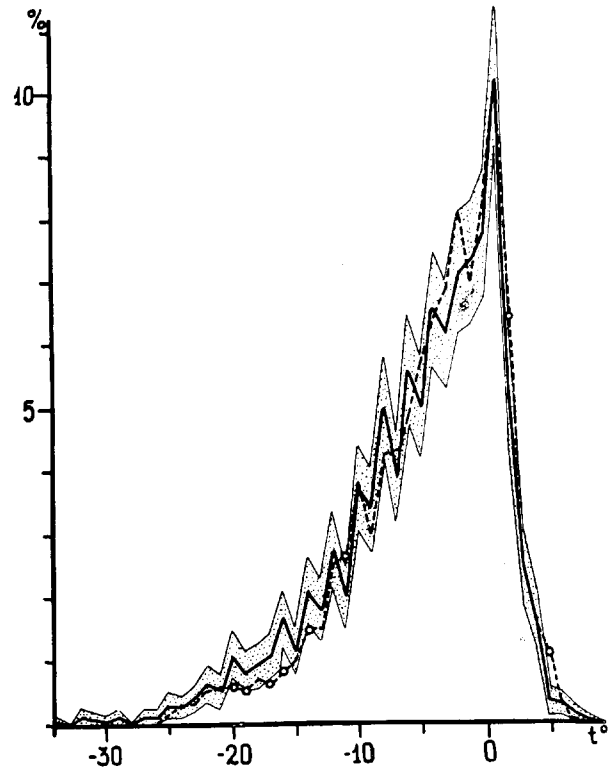


Fig. 28. Distribution of Temperatures for Tartu, December.

calculated for the first period (1870—1900) by means of the formula

$$\sigma_i = 100 \sqrt{\frac{p q}{S}},$$

where  $S$  denotes the whole number of observations,  $p$  — the probability that an observation belongs to the  $i$  group,  $q$  — the inverse probability ( $q = 1 - p$ ). In order to obtain a better survey of the difference in the distributions of temperature in both periods, a four  $\sigma$  broad dotted stripe has been drawn in the figures for each month.

We see that several times in each month the dotted line of frequencies for the last period lies outside the four  $\sigma$  broad stripe. These outlying parts in the diagrams are marked by circles.

We know from the theory of probabilities that 95% of all cases must lie within four  $\sigma$  and only 5% can be outside, if the event takes place without any extraneous influence. As seen from Fig 17, for January the distribution of temperature consists of 38 different groups. The frequencies of the last period (1901—1933) can lie outside the four  $\sigma$  stripe only twice, if the range of temperature has not undergone any extraneous influence. In reality we see that the frequencies of temperature of the last 32 years lie 7 times outside the limits of four  $\sigma$ . The following table shows how many times the frequencies of the last period happen to lie outside four  $\sigma$  in each month.

Table 3.

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
7	10	12	11	5	16	6	10	7	9	5	8

By means of the following term

$$P = \frac{\int_0^{\infty} e^{-\frac{1}{2}x^2} x^{n-1} dx}{\int_0^{\infty} e^{-\frac{1}{2}x^2} x^{n-1} dx}$$

given by Pearson we can easily find the probability that such a difference in the distribution of temperature for both periods has occurred casually without any extraneous influences. In the above given term  $e$  is the basis of the natural logarithms

$$\chi = \Sigma \left( \frac{d^2}{\mu} \right),$$

where  $d$  is the difference between the frequencies of temperature of both periods for every group,  $\mu$  denotes the frequency for the corresponding group of the former period, and  $n$  the number of groups less one. For January the reckoning gives for the required probability  $7 \times 10^{-9}$ , which means that such a difference can take place casually without any extraneous influence only once in a period of  $4 \times 10^9$  years. The mentioned probability ( $P$ ) with its corresponding number of years ( $N$ ) has thus been reckoned for each month and given in the following table.

Table 4.

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
$P$	$7 \times 10^{-9}$	$6 \times 10^{-19}$	$9 \times 10^{-12}$	$9 \times 10^{-16}$	$6 \times 10^{-3}$	$7 \times 10^{-26}$	$2 \times 10^{-7}$	$5 \times 10^{-11}$	$2 \times 10^{-7}$	$1 \times 10^{-5}$	$9 \times 10^{-3}$	$3 \times 10^{-7}$
$N$	$4 \times 10^9$	$5 \times 10^{19}$	$3 \times 10^{12}$	$3 \times 10^{16}$	$5 \times 10^3$	$4 \times 10^{26}$	$2 \times 10^8$	$6 \times 10^{11}$	$2 \times 10^8$	$3 \times 10^6$	$3 \times 10^3$	$1 \times 10^8$

This table shows that the longest period belongs to June, when the frequency of temperature for the last period (1901—1933) lies 16 times outside the four  $\sigma$  stripe.

In the same way we have obtained the annual frequencies of temperature for both periods, which are graphically given in Fig. 29.

In diagram 29, the full line represents the distribution of temperature for the first period (1870—1900), the dotted line — that for the second period (1901—1933), and the dotted stripe — the four  $\sigma$  for the first period. The above mentioned probability ( $P$ ) is only  $2 \times 10^{-48}$ , which shows that such a difference can occur casually only once in a period of  $10^{49}$  years.

These probabilities being infinitely small for every month as well as for the whole year, we conclude that a change of climate has taken place during these 64 years. There follows an attempt to characterize more precisely this change in the annual variation of air temperature.

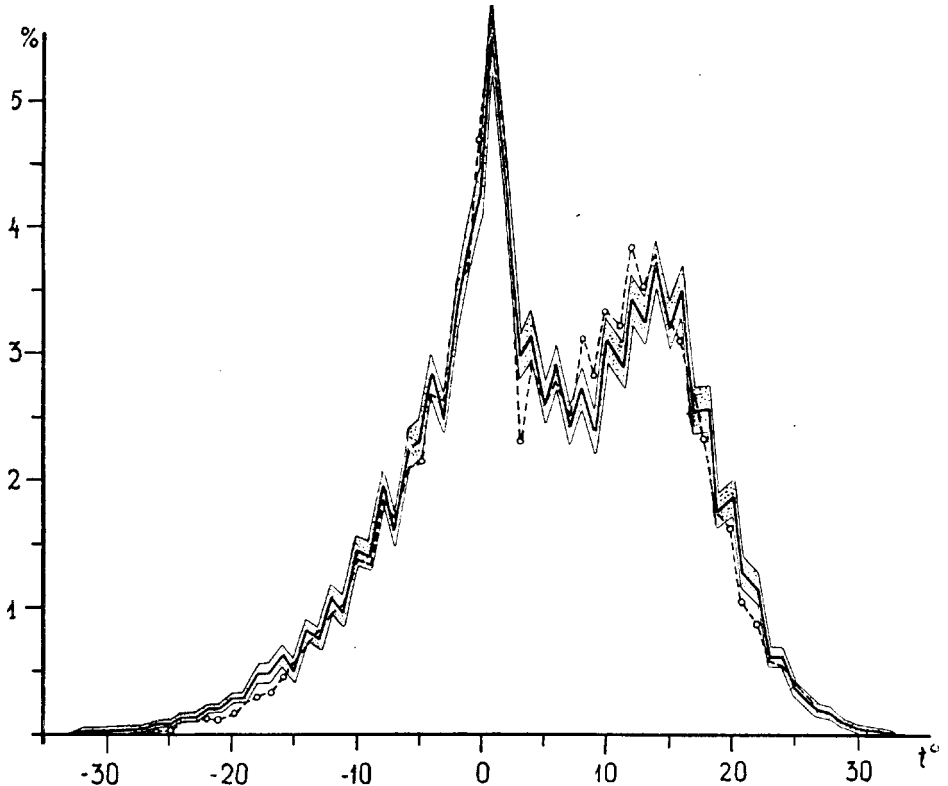


Fig. 29. Yearly Distribution of Temperature for Tartu.

As seen from the diagrams 17—28, the frequencies of low temperature have increased during the summer months and decreased during the winter months, which suggests that the points of highest and lowest temperature have been shifted in the annual variation. In order to determine more exactly this retardation, the moments of the highest and lowest temperatures have been deduced for each ten-years' period. This calculation has not been carried out by means of harmonic analysis, but the third degree polynome has been applied:

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3,$$

where  $a_0, a_1, a_2, a_3$  are constants determined by means of least squares. Harmonic analysis was not used because it forces us to take the average temperature of all the months equally into consideration, whereas in our case we are concerned only with the maximal and minimal temperatures of the annual variation. In determining the maximal point the average temperatures of May, June, July, August, and September have been taken into consideration, whereas for the minimal point — those of De-

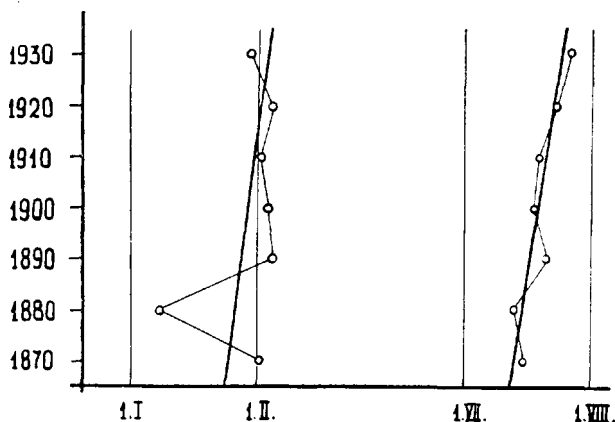


Fig. 30. Retardation in the Annual Range of Max. and Min. Temperatures for Tartu.

cember, January, February, March, and April. The moments thus obtained for each ten-years' period have been graphically represented by circles in the following diagram, where the abscissa denotes the period of time in months, the ordinate — the ten-years' periods.

By means of least squares the two corresponding approximate lines have been drawn across these circles. The reckoning shows that the retardation of the maximal point is 10.2 days with its mean error 0.8 days, and the corresponding retardation of the minimal point is 9.7 days with its mean error 3.2 days.

A decrease of annual amplitude being observed in the annual variation of temperature<sup>1)</sup>, we may admit that in general the climate of Tartu in the years 1870—1933 has become more marine.

1) K. Frisch. Die Veränderungen der klimatischen Elemente nach den meteorologischen Beobachtungen von Tartu 1866—1930. Acta et Commentationes Universitatis Tartuensis A XXIII. 5.