ATOMIC COLLISIONS AND RADIATION OF METEORS

BY

ERNST ÖPIK

TARTU 1933

Printed by C. Mattiesen Incorp., Tartu, 1933.

Abstract.

Relatively small meteors, not more than a few cm in diameter, are considered in this paper. Collisions of meteor atoms with air molecules represent the chief source of radiation, the so-called impact radiation; for larger meteors, of magnitude zero and brighter, temperature radiation produced by mutual collisions of the molecules of the coma, or the expanding cloud of vaporized meteoric material, becomes appreciable. Both give rise to a linear emission spectrum. Electronic collisions as a source of continuous radiation, and the continuous radiation of the meteoric nucleus are considered as unimportant. A method is developed for calculating the amount of visible radiation produced by a meteor of given velocity, mass, and chemical constitution. The method is applied to a typical case, by assuming a certain average distribution of the excitation (radiation) levels. Considerable deviations from the typical case, due to variance in chemical constitution, may be expected.

The minimum mass of a meteor is calculated from its luminosity on the assumption that all the kinetic energy is transformed into black body radiation of 6000°K.

The heat factor, by which the minimum mass is to be multiplied to obtain the probable mass, is calculated for the typical case, and found equal to from 20 to 400, according to size and velocity of the meteor; it decreases with increasing mass and velocity, except for very small telescopic meteors.

Thermodynamic equilibrium is generally not reached in the coma of a small meteor, the expansion and mixing with the atmospheric gases going on too rapidly. The radiating gases are in a state of underexcitation, in spite of the high energy of the collisions; radiation must occur chiefly from collisions of low relative velocity — secondary collisions, and because of the underexcitation and the low inelastic efficiency of the collisions,

ultimate lines are expected to be the most prominent feature of the spectrum. Impact radiation will yield lines of the various ionized states of the same atom, whereas temperature radiation will produce lines of the neutral and of the first ionized states, in the first place of the more easily ionized elements. The average spectral energy distribution (averaging the emission lines over more extended spectral limits) is much more widespread than a black body distribution, showing strong excess both in the red and the violet for a given visual brightness; the richness in ultra-violet radiation must be even greater than that expected in the typical case, due to the fact that many ultimate lines of the elements are found in the ultra-violet.

A physical theory of dissipation of energy and efficiency of inelastic transitions in atomic collisions is developed; the theory gives the right order of magnitude for all velocities, and almost perfect agreement in the case of α-particles in air. The theory is based on the hypothesis that the forces of interaction in atomic collisions, as function of the relative distance, are determined in terms of the electronic energy levels of the two interacting atoms, the chief assumption being that the potential energy of resistance to interpenetration of two electronic orbits is equal to the smaller of the two ionization potentials.

The present, more general sketch is supposed to represent a basis for further detailed numerical researches.

Atomic Collisions and Radiation of Meteors.

The theory of radiation of meteors is especially important in estimating meteor masses, the estimate depending upon the ratio of visible radiation to the total energy of the meteor. The present paper deals with a special chapter of a more general theory of meteor phenomena, some speculations as to interatomic forces being also introduced. Only meteors of relatively small dimensions which do not reach the ground will be considered.

More or less extended physical theories of meteors have been proposed several times: by Lindemann and Dobson [1]), Sparrow [2]), Maris [3]), and others; some early speculations were offered by the writer [4]). All these authors agree in one point, namely — that the radiation from the solid or from the liquid nucleus is insignificant, and that the major part of the radiation of the meteor is produced by the collision of air molecules with the gaseous constituents evaporated from the nucleus, a view supported by the evidence of known meteor spectra, in which emission lines are the most prominent feature. As to the details, Lindemann and Dobson's theory cannot be accepted in its full extent on account of a mistake in the adiabatic formula for the temperature of a compressed gas. According to that formula the major part of the kinetic energy of the air molecules relative to the meteor is not taken into account, because the air, accelerated to the speed of the meteor and moving with it, has zero energy relative to the meteor; therefore according to the formula used by Lindemann and Dobson the internal energy and energy of compression represent only a small fraction of the initial relative kinetic energy. This misconception has been shown very clearly by Sparrow [5]); although Lindemann, in his reply to Sparrow [6]), finds the arguments of the latter not convin-

1) Proceedings Royal Society, **102**, 111 ff, 1923.

2) Astrophysical Journal, **63**, 90 ff, 1926.

3) Terrestrial Magn., **34**, 309 ff, 1929 (Reference according to Science Abstracts).

4) Tartu Publications, **25**.$_1$, 32—37, 1922. And earlier in Russian periodicals.

5) *Loc. cit.*

6) Astrophysical Journal, **65**, 117 ff, 1927.

cing, we must confess that the error is nevertheless there. An accurate formula for the temperature of compressed air has been given by Epstein[1]); applying this formula we find that the flow of heat from a hot air cap toward the nucleus of the meteor is much greater than that assumed by Lindemann and Dobson; consequently the densities of air along the visible path of the meteor are much smaller, and their theory of the "hot" upper atmosphere becomes invalid.

On the other hand, Sparrow's conception of meteor phenomena must be regarded as a very sound one in its general features, notwithstanding the neglect of some important details, such as the amount of heat absorbed in the nucleus before the beginning of vaporization; his argument in favor of the non-formation of an air cap and the direct bombardment of the nucleus by the air molecules is made even more valid, if we consider that the effective free length of path of molecules at meteoric velocities is many times greater than at ordinary molecular velocities; the quantitative data on this subject may be found below. This circumstance invalidates another qualitative feature on which Lindemann and Dobson's theory rests.

1. Forces of interaction in atomic collisions. There are not enough experimental data relating to the dissipation of energy and generation of radiation in atomic collisions; the data are lacking or scarce precisely for the range of velocities and the elements most likely to occur in meteors. It seemed that a theory, giving the size of the target area in a given collision and the probability of a successful inelastic collision "of the first kind", may fill up the gap in the experimental evidence. Such a theory depends altogether upon the assumed law of forces of interaction between two approaching atoms, or molecules. In the following discussion a practical solution of the problem is proposed, representing the experimental facts within a fair order of magnitude, from low molecular velocities up to velocities of alpha-particles. We have to introduce certain simplifications, which however must not extend over essential details; thus, the conception of rigid spheres with constant radius, or even with an effective radius depending upon the relative velocity only, is of no use for our purpose.

1) Proceedings Nat. Acad., **17**, 532, 1931.

We neglect altogether, as of smaller importance at the velocities concerned, forces of attraction acting between molecules at greater distances; also we disregard any possible asymmetry in orientation. In other words, we shall consider only the average force of repulsion at a given distance.

We assume that the two atoms create a mutual field of central force, with a potential energy of repulsion equal to $V(x)$, x being the relative distance of the nuclei; $V(x)$ is supposed to vanish at distances greater than or equal to the average distance of the molecules in a liquid state.

The trajectory of one of the atoms relative to the other is a curve, convex to the center of force; for the sake of simplicity we substitute for this curve two straight lines, crossing at the point of closest approach, and assume that instead of acting gradually, all the energy $V(x_0)$ acts suddenly at the distance of closest approach, x_0; this is evidently equivalent to a substitution of the continuous field of force by a fictitious rigid sphere of radius x_0 and radial energy of interaction $V(x_0)$; from the spheres of the kinetic theory of gases this sphere differs by having the radius, x_0, dependent upon both the relative velocity and the angle of impact.

Let ξ denote the radius of the target, or the distance of one of the atoms from the undeflected rectilinear path of the other; from the laws of conservation of momentum, of energy, and of angular momentum the following equation may be derived:

$$E_r \left(1 - \frac{\xi^2}{x_0^2}\right) = V(x_0) \quad \ldots \ldots \quad (1),$$

where E_r is the relative kinetic energy,

$$E_r = \frac{m_1 m_2 w^2}{2(m_1 + m_2)} \quad \ldots \ldots \quad (2);$$

here m_1, m_2 are the masses; w, the relative velocity.

$V(x_0)$ is the relative energy of the collision; this represents the maximum amount available for inelastic transformations. This quantity cannot exceed E_r.

Equation (1) permits computation of the target area, $\pi\xi^2$, for a given relative energy of the collision, $V(x_0)$, when the function $V(x)$ is known.

In our special problem we have to consider the effect of successive collisions of a moving particle m_2 with particles m_1

of a medium whose molecules have small relative velocities, so that we may assume, as a good approximation, that all the m_1 are at rest with respect to each other. We choose our coordinates of reference as fixed relative to the medium, or, which is the same, as fixed relative to the original state of m_1 before the collision. In the case of a perfectly elastic collision, the amount of kinetic energy transferred from m_2 to m_1 is

$$\Delta\left(\frac{m_2 w^2}{2}\right) = - V(x_0) \cdot \frac{4\, m_2}{(m_1 + m_2)} \quad . \quad . \quad . \quad . \quad (3).$$

In the case of a completely inelastic collision, when all the amount $V(x_0)$ is absorbed by the inelastic transition, the total loss of kinetic energy of m_2 is

$$\Delta\left(\frac{m_2 w^2}{2}\right) = - V(x_0) \cdot \left(1 + \frac{m_2}{m_1 + m_2}\right) \quad . \quad . \quad . \quad (4),$$

whereas the gain by m_1 is $V(x_0) \cdot \frac{m_2}{(m_1 + m_2)}$.

From these formulae the dissipation of energy may be calculated.

It remains to define the function $V(x)$ 1). The repulsive energy we shall consider as composed of the two parts: $V_1(x)$, the energy of the „quantum forces" due to the presence of electrons, and $V_2(x)$, the electrostatic energy. Thus,

$$V(x) = V_1(x) + V_2(x) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5);$$

V_1 is by far the most important of the two.

Let us define r_n, the reduced radius of an electronic orbit, by the equation:

$$I_n = \frac{7 \cdot 15 \times 10^{-8}\, n}{r_n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6);$$

here I_n is the ionization potential in volts; n, the degree of ionization (or the ordinal number of the electron counted from outside), r_n being given in cm.

1) The potential energy function could have been derived by methods of quantum mechanics, *e. g.* applying the method of Heitler and London in a way analogous to the way used by J. C. Slater (Phys. Rev., **32**, 349, 1929) in treating the interaction of helium atoms. The result would have been practically the same except for very low energies and minute details, which are of no importance for our present purpose.

Let us consider two approaching atoms a and a' at a distance $x_1 = r_1 + r_1'$, where r_1 and r_1' are the reduced radii of their outer electronic orbits; I_1 and I_1' the corresponding energies of ionization. Let $I_1' < I_1$. We assume that the "quantum" energy of interaction in this case is equal to the smaller of the two ionization potentials, $V_1(x) = I_1'$, and that the "weaker" electron, the one with smaller energy, is "knocked out" — that is, it is forced to move somewhere outside, and its "quantum resistance" to the intruding atom is broken down. At further approach, at a distance $x_2 = r_1 + r_2'$ the atom a meets the resistance I_2' of the atom a'; let now $I_2' > I_1$. We assume $V_1(x_2) = I_1' + I_1$, and so forth.

The relative position of the two atoms, when their distance is equal to the sum of the reduced radii of two of their electronic orbits, we will call the "contact" of these orbits.

Thus, generally, considering the k^{th} in a sequence of successive contacts, when the distance x_k is equal to the sum of the reduced radii of the outer not yet knocked out electrons of the two atoms, we assume that $V_1(x_k)$ for the given contact exceeds $V_1(x_{k-1})$ of the preceding contact by an amount equal to the smaller of the two ionization energies of the interacting electronic orbits, and that the "weaker" of the two electrons is at the same time "knocked out". In the case of equal ionization energies we still assume that only one of the electrons is knocked out. Thus, $V_1(x_k) = \sum_1^k I(i)$ . . . . (7), where $I(i)$ denotes the ionization energy of the "weaker" electron in a contact. As to the "knocked out" electron, it may return, after the collision, to its former position without radiation, in which case we have a perfectly elastic collision; or it may stay on a higher energy level and fall back into the lower state with emission of a quantum; or the "knocking out" may result in separating the electron altogether from the atom; the latter will be ionized. The energy of interaction, $V_1(x)$, however, is assumed equal to the ionization energy, without regarding whether the collision really results in ionization, excitation, or is perfectly elastic.

Further, we assume that for a bare nucleus V_1 is zero, or that the nucleus without bound electrons is not affected by the "quantum forces" of the electronic orbits of the other atom; only electrostatic forces will be taken into account in this case.

The electrostatic energy of interaction is defined by the following formula:

$$V_2(x_2) = V_2(x_1) + 7.15 \times 10^{-8} e_1 e_2 \left(\frac{1}{x_2} - \frac{1}{x_1}\right) \quad \text{. . . . . (8)},$$

where e_1 and e_2 are effective numbers of elementary charges of the two atoms. Formula (8) evidently considers the electronic charges either concentrated in the center, or distributed evenly over a spherical shell — a conception fulfilled statistically, if not in individual cases. As to e_1 and e_2, we assume that the knocked out electrons exert zero force on both the colliding atoms and upon their electrons not yet knocked out; thus, knocking out of an electron from the atom ***a*** increases its effective charge e_1 by one positive unit. Therefore, between two consecutive contacts, e_1 and e_2 remain constant, and formula (8) is to be used, for the computation of V_2, stepwise for each contact separately.

The effective molecular or atomic radii, as given by the kinetic theory of gases, are much greater than the reduced radii of the outer electronic orbits; at the same time, the average energy involved in molecular collisions at ordinary temperatures is of the order of 0.04 volt, or much smaller than the smallest of the ionization potentials; also, we know from the behaviour of the coefficient of viscosity of gases that the effective molecular diameter decreases with increasing temperature, that is, with increasing energy of the collisions. This indicates that the energy of interaction does not attain suddenly its value $V(x_1)$ at the first contact, but that from the distance x_1 outward it must decrease gradually. By analogy we assume also that between two contacts, $k-1$ and k, $V(x)$ changes gradually.

The rule for constructing the potential function $V(x)$ is now given. For the consecutive contacts $x_1, x_2, \ldots x_k \ldots$ we compute, according to formulae (6), (5), (7), (8), the values of $V(x)$; further, for $x_m = \frac{\sigma_1 + \sigma_2}{2}$, where σ_1 and σ_2 are the molecular diameters according to the kinetic theory of gases, we assume $V(x_m) = 0.036$ volt-e, which is the average relative energy of two molecules at 273° K, and from the known (or assumed) dependence of the coefficient of viscosity upon temperature we determine $\left.\frac{\partial V}{\partial x}\right|_{x = x_m}$ [1]), or the slope of $V(x)$ at $x = x_m$, taking into account that $V(x)$ is proportional to the temperature.

The curve $V(x)$ is then traced through the given points, being also fitted to the assumed slope at $x = x_m$; the curve is led exactly through the points, without smoothing except between the points. Very little is thus left to arbitrariness in tracing the curve. The greatest uncertainty arises from an incomplete knowledge of the ionization energies, which in many cases must be guessed; but this uncertainty does not influence the order of magnitude of the results.

Table I contains examples of adopted functions $V(x)$, assumed according to the rules explained above.

TABLE I

Energies of Interaction for Some Pairs of Colliding Atoms [2])

a) He : N

x	2.50	2.00	1.50	1.00	0.790	0.779	0.750	0.720	0.655	0.390	0.369
$V(x)$	0.036	0.22	0.68	2.8	14.4	38.7	68.6	116.5	173.6	218	226
x	0.083	0.075	0.05	10^{-2}	10^{-3}	10^{-4}	10^{-5}				
$V(x)$	894	1004	1670	9700	10^5	10^6	10^7				

b) He+ : N

x	2.47	1.97	1.47	0.97	0.761	0.750	0.720	0.655	0.390	0.369
$V(x)$	0.036	0.22	0.68	2.8	14.4	44.0	91.9	149.0	193	202
x	0.083	10^{-2}	10^{-3}	10^{-4}	10^{-5}					
$V(x)$	869	9700	10^5	10^6	10^7					

c) He++ : N

x	0.496	0.485	0.455	0.390	0.369	0.083	0.075	0.05	10^{-2}
$V(x)$	0	0.6	4.6	26.0	34.6	702	812	1480	9500
x	10^{-3}	10^{-4}	10^{-5}						
$V(x)$	10^5	10^6	10^7						

d) N : Fe

x	2.70	2.50	2.25	2.00	1.75	1.50	1.414	1.366	1.355	1.310
$V(x)$	0.036	0.093	0.35	0.74	1.5	4.0	7.8	22.2	28.8	65
x	1.152	1.122	0.965	0.900	0.833	0.812	0.773	0.487	0.464	
$V(x)$	97	141	197	273	356	457	564	851	1029	
x	0.376	0.369	0.348	0.339	0.330	0.321				
$V(x)$	1392	2237	2655	3082	3557	4085				

1) *cf* Jeans, Dynamical Theory of Gases, p. 284 (Cambridge, 1925).

2) $V(x)$, in volts; the relative distance, x, in Ångströms.

Where electrons with higher ionization energies yield a reduced radius greater than the radii of those with lower energies, as apparently happens in the closed outer group of the neon and argon cores, the first contact was assumed to take place with the orbit of the greatest reduced radius, and it was assumed that the corresponding electron might be knocked out only together with all the other electrons of smaller energies.

2. Dissipation of kinetic energy by a high speed atom. In considering processes of radiation produced by an individual high speed atom, we are justified in neglecting molecular velocities to temperatures of several thousand degrees; the molecules of the medium are practically at rest, and formulae (3) and (4) of the preceding section refer to this case. Denoting the ratio of masses by μ,

$$\mu = \frac{m_2}{m_1},$$

and the kinetic energy by

$$E_2 = \frac{1}{2} m_2 w_2^2,$$

formula (2) for the relative energy is transformed into

$$E_r = \frac{E_2}{1+\mu} \quad . \quad . \quad . \quad . \quad (2'),$$ and similarly

formulae (3) and (4) into

$$\Delta E_2 = - V(x_0) \cdot \frac{4\mu}{1+\mu} \quad . \quad . \quad . \quad . \quad (3') \quad . \quad . \quad \text{(elastic)},$$

$$\Delta E_2 = - V(x_0) \cdot \left(1 + \frac{\mu}{1+\mu}\right) \quad . \quad . \quad . \quad . \quad (4') \quad . \quad \text{(inelastic)}.$$

In the latter case, the kinetic energy gained by m_1 is

$$\Delta E_1 = + V(x_0) \cdot \frac{\mu}{1+\mu} \quad . \quad . \quad . \quad . \quad (4'').$$

For our purposes it is convenient to measure the kinetic energy in volt-electrons (volts). Table II gives these energies for different atoms and for different velocities of the order which may occur in meteors.

TABLE II

Kinetic Energy of Atoms

Velocity, km/sec

w	0.92	1.30	1.85	2.61	3.70	5.22	7.40	10.44
				E_2 = kinetic energy, volts				
Atom								
H	0.0046	0.0092	0.018	0.037	0.074	0.148	0.295	0.59
N	0.06	0.12	0.25	0.50	1.0	2	4	8
Fe	0.25	0.50	1.0	2	4	8	16	32

w	14.8	20.9	29.6	41.8	59.2	83.6	118.4	167.2	236.8
Atom					E_2				
H	1.15	2.3	4.6	9.2	18.4	36.9	73.8	147.5	295
N	16	32	64	128	256	512	1024	2048	4096
Fe	64	128	256	512	1024	2048	4096	8192	16384

The rate of dissipation of energy is given by $\frac{\partial E_2}{\partial \lambda}$, or by the loss of energy over unit length of path, volt cm^{-1}; λ is the actual length of the trajectory; it may differ very much from the apparent length of penetration, λ', counted along a straight line between two actual positions of the moving molecule.

Using the notations of this and the preceding sections, the rate of dissipation of energy is given as follows:

$$\frac{\partial E_2}{\partial \lambda} = \pi \nu \int_{\xi=0}^{\xi=(\infty)} f(\mu)\ V(x_0)\, d(\xi^2) \quad \ldots \ldots \quad (9),$$

where ν is the number of atoms per cm^3, and $f(\mu)$ is equal to $f_e = \frac{4\mu}{1+\mu}$ for elastic, and $f_i = 1 + \frac{\mu}{1+\mu}$ for inelastic collisions; for partly inelastic collisions an intermediate value of $f(\mu)$ is to be taken; generally $f(\mu)$ may be regarded as a function of $V(xo)$ and E_r, limited by the extreme values f_e and f_i; ξ is determined by x_0, through equation (1).

The maximum possible amount of inelastic energy dissipated is found from (9), putting $f(\mu) = 1$; the true amount of inelastic energy may be smaller, or nil.

With the aid of Table I, and formulae (9) and (1) for high velocities of the order of those of alpha-particles, the values of $\frac{\partial E_2}{\partial \lambda}$ are found to be practically constant as follows:

	$He:N$	$He^+:N$	$He^{++}:N$	
$-\frac{\partial E_2}{\partial \lambda}$ elast.	$21,7.10^5$	$18,3.10^5$	$4,9.10^5$	$\frac{\text{volt}}{\text{cm}}$
$-\frac{\partial E_2}{\partial \lambda}$ inelast.	$29,7.10^5$	$25,1.10^5$	$6,7.10^5$	

These figures refer to a helium atom moving in atomic nitrogen at 1520 mm pressure, so that the number of nitrogen atoms is the same as in molecular nitrogen at atmospheric pressure. For the energy of a helium atom ranging from 8.10^5 to 8.10^6 volts these figures change only by a fraction of one percent. As there is not a very great difference between the elastic and the inelastic cases, and as the collisions of alpha-particles are known to be practically "perfectly inelastic", we may adopt the inelastic set of figures. For comparison, from the curves of dissipation of energy of alpha rays of RaC as given by Kapitza [1]), the following rates of dissipation are derived:

energy, E_2, volt		8.10^5	8.10^6
observed $-\frac{\partial E_2}{\partial \lambda}$	$\frac{\text{volt}}{\text{cm}}$	$18,5.10^5$	$8,4.10^5$
computed		$12,9.10^5$	$6,7.10^5$
computed: observed		0,70	0,80

How the computed values were found is explained below.

We must take into account that an alpha-particle moving in air is apt to catch one or two electrons and lose them again, so that it partly moves as He^{++}, He^+ or He. Both theoretical considerations and observations indicate that the free length of path for taking up a charge increases with increasing speed, so that the higher the velocity, the greater will be the percentage of collisions that happen during the higher ionized state of the moving atom. From Rutherford's observations on the free length of path of recharge of alpha-particles in air one may infer that at $E_2 = 8.10^5$ volts, 67 percent and at $E_2 = 8.10^6$ volts, 99.6 percent of the collisions are produced by He^{++}, the rest being ascribed to He^+, whereas the share of He is practically nil. The increasing fraction of He^{++} collisions may explain the observed decrease of $\frac{\partial E_2}{\partial \lambda}$ with increasing velocity. With Rutherford's data for the ratio $\frac{He^{++}}{He^+}$, the "computed" values of the preceding table were found. It may be added that within the limits

1) Proc. Royal Soc., **102**, 48, 1922.

covered by the table the observed range, which is proportional to $E_2 \Big/ \frac{\partial E_2}{\partial \lambda}$, changes with the 2.68 power of velocity, whereas the computed range is proportional to the 2.58 power. Perfect agreement between observation and calculation may be attained by changing slightly the relative frequency of $He^{++} : He^{+}$ collisions. Table III gives these effective ratios, which were assumed conventionally for the calculation of the number of ions in the following section.

TABLE III

Effective Fraction *(p)* of He^{++} Collisions for Alpha-Particles in Air

E_2, 10^6 volt	8.2	5.7	4.1	2.5	1.65	0.82
$-\frac{\partial E_2}{\partial \lambda}$, 10^6 $\frac{\text{volt}}{\text{cm}}$	0.84	0.97	1 25	1.60	1.77	1.85
p	0.91	0.84	0.69	0.50	0.40	0.36

A remarkable feature of the dissipation of energy at high velocities is that the major part of the energy is lost by very small fractions. This is opposite to what happens in collisions of rigid spheres of constant radius, where the major part of the energy is transferred in large fractions of the relative energy, *Er*. The following example may serve as illustration.

TABLE IV

Fraction of Energy Dissipated within Given Limits of the Energy of Collision
$E_2 = 8.10^5$ volt

	$V(x_0) : Er$								
	0 to $1{,}5.10^{-5}$	$1{,}5.10^{-5}$ to 0,0001	0,0001 to 0,001	0,001 to 0,003	0,003 to 0,01	0,01 to 0,1	0,1 to 0,5	0,5 to 1,0	Sum
	Fraction of energy								
$He : N$	0,034	0,016	0,818	0,075	0,037	0,018	0,0009	0,00015	0,999
$He^{+} : N$	0,040	0,023	0,783	0,086	0,045	0,022	0,0011	0,00018	1,000
$He^{++} : N$	0,006	0,099	0,461	0,209	0,142	0,080	0,0041	0,00066	1,002
Rigid spheres of constant radius	$2{,}25.10^{-10}$	10^{-8}	10^{-6}	8.10^{-6}	8.10^{-5}	0,0099	0,24	0,75	1,000

We may say in general that our theory, representing very well the rate of dissipation of energy at velocities of alpha-particles and being made to fit at ordinary molecular velocities, gives a fair approximation also for intermediate velocities, the discrepancy not exceeding 2:1 and never affecting the order of magnitude. The following data may show how well the computed values represent what happens at meteoric velocities (from 8 to 50 km/sec).

Relative Change of Mean Free Path with Velocity.

Relative energy, $Er = E_2/(1+\mu)$, volt	3	10	30	100	Mass Ratio
Ratio of mean free path to kinetic theory value					
Computed, $Fe:N$	3.1	4.1	5.4	6.4	4
Observed, $K^+:N_2\,(N)$	2.3	3.0	3.6	4.2	1.4 (2.8)
Observed, $K^+:A$	1.5	2.5	3.8	5.0	1.0
Observed, $K^+:He$	4.9	7.6	12	—	10.

The observed values are according to F. M. Durbin[1]) and are derived essentially from deflections in angle; they are therefore not well comparable with the values computed for the dissipation of energy, but they show that the computed values give perhaps more than the mere order of magnitude.

3. Ionization by atomic collisions. — The chief attention of experimental and theoretical physicists has been hitherto directed to inelastic collisions of atoms with electrons. In the process of radiation of meteors these collisions must play an insignificant part compared with collisions of atoms with atoms or with molecules. It was therefore necessary to work out a quantitative model of such collisions, on the basis of the theoretical considerations in the two preceding sections, supplemented by existing experimental evidence.

Ionization is the easiest to observe of all kinds of inelastic energy, and is therefore most adapted to furnish a test of the theoretical considerations. Excitation yields probably to the same rules as ionization, but quantitative tests are not so easy to obtain. A few words may be said about the mechanism involved in inelastic collisions of two atoms. For the sake of

1) Physical Review, **30**, 844, 1927.

simplicity we limit our attention to ionization only, although the considerations are quite general. The following models may be considered.

Model 1. The outer electron is thrown out as the result of the recoil of the nucleus, in the collision of two nuclei, or atomic cores. For alpha-particles in air it would correspond to a maximum fraction of ionization energy of 1/25000 only, whereas observations give a fraction of the order of 1/2. Also, according to this hypothesis, the minimum translational energy at which ionization begins should be of the order of 5.10^5 volts, whereas in the collision of alkali ions with inert gases appreciable ionization sets in at 110—300 volts.

Therefore, there is no doubt that the part played by Model 1 in producing ionization is negligible and need not be taken further into account.

Model 2. An electron is thrown out by an elastic collision with the nucleus of the other atom. This model is to be discarded exactly for the same reasons as Model 1.

Model 3. The model cannot be described precisely. The essential is that all the relative energy of the collision, $V(x_0)$, which depends upon the mass-ratio of the two colliding atoms as a whole, and not upon the masses of the single electrons, is available for ionization. The electron is not simply a satellite moving on its orbit under the force of electrostatic attraction, and otherwise free, but is much more closely bound to the whole system by "quantum forces" so that inertia is not the only way of transmitting energies of collision.

The present model may have two variants:

3^a — That only forces of interaction due to the presence of electronic orbits, $V_1(x)$ of Section 1, may produce ionization; this variant is to be rejected because in this case He^{++} particles should be unable to ionize except by the minute amounts corresponding to Models 1 and 2, and the ionization by alpha-particles would be considerably smaller than the observed one. Thus we have to accept the only possible variant:

3^b — That all the relative energy of a collision, $V(x) = V_1(x) + V_2(x)$, including the electrostatic fraction of the energy, is available for inelastic changes in the electronic energy levels.

In the case of the n^{th} degree of ionization, the inelastic energy is given by

$$H = I_1 + I_2 + \ldots + I_k + \ldots + I_n + \varepsilon_n \quad . \; . \; . \quad (10),$$

where I_k is the ionization potential of a k^{th} electron, and ε_n is the energy of a possible excited level of the n^{th} ionized state; putting $\varepsilon_n = 0$ we get a minimum value of the inelastic energy, which however is in most cases close to the true value, because ε_n is only a fraction of I_{n+1}. Dealing with ionization alone, we need to consider only this fraction of the inelastic energy,

$$H_i = \Sigma_1^n \, I_k \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10').$$

The maximum possible degree of ionization is determined by the inequality

$$H_i \leqslant V\,(x_0) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11).$$

To explain ionization by alpha-particles, we must assume that maximum ionization really takes place. On the other hand, experiments indicate that at low velocities the ionization is smaller than the maximum possible. From a comparison of experimental data relating to different velocities[1]), to 20000 volts, chiefly from observations of ionization of rare gases by alkali ions, it was found that the order of magnitude of the observed number of ions is well represented on the assumption that the fraction of successful inelastic collisions of a given degree of ionization, or the coefficient of efficiency, k, is determined by

$$k = 0{,}026 \sqrt{\frac{E_r}{H_i}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (12),$$

under the restriction $k \leqslant 1$. Formula (12) means that for a given pair of atoms the efficiency is proportional to the relative velocity. When k exceeds 1 according to (12), we must assume it equal to 1.

As the result of an inelastic collision, ions of different degrees of ionization are created. However, in successive collisions with neutral molecules, a redistribution of the charge takes place, so that finally only single-charged ions remain; in air of atmospheric density this process is completed within less than 10^{-9} seconds.

---

1) References in connection with Table VI.

The maximum number of singly charged ions produced by a highly charged ion is not equal simply to the integer of the ratio of their energies, but is smaller because in the consecutive collisions there is not available a sufficient number of molecules to take up at once all the extra charge; the redistribution of the charge must thus proceed by steps; in this process some energy is lost in the form of residues, each of which is probably smaller than the energy of first ionization; part of the loss goes into dissociation of the molecules. The redistribution may take place in different ways, the resulting number of ions being practically the same. For the case of alpha-particles in air, the data of Table V were calculated on the basis of a particular model of the redistribution of charges, where dissociation by recoil was also taken into account.

TABLE V

Number (F_i) of Singly Charged Ions Formed in Nitrogen through Redistribution of Charge

	Primary ion								
	N_I	N_{II}	N_{III}	N_{IV}	N_V	N_{VI}	N_{VII}	$He_{II} \to He_I$	$He_{II} \to He$
Energy, volts	14.4	43.9	91.0	164.5	261.5	798	1461	54.1	78.4
F_i	1	2	6	9	14	44	82	2	3

The number of primary ions created per cm of path of a moving atom is found from

$$N_i = \pi \nu k \left(\xi_i^2 - \xi_{i+1}^2\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13),$$

the notations being those of formulae (1), (9), (12). ξ_i is the target radius for $V(x_0) = \Sigma_1^i I_k$ in notations of (10′) and (1).

The final number of singly charged ions is given by

$$N = \Sigma F_i N_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14),$$

where F_i for nitrogen is given by Table V.

For the high velocity alpha-particles we put $k = 1$; also, the target radii ξ_k prove to be practically independent of velocity, for the limits of velocity considered of importance in this case; the same was found for the rate of dissipation of energy at high velocities. With the aid of Table I, the number of ions formed per cm by He^{++} was computed as follows:

Ion	N_I	N_{II}	N_{III}	N_{IV}	N_V	N_{VI}	N_{VII}	Sum
H_i	14.4	43.9	91.0	164.5	261.5	798	1461	—
$log\, \xi_i^2 + 16$	−0.76	−0.92	−1.16	−1.44	−1.64	−2.20	−2.58	—
N_i	910	870	553	228	282	62	45	—
F_i	1	2	6	9	14	44	82	—
$F_i N_i$	910	1740	3318	2052	3948	2728	3690	18336

In a similar way, the number of ions generated by He^+ per cm was calculated and found equal to 85900. The amount of energy spent per cm in primary ionization is:

$He^{++} = 328000 \frac{\text{volt}}{\text{cm}}$ or 49 per cent of the whole dissipated energy;

$He^{+} = 1586000 \frac{\text{volt}}{\text{cm}}$ or 63 per cent of the whole.

The maximum ratio of inelastic energy to the whole dissipated energy is $\frac{1+\mu}{1+2\mu}$ (equation 4′), in the case of a "completely" inelastic collision; in the present case, with $\mu = \frac{4}{14}$ [1]), the ratio is 81 per cent. Taking into account that in addition to ionization excitation and dissociation occur, we conclude that the collisions of alpha-particles approach very closely the conditions of complete inelastic encounters, as was anticipated in Section 2.

The average energy spent in generating one singly charged ion is calculated as follows:

$$\text{by } He^{++},\ \frac{670\,000}{18336} = 36.5 \text{ volts};$$

$$\text{by } He^{+},\ \frac{2\,510\,000}{85\,900} = 29.3 \text{ volts}.$$

Assuming the effective He^{++} ratio according to Table III, we may compute the total number of ions generated on the path of the alpha-particle by mechanical quadratures, taking into account the change of the said ratio with decreasing velocity. The results, as compared with experimental data, are as follows:

Source	Radium C	Uranium 1
Initial energy, volt	$8{,}2.10^6$	$4{,}06.10^6$
Total number of ions per particle, computed	243600	125400
ditto observed	237000	126000
Energy per ion, computed	33,7	32,4
ditto observed	34,6	32,3

1) Not 4/28.

The agreement is very close, even the minute increase in the average energy spent per ion with increasing speed is predicted, at least in sign.

Applying the same methods, and assuming the efficiency according to equation (12), different experimental data were compared with the theory. The results are contained in Table VI. It must be noticed that the numerical coefficient of formula (12) was derived as a mean value from the same experimental data.

TABLE VI

Number of Ions Generated per cm at 760 mm and 0° C

Atom	E_2 volts	Velocity km/sec	Gas	Number of ions		Ratio Obs : Comp	Remarks
				Observed	Computed		
Li+	750	143.	He	300	170	1.76	1
„	„	„	Ne	570	780	0.73	1
„	„	„	A	2970	2580	1.15	1
Na+	750	79.0	He	150	140	1.07	1
„	„	„	Ne	1560	780	2.00	1
„	„	„	A	1370	2740	0.50	1
K+	750	60.6	He	80	210	0.38	1
„	„	„	Ne	1180	1140	1.04	1
„	„	„	A	9350	3580	2.61	1
Rb+	750	41.0	He	40	90	0.44	1
„	„	„	A	4780	2830	1.68	1
Cs+	750	32.9	He	0	0	—	1
„	„	„	Ne	680	740	0.92	1
„	„	„	A	2820	2540	1.11	1
He, He+	2100	1000.	He	2830	2800	1.01	2
„	15000	840.	„	2400	2600	0.93	2
„	11000	720	„	1850	2000	0.92	2
„	7500	600	„	1240	1700	0.73	2
RaC α-part.	$8{,}2.10^6$ to 0	20000	air	33400	34300	0.97	3
U_1, α-part.	$4{,}06.10^6$ to 0	14000	air	50500	50300	1.00	3

Remarks: 1. Observed data according to Sutton and Mouzon, Phys. Rev., **37**, 379, 1931. For the same ion and gas the relative change of the number of ions with velocity below

750 volts is much better represented than the absolute number of ions. The accuracy of the observed numbers may be about ± 80.

2. Observed data by Rudnick, Phys. Rev., **38**, 1342, 1931.

3. These numbers are average values over the whole range of the corresponding alpha-particle.

In Table VI, the largest part of the deviations is to be attributed to the uncertainty in the efficiency factor k; the uncertainty in the assumed higher ionization energies of the rare gases and of the alkalis probably also affects the results to some extent. Omitting the data for alpha-particles, where $k = 1$ is practically certain, the rest of the data show a probable deviation in log k equal to ± 0.19, or a probable error in the ratio 1,55 : 1. This may be regarded as a measure of the maximum statistical uncertainty of equation (12).

We may assume that excitation to higher electronic levels obeys the same rules as does ionization, at least as far as the order of magnitude is concerned; only for H_i in (12) we substitute H, as given by equation (10).

4. Physical conditions of the radiation of meteors. — Several of the statements made in this section are based on unpublished results of a general physical theory of meteors. The statements are partly based on estimates of the probable masses of meteors of different brightness and velocity, appearing at the end of this paper. The numerical estimates are supposed to give the order of magnitude, within a factor of 2, approximately.

In the meteor we may distinguish the following principal elements: the nucleus, the compressed air cap in front of the nucleus (when it exists), and the coma consisting of the products of vaporization of the nucleus, mixed with the intruding molecules of the air [1]). The coma, through the effect of retardation, extends backwards and at the same time grows wider with increasing distance from the nucleus.

In the introduction it was stated that the compressed air cap does not exist for ordinary meteors of small size. More

1) In the following we consider chiefly nitrogen. If atomic oxygen, according to Chapman, is the chief constituent, this would not make a great difference. However, a hydrogen atmosphere would lead to somewhat different conclusions.

precisely, we may say that according to Epstein's[1]) theory, the maximum air mass in the air cap is approximately equivalent to a layer of undisturbed air in the surrounding atmosphere of thickness $\frac{3}{2} R$, R being the effective radius of the nucleus. Also, from other considerations, the maximum air mass in the cap is attained when the radius has decreased (by vaporization) to about four sevenths of its original value. This maximum air mass is equal to

$$a_m = \frac{1{,}85\, R_0^2}{w^2\, Sec\, z}\, \frac{gr}{cm^2} \quad \ldots \ldots \quad (15),$$

Ro being the initial radius in cm; w, the velocity cm/sec; and z, the zenith angle of incidence.

With the free path defined as the path reducing the energy in the ratio 1 : 2, and computed from the theory given in Sections 1 and 2, we find limiting values of Ro at which a_m is equal to the free path; these values are given in Table VII (free path λ_0, refers to normal conditions N. T. P.).

TABLE VII

Limiting Radius R_0, for the Formation of an Air Cap. Sec $z = 1.85$

w, km/sec	14.8	29.6	59.2	118.4
Free path, λ_0, cm $\times 10^5$	8.4	11.7	14.1	21.0
R_0, cm	4.9	11.6	25.6	63
Apparent magnitude (140 km distance)	—6.5	—11	—16	—20
Density of atmosphere at point of disappearance, $\frac{gr}{cm^3}$	1.1×10^{-6}	0.66×10^{-6}	0.35×10^{-6}	0.22×10^{-6}

For other initial data, Ro is to be changed in the ratio $w \sqrt{\lambda_0\, Sec\, z}$. The density of air in the end point is given by

$$\delta_e = 4.34 \times 10^6 \cdot \frac{R_0}{w^2\, Sec\, z} \quad \ldots \ldots \quad (16).$$

From the table we infer that the formation of air caps is limited to fireballs of quite unusual brightness. Thus in meteors appearing in our regular observations no air cap can be formed.

The density of vapors in the coma near the nucleus is very much greater than the density of surrounding air; even in the

1) Proc. Nat. Acad., **17**, 532, 1931.

case of ordinary meteors the coma may be impenetrable for air molecules, and still more so for the molecules of the coma itself, because of the initially low temperature and corresponding small free path of the molecules of the coma[1]). At greater distances from the nucleus the penetrability of the coma increases; at first it holds together as a compact expanding cloud, but later the coma is dissolved into a cluster of single atoms, shooting more or less in different directions through the atmosphere in which they are so diluted that they have little chance of meeting one another.

The radiation of the meteor consists of the radiation from the nucleus, and from the coma. The radiation from the nucleus consists again of two parts: a) Regular temperature radiation, the temperature being limited by the temperature of vaporization. With the most generous estimates, visible radiation from this source cannot attain on the average as much as 1 per cent of the radiation of the coma for the slowest meteors, and thus may be neglected. b) Impact radiation, from the points of impact of the air molecules on the nucleus. This kind of radiation, probably partly continuous, partly linear emission, can by no means be treated as black body radiation, and the application of any kind of "effective temperature" in this case has no meaning. It may be treated in the same way as impact radiation from the coma; the visible impact radiation may be several times greater than the regular temperature radiation of the nucleus, but it must be small compared with the radiation of the coma — not more than ten per cent for slow meteors, and much less for fast ones.

The chief source of radiation is the coma. It may be shown that the conception of temperature cannot be applied to the coma without reservation: in the case of small meteors the change in energy content goes on faster than the equipartition of energy, so that the mixture is not in thermodynamic equilibrium. With respect to the molecules originating from the nucleus (of small initial kinetic energy relative to the coma), mutual equipartition of energy practically takes place in the denser portions of the coma; but the fast air molecules shooting through the coma, and some of the molecules of the coma which have

1) Free path increases with temperature; compare end of Section 2; also Kinetic Theory of Gases.

acquired greater velocities after being hit by the air molecules, have too great a free path and too few collisions during the life time of the coma. The conditions are especially unfavorable for electrons to acquire velocities corresponding to the law of equipartition. Because of the large velocity of equipartition, an electron originally at rest must undergo about one thousand encounters with atoms to reach the average required velocity; before this can happen, the coma will be dispersed into the atmosphere. A rough calculation indicates that for meteors fainter than magnitude —2 to —5, equipartition of energy with respect to electrons does not take place even in the densest portion of the coma.

If however we use the term temperature for the coma, it will be in the sense of mean energy content, without regard to the law of distribution of velocities.

The radiation of the coma consists again of two principal parts: impact radiation, and temperature radiation. Impact radiation is produced by collisions of high relative velocity, between air molecules and molecules of the meteor, and is calculated according to the rules given in Sections 1, 2, and 3.

The temperature radiation is produced by the compact cloud of the coma. For the reason already given, electronic collisions cannot play an important part in producing radiation, because there is not time enough for their velocities to reach the necessary size; also, the number of free electrons will be smaller than the number expected from equilibrium considerations, because the efficiency of atomic collisions in producing ionization is small, and the approach toward an equilibrium state proceeds very slowly. The coma is in a state of "underexcitation", especially during its first phase of existence until the maximum temperature is reached; this phase is at the same time the chief source of radiation. Radiation itself cannot play any part in determining the state of ionization of the coma, because if some atom is ionized by radiation of the coma, the radiation must have originated at the expense of a recombination in another part of the coma; in this process only a decrease, never an increase of the energy stored in the form of excitation and ionization can take place. Matter and radiation are so diluted in the coma that even in a direction forming an angle of only 8^0 with the direction of motion of the meteor the absorption by the coma of its own radiation does not attain even a few hundredths of a magnitude [1]); this takes place in spite of the circumstance that the coma must be very strongly elongated in the direction of motion. On account of this circumstance, also, radiation cannot contribute sensibly to the state of excitation of the coma.

The amount of temperature radiation increases with the duration of the coma, which on the other hand depends upon the dimensions of the meteor; for meteors fainter than about first magnitude, the visible temperature radiation is insignificant compared with the impact radiation. For bright fireballs, the total amount of temperature radiation may be equal, or even greater than the impact radiation. Quantitative data will be given later.

The effective degree of excitation in the coma changes in the same way as the importance of temperature radiation; the smaller the meteors and consequently the shorter the life time of the coma, the remoter will be the conditions from the equilibrium state of excitation. Also, the efficiency of inelastic collisions, according to equation (12), increases with decreasing energy of excitation. Therefore ultimate lines, corresponding to the lowest state, are expected to be especially prominent. For larger meteors, the probability of the appearance of excited lines increases. But even in the case of fireballs, the ultimate lines must be unusually strong, compared with the penultimate [2]) and other excited lines, on account of the "underexcitation" mentioned above. At the same time, owing to the great energy of collisions, the impact radiation may produce lines belonging to high degrees of ionization; thus, a meteor spectrum may show strong "superexcitation" as regards the degree of ionization, and "underexcitation" with respect to the relative probability of energy levels of the neutral state.

Exchange of charge in mutual collisions of the molecules in a coma tends to reduce the number of ions of high potential or of multiple charge, substituting for them easily ionized singly charged atoms; thus, temperature radiation is likely to represent lines of the neutral state, and of the first ionized state, with preference for the first ionized states of the easily ionized elements.

5. Dissipation of energy and impact radiation in a typical case. — For the order of magnitude,

1) E. Öpik, Tartu Public., **25.**$_1$, p. 28, 1922.

2) For terminology, *cf.* H. N. Russell, Astroph. Journ., **61**, 3, 1925.

the following simplifying assumptions may suffice: a) Ionization potentials and the law of atomic interaction as for nitrogen and iron, Table I, d; these data are more or less typical. b) Excitation, or radiation potentials evenly distributed within the given interval of ionization energy, and radiation of the type of ultimate lines[1]). c) The short life time of an excited state, as compared with the time between two collisions.

Assumption b) needs some explanation. The chemical composition of meteors is not so very diversified: on the average about 91 per cent of the stony meteorites is made of only four elements — oxygen, magnesium, silicon, and iron[2]), and in iron meteorites 90 per cent is iron alone. Thus, probably only these four elements, together with the constituents of the air, determine all the radiative properties of the meteors. But the different ionized states of one and the same element make the composition, from the standpoint of radiative properties, equivalent to a mixture of a much greater number of elements. Assumption b) means that, in a collision producing an atom of the n^{th} degree of ionization, the probability of a radiation potential to be produced between ε_n and $\varepsilon_n + \Delta\varepsilon_n$ is equal to

$$p_\varepsilon = \frac{\Delta\,\varepsilon_n}{I_n} \quad \ldots\ldots\ldots\ldots \quad (17),$$

in the notations of equation (10).

An idea of what may be found in an arbitrary mixture of different types of atoms may be derived from Table VIII.

TABLE VIII

Distribution of Excitation Potentials in H. N. Russell's List of Ultimate and Penultimate Lines[3])

$\varepsilon_n : I_n$, or ratio of excitation to ionization potential

	0—0.1	0.1—0.2	0.2—0.3	0.3—0.4	0.4—0.5	0.5—0.6	0.6—0.7	0.7—0.8	0.8—0.9	0.9—1.0	Sum
Number	16	29	22	28	19	20	10	7	7	2	160
Percentage	10.0	18.1	13.8	17.5	11.9	12.5	6.3	4.4	4.4	1.2	100

The distribution is not uniform, partly due to atomic, partly perhaps to observational selection; high excitation poten-

1) By neglecting lines of higher levels, we obtain a minimum estimate.

2) *Cf.* T. and W. Noddack, Naturwissenschaften, **18**, 757, 1930.

3) Astroph. Journ., **61**, 223, 1925. In constructing the table, some recent data for the ionization potentials were used.

tials are less frequent in the list than low and moderate ones. But for the calculation of the amount of visible radiation, the interval $\frac{\varepsilon_n}{I_n}$ from 0.1 to 0.4 is the only important one, and judging by the table the true amount of visible radiation may be slightly greater than the amount calculated according to formula (17). However, the considerable part played by atmospheric gases in meteoric radiation will increase the probability of high values of $\frac{\varepsilon_n}{I_n}$, at the expense of low and moderate values, due chiefly to iron; therefore it seems that the assumption of uniform distribution may represent the actual case of meteoric radiation better than the non-uniform distribution of Table VIII.

TABLE IX (abbreviated)

Energy Distribution *(Z)* and Square of Target Area Radius (ξ^2) for Atomic Collisions $N:Fe$

$\mu\ (N:Fe) = 0.25$; $\mu\ (Fe:N) = 4$. W_r = rel. velocity, km/sec; E_r = relative energy, volts

$V(x_0)$	$W_r = 14.8$, $E_r = 12.8$		$W_r = 20.9$, $E_r = 25.6$		$W_r = 29.6$, $E_r = 51.2$		$W_r = 41.8$, $E_r = 102.5$		$W_r = 59.2$, $E_r = 205$		$W_r = 83.6$, $E_r = 410$	
Volt	$10^{16}\cdot\xi^2$	Z	$10^{16}\cdot\xi^2$	Z	$10^{16}\cdot\xi^2$	Z	$10^{16}\cdot\xi^2$	Z	$10^{16}\cdot\xi^2$	Z	$10^{16}\cdot\xi^2$	Z
0	. . .		. . .		. . .		. . .		. . .		. . .	
		0.064		0.036		0.020		0.011		0.0061		0.0040
0.80	3.60		3.72		3.78		3.81		3.82		3.83	
		0.069		0.037		0.020		0.011		0.0064		0.0039
1.6	2.59		2.78		2.87		2.92		2.94		2.95	
		0.110		0.056		0.029		0.015		0.0085		0.0051
3.2	1.78		2.07		2.22		2.30		2.33		2.35	
		0.206		0.085		0.038		0.019		0.0100		0.0055
6.4	1.02		1.53		1.79		1.91		1.97		2.03	
		0.552		0.172		0.056		0.022		0.0072		0.0045
12.8	0.00		0.98		1.47		1.68		1.84		1.90	
				0.614		0.190		0.057		0.024		0.012
25.6	. . .		0.00		0.925		1.38		1.62		1.73	
						0.650		0.192		0.066		0.025
51.2	. . .		. . .		0.000		0.885		1.32		1.55	
								0.677		0.292		0.152
102.5	. . .		. . .		. . .		0.000		0.660		0.989	
										0.581		0.287
205	. . .		. . .		. . .		. . .		0.000		0.459	
												0.500
410	. . .		. . .		. . .		. . .		. . .		0.000	

As to assumption c), it is probably not always fulfilled. For naked eye meteors the effective time between two molecular collisions which may influence the electronic states, or the state of ionization, is of the order of 10^{-6} sec., thus long compared with the life time of most excited states; but in the densest portion of the coma just adhering to the nucleus, for meteors of about the first magnitude, the time between two collisions may attain 10^{-9} sec. and less, and the assumption is no more valid in this case. However, the failure of this assumption, having some bearing on the appearance of the emission spectrum, does not influence greatly the order of magnitude of the calculated amount of radiation.

Table IX contains theoretical data relating to collisions of nitrogen and iron, on which further calculations were based. The target area in cm² is equal to $\pi \xi^2$; Z is the fraction of total energy dissipated within given limits of $V(x_0)$, the relative energy of the collision.

A comparison with Table IV suggests that collisions $N:Fe$ at meteoric velocities resemble much more closely the dissipation of energy by rigid spheres, than the dissipation of energy of alpha-particles. Nevertheless, the same kind of deviation from rigid spheres is found: a certain excess of transfer of energy by small amounts.

Table X contains data relating to the dissipation of energy by a *Fe* atom moving in a nitrogen medium. The data are calculated according to formula (9), with $f(\mu) = \frac{4\mu}{1+\mu} = 3{,}2$ corresponding to elastic collisions, and $E_r = \frac{E_2}{1+\mu} = 1/5 \; E_2$ (notations of Sections 1 and 2).

TABLE X

Rate of Dissipation of Energy $\left(-\frac{\partial E_2}{\partial l}\right)$ and Range (L) for an Iron Atom moving in Nitrogen (N_2) at 760 mm and 0° C (the range L is the length of path until the velocity decreases to 3.7 km/sec.)

$W_2 \frac{\text{km}}{\text{sec}}$	3.70	5.22	7.40	10.44	14.8	20.9	29.6	41.8	59.2	83.6	118.4	167.2	236.8
$E_r = \frac{1}{5} E_2$, volt	0.8	1.6	3.2	6.4	12.8	25.6	51.2	102.5	205	410	820	1640	3280
$-\frac{\partial E_2}{\partial l} \times 10^{-5} \frac{\text{volt}}{\text{cm}}$	1.44	2.24	3.51	5.72	9.40	16.3	29.2	53.6	93.0	150	234	323	415
$L \times 10^5$ cm	0.0	2.4	5.3	8.8	12.4	18.0	23.9	30.2	37.3	45.8	56.3	70.9	93.0

For a nitrogen atom moving in an iron medium of the same number of atoms, or of density about four times the density of atmospheric air, the range is the same, and the rate of dissipation of energy four times s m a l l e r than the corresponding values of Table X; this may sound somewhat paradoxical, but it follows from the law of elastic collisions. A certain constant number of collisions is required to reduce the velocity in a given ratio; the number of collisions is the same, whether m_2 moves in a medium of m_1, or m_1 — in a medium of m_2. Of course, the heavy *Fe* atom will be less deflected than the light *N* atom, and although their true trajectories will be of equal length, the length measured along a straight line joining the terminal points will be shorter for the *N* atom moving in a *Fe* medium.

The average amount of inelastic energy per collision is the same, whether *N* moves in *Fe* or vice versa (for a given velocity); the numbers of collisions being also equal, the total amount of inelastic energy will be the same, if secondary collisions [1]) are not taken into account; the total amount of kinetic energy dissipated per atom is, however, equal to the initial kinetic energy and therefore is proportional to the atomic weight, and for *Fe* four times greater than for *N*; thus, with a constant amount of inelastic energy generated per atom, a single *Fe* atom moving in a nitrogen atmosphere may be expected to generate four times less inelastic energy in proportion to its kinetic energy than a nitrogen atom in a cloud of iron vapors.

The density of air along the visible path of a meteor may be estimated between 10^{-7} to 10^{-9} $\frac{\text{gr}}{\text{cm}^3}$, or 10^{-4} to 10^{-6} of atmospheric air. For $W_2 = 59$ km/sec this gives a range of the *Fe* atom from 3.7 to 370 cm. Evidently, direct penetration of atmospheric air by ionized or ionizing atoms from the meteor is quite inadequate to explain the appearance of meteor trains, which may extend to a distance as great as 1 km from the original track. For the explanation of trains, we have to assume either short wave radiation capable of producing ionization of atmospheric molecules, which, in recombining, emit visible radiation [2]),

1) Those produced by atoms of the medium primarily hit by the moving atom.

2) *cf.* C. C. Trowbridge, Astroph. Journ., **26**, 95, 1907.

or a general outward motion of the surrounding atmosphere produced by the expansion of the meteor vapors. By estimating the velocity of expansion of the trains, calculations indicate that notwithstanding the great volume of air set in motion, the energy necessary to produce it is of the same order of magnitude as the radiated energy; thus, the explanation may be plausible: in the latter case we should expect also metallic radiation from the train.

To estimate the order of magnitude of visible impact radiation, we shall consider at first only two of the simplest cases — that of a nitrogen atom moving in a *Fe* cloud, and of a *Fe* atom in a nitrogen (monatomic) atmosphere. The real case is of a more complicated mixture; by weighting the two extreme cases in a certain proportion, we may obtain an approximation of the real case.

On the basis of the theory and data presented above, notably with the aid of formulae (12) and (17), the amount of visible impact radiation for the typical case was calculated. Table XI contains the result. The visible radiation is here defined as the energy between 4500 and 5700 Ångströms, or ε_n between 2.17 and 2.75 volts; thus in (17) we put $\Delta \varepsilon_n = 0.58$ volts. For the sun, the fraction of spectral energy comprised within these limits is 0.20. Denoting by β the fraction of the kinetic energy of the meteor converted into visible radiation, we have for the heat factor of the meteor the expression

$$h = \frac{0{,}20}{\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (18).$$

By this factor the minimum mass of the meteor must be multiplied, computed on the assumption that all the kinetic energy of the meteor is converted into radiation of the same spectral energy distribution as the sun's distribution.

The spectral energy distribution of impact radiation[1]) may also be calculated; the result depends, of course, partly upon our assumption as to the uniform distribution of excitation levels, but it chiefly follows from the intrinsic nature of the inelastic collisions. The distribution proves to be much more widespread than black body radiation.

1) Although an emission spectrum of linear character, we may estimate the general energy distribution by averaging the energy over certain not too narrow limits of wave lengths.

TABLE XI

Inelastic Energy and Impact Radiation. Typical Case of N in Fe, $\mu = 0.25$

Velocity W_2, km/sec			7.40	10.44	14.8	20.9	29.6	41.8	59.2	83.6
E_2, volts[1]			4	8	16	32	64	128	256	512
Primary collisions only[2]	Total inelastic energy, s	10^{-3} units of the dissipated energy	45.0	45.0	45.3	55.9	50.7	51.2	53.8	57.3
	Total visible		7.2	4.5	3.3	2.85	2.25	2.08	1.94	1.90
Total amount of visible energy in 10^{-3} units of initial kinetic energy (velocity decreasing from initial value W_2 to 0)	Primary collisions β_1		1.4	3.6	3.8	3.4	3.0	2.6	2.5	2.2
	Secondary collisions β_2		0	0.05	1.1	3.1	5.5	8.1	10.3	12.3
	Total β		**1.4**	**3.6**	**4.9**	**6.5**	**8.5**	**10.7**	**12.8**	**14.5**

As to the color of meteors, even disregarding the actual irregularity in the distribution of emission lines, the above mentioned peculiarity in the general distribution of energy will make the definition of color strongly dependent upon the spectral regions used for that purpose. Defining the color-temperature as the black body temperature yielding the given value of the spectrophotometric gradient, $\frac{\partial \log E\lambda}{\partial \lambda}$, the color temperature of impact radiation is represented, within the range of visible, photographic and near ultra-violet spectrum, approximately by

$$T_c = \frac{4400^0}{\lambda} \quad \ldots\ldots\ldots\ldots \quad (19),$$

λ being given in microns; the expression is valid for practically all the range of meteor velocities.

For the visual region, $\lambda = 0.55$, we have $T_c = 8000^0$, which may be compared with 7000^0 as estimated from observations of Perseids[3]). For the photographic region, $T_c = 10000—11000^0$; for the ultra-violet, $15000—20000^0$.

1) For $E_2 = 2.0$ volts, the visible impact radiation $\beta = 0$.

2) $W_2 =$ const.

3) Tartu Observatory Publications, **25**.₁, p. 32, 1922.

The expected richness in short wave radiation, which seems to be confirmed by existing meteor spectra, favors meteor photography. The observed strong coloration of some meteors cannot, of course, be explained by the general distribution of energy, and must be due to the strength of certain individual emission lines.

6. Impact radiation from a meteor. — Table XI gives the radiation generated by a single atom of initial velocity W_2, decelerated by a certain medium such as the meteor coma; the coma itself, however, undergoes deceleration as the result of capture of atmospheric molecules; thus the initial velocity of the atoms relative to the coma changes, and we have to average the data of Table XI over the whole range of the velocity decreasing from the initial velocity of the meteor to zero. We may try to have a better approximation than Table XI, by taking into account the prominent role of atmospheric nitrogen[1]) in meteor radiation. Nitrogen, on account of its high excitation potentials, is not very likely to be a source of visible radiation. Neglecting altogether radiation from nitrogen in the following discussion, we obtain a minimum estimate of the impact radiation.

We disregard also the diatomic character of the air molecules; although it seems that dissociation at impact necessarily takes place, the energy involved in dissociation amounts only to fourteen per cent for $W_2 = 20.9$ km/sec and about two per cent for $W_2 = 59$ km/sec; these quantities, although not negligible, will not influence the order of magnitude.

It is still possible that some of the observed "continuous" background in meteor spectra is due to bands produced by the recombination of air molecules.

Let us consider a detached cloud of vaporized meteoric material; it is gradually slowed down by the addition of air molecules supposed to be nitrogen. Denoting atoms of the cloud by the symbol *Met.*, we have the following possible combinations of collisions:

a) Primary Collisions of Atmospheric Nitrogen

1) with *Met.*; radiation according to Table XI;

2) with admixed N; visible radiation zero.

1) Or perhaps oxygen.

b) Secondary Collisions

1) of *Met.* with *Met.*; assumed double amount of visible radiation of Table XI;

2) of *Met.* with N; radiation as in Table XI;

3) N with N; no visible radiation.

Denoting the proportion in mass of admixed N to *Met.* by u, which corresponds to a ratio $4\,u$ of the numbers of atoms, we find approximately that the visible radiation of Table XI must be multiplied:

for primary collisions, by $q_1 = \dfrac{1}{1+4\,u}$ . . . . (20),

and for secondary collisions, by $q_2 = \dfrac{2+4\,u}{1+4\,u+16\,u^2}$ . . (21).

The relative velocity is $W_2 = \dfrac{W_0}{1+u}$ . . . . (22),

the energy of the N atoms relative to the coma is

$$E_2 = \frac{E_0}{(1+u)^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (23),$$

and the amount of kinetic energy liberated per unit mass of the cloud is

$$Q_u = \frac{W_0^{\,2}}{2} \cdot \frac{u}{(1+u)} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (24),$$

$$\text{or } d\,(Q_u) = \frac{W_0^{\,2}}{2} \cdot d\left(\frac{u}{1+u}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (24'),$$

where W_0 is the initial velocity of the meteor, and E_0, the kinetic energy of the impinging atom (N) corresponding to W_0. The probable fraction, β, of visible radiation generated by the decelerated cloud of meteoric vapors may be computed from

$$\overline{\beta} = \int_{u=0}^{u=\infty} (\beta_1\,q_1 + \beta_2\,q_2)\; d\left(\frac{u}{1+u}\right) \quad . \; . \; . \; . \; . \; . \quad (25),$$

in notations of Table XI and those of the preceding formulae. The results of the computation are given in Table XII.

TABLE XII

Ratio ($\bar{\beta}$) of Visible Impact Radiation (4500 to 5700 Å) to Kinetic Energy of a Meteor (W_0 is the initial velocity, km/sec)

W_0	5.22	7.40	10.44	14.8	20.9	29.6	41.8	59.2	83.6
$\bar{\beta} \times 10^3$	0	0.08	0.53	1.14	1.90	2.90	4.10	5.30	6.42
Heat factor, h	(∞)	2500	380	175	105	69	49	38	31

The heat factor, computed according to (18), refers to impact radiation only, and thus applies to small meteors. For large meteors the heat factor will be smaller, on account of temperature radiation.

From the table we infer that the heat factor decreases, or $\bar{\beta}$ increases with velocity; high velocities are more efficient in producing visible radiation, than low velocities. Thus, the total amount of visible light generated by a meteor of a given mass is expected to increase faster than with the square of the velocity, about as $w_0^{\frac{5}{2}}$ for the range of meteoric velocities.

For meteors of very small size, when the size of the coma is smaller than the free path of its own molecules, the coma cannot be regarded as a compact cloud; the secondary collisions will take place with air molecules of the practically undisturbed atmosphere. The case will correspond evidently to the reverse of Table XI — to a single *Fe* atom moving in a nitrogen atmosphere. In agreement with what was said on p. 30, and because secondary collisions are inefficient, $\bar{\beta}$ may be estimated equal to $\frac{\beta_1}{4}$ of Table XI. Table XIII contains these values.

TABLE XIII

Ratio of Visible Impact Radiation to Kinetic Energy for Very Small Meteors

W_0	5.22	7.40	10.44	14.8	20.9	29.6	41.8	59.2	83.6
$\bar{\beta} \times 10^3$	0.	0.35	0.90	0.95	0.85	0.75	0.65	0.62	0.55
Heat factor, h	(∞)	570	220	210	235	270	310	320	360

For the range of meteoric velocities, $\bar{\beta}$ is smaller in Table XIII than in Table XII, and decreases with velocity; thus, small masses are probably less efficient in producing visible radiation than larger masses, the ratio of efficiencies amounting to almost

10:1, or from two to three magnitudes[1]). Hence generally, we may expect that for constant velocity the visible efficiency increases with increasing mass, or that the total amount of visible radiation changes faster than simply with the mass. For meteors of high luminosity, this phenomenon is strengthened by the increasing amount of temperature radiation with increasing mass.

The transition from Table XII to Table XIII may begin at the fainter visual meteors and may be completed for telescopic meteors of a certain magnitude m; over the interval of transition, the increment of meteor numbers with magnitude may be expected to weaken, with a more or less sudden increase at and below m — facts that seem to be supported by preliminary observational evidence[2]).

7. Temperature radiation from atomic collisions. — The word temperature is to be used here in a restricted sense, as explained in Section 4; it represents the average translational energy of the molecules, V, and we shall measure it in volts per molecule. The absolute temperature, T, is related to V as follows:

$$T = 7740^0\ V \quad \quad . \ . \ . \ . \ . \ . \ . \ . \quad (26).$$

The following calculations are of practical importance for a gas of feeble ionization, where electronic collisions play a subordinate role in producing radiation; the meteor coma presents doubtlessly such a case.

It is possible here to give the line of argumentation only in general outlines. The properties of molecular collisions were assumed identical with those calculated for $N:Fe$, and the molecular weight was put equal to 56, corresponding to a pure iron atmosphere. The average relative energy of two molecules of a random distribution of directions is $E_r = \frac{2V}{1+\mu} = V$ when $\mu = 1$, that is, when equal molecules are considered. According to the data of Table X, the rate of dissipation of energy by

1) This conclusion depends altogether on the neglect of visible radiation from atmospheric molecules. If some radiation of this kind is present, the ratio will be smaller. Qualitatively, however, the phenomenon will be the same.

2) E. Öpik, Tartu Publications, **27**.2, p. 5, Table II, 1930; also, according to S. L. Boothroyd, from unpublished telescopic observations at the Arizona Meteor Expedition.

one molecule is very closely represented by $\frac{\partial E}{\partial t} \sim - E^{\frac{3}{4}}$; the rate of dissipation of energy by a single molecule per second is proportional to the velocity, $E^{\frac{1}{2}}$, thus $\frac{\partial E}{\partial t} \sim - E^{\frac{5}{4}}$. The fraction of the energy, dissipated by elastic collisions, remains in the gas, and thus does not lower the temperature; the inelastic fraction, s, is the only one that comes into consideration. This fraction is ultimately radiated; thus, for the decrement of temperature we have $\frac{\partial V}{\partial t} \sim - s V^{\frac{5}{4}}$, because $V \sim E$.

From Table XI we find that s slightly changes with E, thus with V, about $s \sim V^{0.05}$. Taking into account that the rate of dissipation, depending upon the number of collisions, is proportional to the density ϱ of the gas, we have

$$\frac{\partial V}{\partial t} = - \varrho\, V^{1,3} \times \text{const.}$$

The constant includes a factor $\frac{N}{2}$, where N is the number of molecules per unit volume of standard density; one half of it is to be taken because otherwise each collision would have been counted twice. Another factor, $k = 1.07$, is included to allow for the fact that the average value of $V^{1.3}$ is slightly greater than $(\overline{V})^{1.3}$, in the case of a spread in the velocities (which was assumed Maxwellian for this particular calculation of k).

With the numerical constants, the formula for the loss of translational energy through inelastic atomic collisions is:

$$\frac{\partial V}{\partial t} = - 1.16 \times 10^{8} \left(\frac{\varrho}{\varrho_0}\right) V^{1.3} \quad \ldots \ldots \ldots \quad (27),$$

where ϱ is the density, and ϱ_0 the normal density at 273° K and 760 mm pressure. We remind the reader that the formula is based on the empirical law expressed by equation (12), and is supposed to be valid for a feebly ionized gas. When the temperature is so high that the equilibrium state requires a high degree of ionization, the formula applies only to the transition phase from an aggregate of neutral atoms toward the equilibrium state; as the degree of ionization increases, the coefficient of (27) decreases and must reach zero for complete ionization.

From this standpoint formula (27) may be regarded as representing radiation produced by atomic collisions, in volt-electrons per molecule. Writing the radiation formula in the form

$$Q = \varkappa \sigma T^4,$$

where σ is Stefan's constant, and $\varkappa$ is the mass-coefficient of absorption, we find that formula (27) is equivalent to the following expression for the "atomic" mass-coefficient of absorption:

$$\varkappa_a = 0.85 \times 10^{14} . P T^{-3.7} \quad \ldots\ldots \quad (28),$$

where P is the pressure in dynes/cm^2, T the temperature. Milne [1]) suggests for the effective mass-coefficient of absorption of stellar atmospheres a value which, expressed in the same units, is

$$\varkappa_e = 0.85 \times 10^{18} . P T^{-4.5} \quad \ldots\ldots \quad (29)$$

It is interesting to note that the order of magnitude of both expressions is rather close, for temperatures of the order of those found in stellar atmospheres:

T	2500°	5000°	10000°	20000°	40000°	80000°	160000°
V	0.32	0.65	1.29	2.59	5.18	10.3	20.7
$\varkappa_a / P$	23	1.75	0.13	0.010	$8.0 \cdot 10^{-4}$	$6.2 \cdot 10^{-5}$	$4.7 \cdot 10^{-6}$
$\varkappa_e / P$	430.	19.	0.85	0.038	$17 \cdot 10^{-4}$	$7.4 \cdot 10^{-5}$	$3.3 \cdot 10^{-6}$

There seems to be no doubt that in the outer layers of a star where the degree of ionization is not too high, atomic collisions may play a not insignificant role in producing radiation.

Applying the theory described above to the meteor coma, it is found that the greater the meteor, the more important temperature radiation becomes compared with impact radiation. The results are as follows:

W_0, km/sec	14.8	41.8	83.6
Ratio of temperature radiation to impact radiation	Average apparent magnitude of meteor at 100 km distance		
0 5	+ 1.0	— 3.0	— 6.3
1.0	— 0.3	— 4.3	— 7.6
2.0	— 2.5	— 6.5	— 9.6

These data were derived by considering the time of expansion of the coma in connection with the general theory of meteor phenomena (unpublished). The general order of magnitude of the results is such that we are led to the conclusion: in most of the meteors visible to the naked eye, temperature

1) Trans Roy. Soc., **228**, 441, 1929.

radiation is insignificant compared with impact radiation. It is interesting to note that with Milne's expression for the mass-coefficient of absorption (from 29), the result is substantially the same.

9. Masses of meteors. — There may be set two rough limits for the mass of a meteor: a minimum mass, calculated on the assumption that all the kinetic energy of the meteor is converted into radiation of the same heat index as the heat index of the sun; and a maximum mass, computed on the assumption that all the radiation is a black body radiation from the nucleus, at the relatively low temperature of its vaporization. These limits may differ very much, in the ratio of 20000 : 1 or so, the ratio depending much upon velocity. The present theory of meteor radiation gives another estimate of the meteor mass, by assuming that only impact radiation is responsible for the visible light of the meteor. This assumption would seem to give a maximum estimate of the mass, because other sources of radiation are neglected; however, the neglect of these other sources is probably of minor importance compared with the uncertain or accidental character of the data on which the estimate is based. We cannot tell yet on what side of the true value our estimate lies. Thus, by multiplying the minimum mass by the heat factors as given in Tables XII and XIII, we obtain a value for the probable mass of the meteor. Taking into account temperature radiation and other circumstances, we may say that generally the probable masses of meteors are from 20 to 400 times their minimum masses, the factor depending upon size, velocity, and chemical composition. As an example, we may state that for a Perseid of second zenithal magnitude, the minimum mass is 0.3 mgr[1]); the heat factor according to Table XII, $W_0 = 56$ km/sec, is 40; the probable mass thus is 12 milligram.

I wish to express my thanks to Dr. Harlow Shapley, director of Harvard College Observatory, for making possible the present research; and to professors H. H. Plaskett, H. N. Russell and J. C. Slater for helpful discussion and comments.

Harvard College Observatory, Cambridge, Mass.

April 24, 1932.

1) Tartu Publ., **25.**$_1$, 33, 1922.